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# *Application of policy iteration to strategic maintenance scheduling*

Kalle Alaluusua  
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# Problem description

# Maintenance scheduling

Consider a system that consists of multiple critical components.

How to schedule components' maintenance in the **long run** when the maintenance decisions influence

- the state of the system
- future wear-off

Keep **costs low** and **reliability high**

# Maintenance scheduling

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How to schedule components' maintenance in the **long run** when the maintenance decisions influence

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Keep **costs low** and **reliability high**

**Reliability** = ability to perform the **required function** under prevailing **operational conditions** for a stated **time period**

# Maintenance scheduling

## Naïve approach

- **Minimize** expected maintenance **costs** of **single** components
- ⇒ Take reliability measures into account poorly

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## Naïve approach

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## Improvement

Extend the naïve approach and use dynamic programming to

- **Group** maintenance operations of **multiple** components effectively
- Introduce a **reliability threshold** to keep reliability high enough



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# Problem formulation

# System state

- Fixed maintenance interval  $\Delta t > 0$
- For the system to operate, every component must operate
- Components fail according to some known probability distributions

## A discrete time Markov decision process with state variables

- Age  $(a_k)_i$
- Failure state  $(f_k)_i \in \{0, 1\}$

of component  $i$  at maintenance instance  $t_k$

**Reliability** = probability that a system is operational in  $t_{k+1}$  given  $a_k$



# Costs and dependencies

## Costs

- Set-up
- Component specific
- Shutdown
- Downtime

## Dependencies

- Economic
- Structural
- Stochastic

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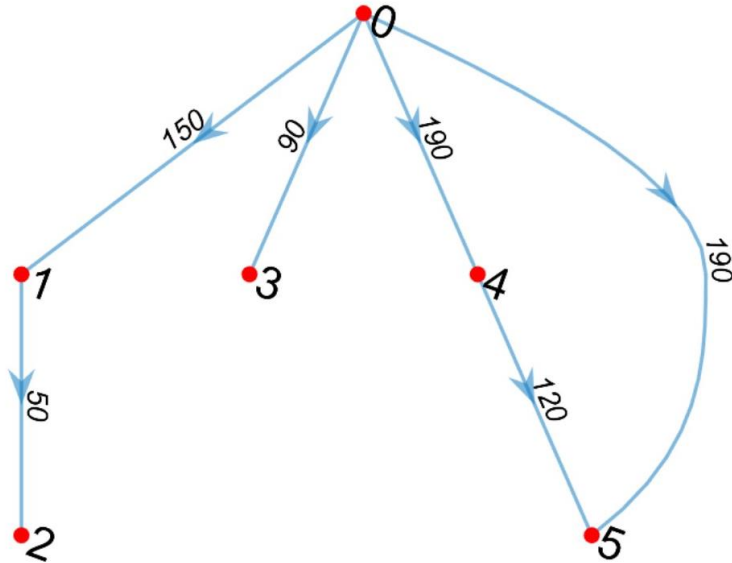


Figure 3.1: Example of a system of five components and corresponding costs of  $c_{ij}$

Leppinen, J. (2020)

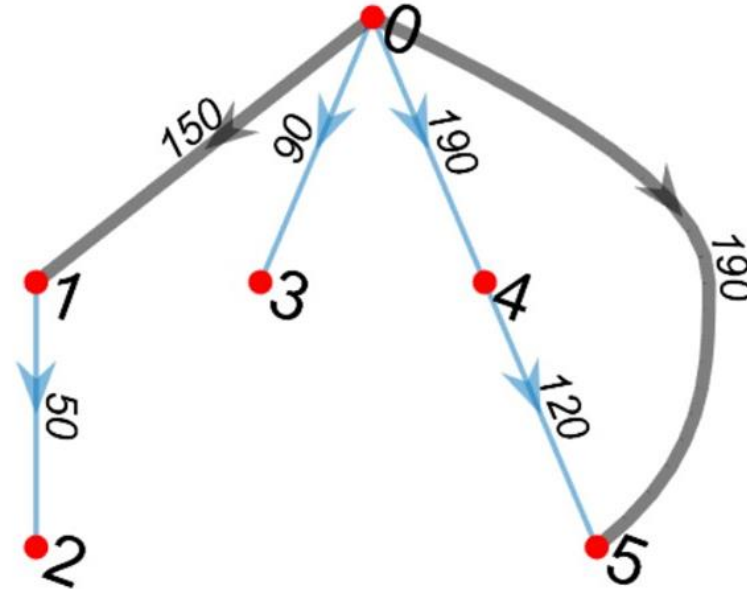
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(a)  $x_k = \{1, 5\}$

Leppinen, J. (2020)

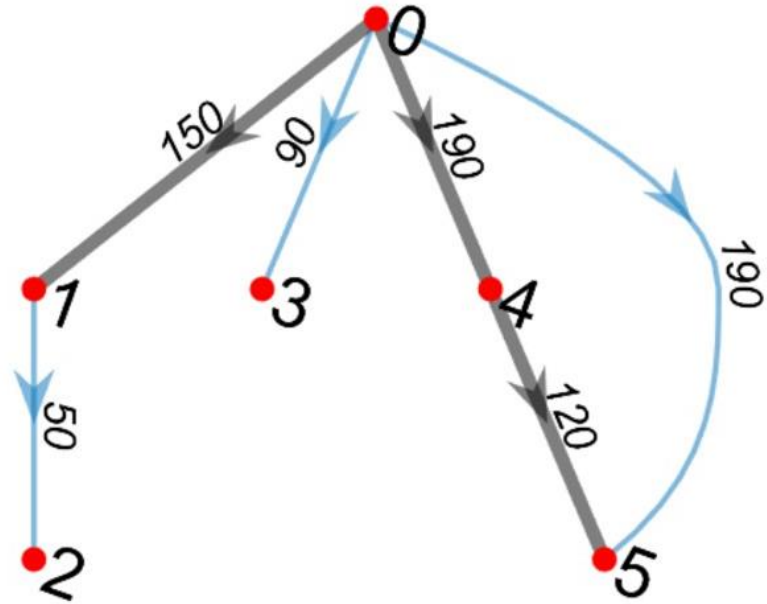
# Costs and dependencies

## Costs

- Set-up
- Component specific
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- Downtime

## Dependencies

- Economic
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- Stochastic



(b)  $x_k = \{1, 4, 5\}$

Leppinen, J. (2020)

# Assumptions

- At most single failure per maintenance period
- Components can only be replaced into new ones
- No downtime cost
- No stochastic dependencies

# Policy

## Portfolio

$$x \in \{0,1\}^N$$

has  $x_i = 1$  when component  $i$  is replaced

## Feasible portfolios

- Fulfil the reliability threshold
- Replace failed components
- Satisfy structural dependencies

# Objective

## Find a stationary policy which

- Is feasible
- Minimizes the long run average cost per time unit

# Policy iteration algorithm

1. *Step: Initialization* Choose a stationary policy  $U$ .
2. *Step: Value-determination step* For the current policy  $U$ , compute the unique solution  $\{g(U), v(U)\}$  to the following system of linear equations:

$$v_{\sigma_i} = c_{\sigma_i}(U_{\sigma_i}) - g + \sum_{\sigma_j \in S} p_{\sigma_i \sigma_j}(U_{\sigma_i}) v_{\sigma_j}, \quad \sigma_i \in S \quad (3.20)$$

$$v_{\sigma_s} = 0,$$

where  $\sigma_s$  is an arbitrarily chosen state.

3. *Step: Policy-improvement step* For each state  $\sigma_i \in S$ , determine a portfolio  $x_k$  yielding the minimum in

$$\min_{x_k \in X_{\sigma_i}} \left\{ c_{\sigma_i}(x_k) - g(U) + \sum_{\sigma_j \in S} p_{\sigma_i \sigma_j}(x_k) v_{\sigma_j}(U) \right\} \quad (3.21)$$

The new stationary policy  $U'$  is obtained by setting  $U'_{\sigma_i} = x_k$  for all  $\sigma_i \in S$ .

4. *Step: Convergence test* If the new policy  $U'$  equals  $U$  the algorithm is stopped with policy  $U$ . Otherwise, set  $U = U'$  and go to step 2.





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# Case study: A ground transportation equipment system

# Components and dependencies

## Components

- Engine 1 (E1), engine 2 (E2), chassis (C) and wheels (W)
- Deteriorate over time and have structural dependencies

## Structural dependencies

- An engine must be dismantled before it can be replaced
- To replace the chassis, the chassis and both engines must be dismantled
- To replace the wheels, the chassis and the engines must be dismantled

# Costs

A fixed set-up cost  $c_0 = 388$  for every operation and component specific costs:

Table 5.1: Maintenance costs of different components

component	symbols	component specific costs		
		dismantle	replacement	corrective surplus
engine 1	1, E1	23	393	300
engine 2	2, E2	28	403	300
chassis	3, C	167	413	160
wheels	4, W	0	1000	613

# Directed graph of cost structure

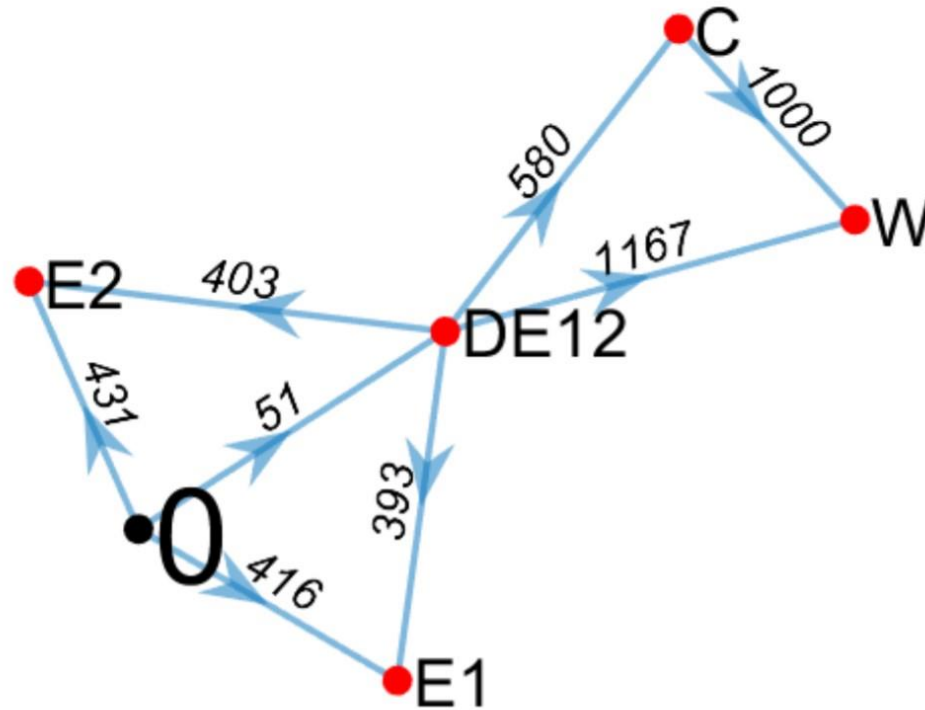
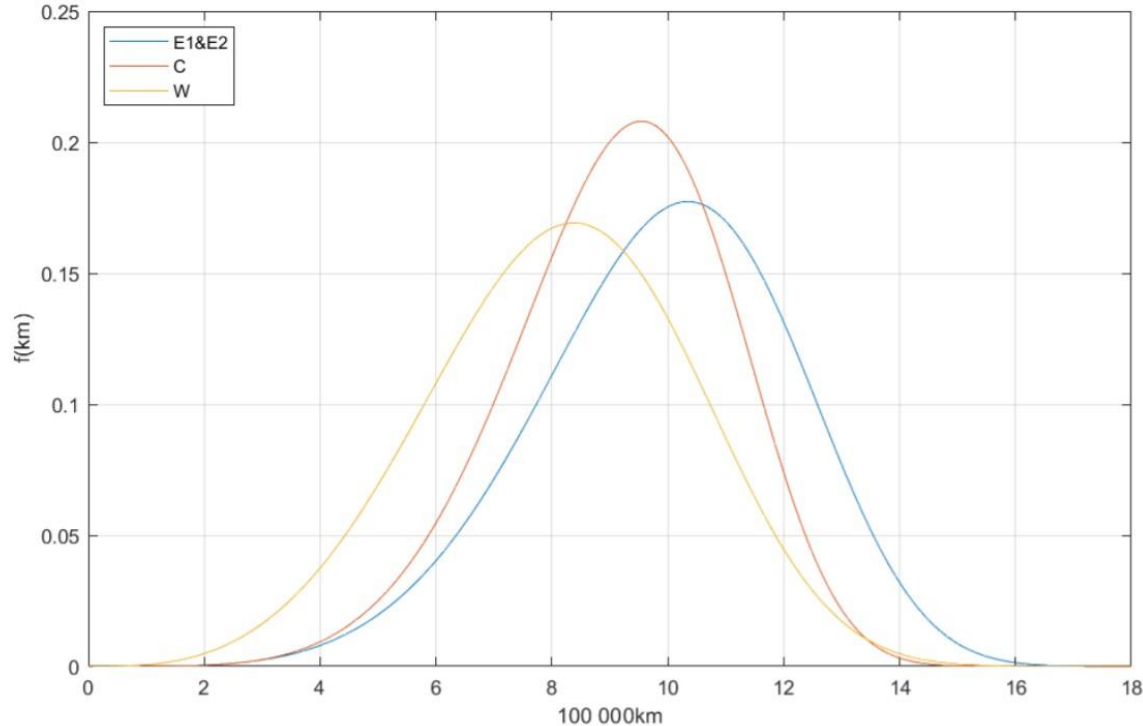


Figure 5.1: Cost structure of the system where the root node is on the left

# Weibull distributed failure probabilities



component	shape $k$	scale $\lambda$
engine 1	5.1	10.8
engine 2	5.1	10.8
chassis	5.5	9.9
wheels	4.0	9.0

Figure 5.2: Failure probability density as a function of distance driven from last replacement

# The results

## Portfolio

$x_{E1}x_{E2}x_Cx_W, x_i \in \{0,1\}$

has  $x_i = 1$  when  
component  $i$  is replaced

Table 5.4: Comparing replacement portfolios when changing reliability threshold as a function of  $(a_k)_{E2}$  and  $(a_k)_W$ , when  $(a_k)_{E1} = (a_k)_C = 75$  and  $f_k = 0$

$(a_k)_2,$ engine 2	$(a_k)_4,$ wheels							
	75	150	225	300	375	450	525	600
$\rho = 0.90$								
75	0	0	0	0	0	0	0001	0001
150	0	0	0	0	0	0	0001	0001
225	0	0	0	0	0	0	0001	0001
300	0	0	0	0	0	0	0	0101
375	0	0	0	0	0	0	0	0101
450	0	0	0	0	0	0	0	0101
525	0	0	0	0	0	0	0	0101
600	0100	0	0	0	0	0	0101	0101
675	0100	0100	0100	0	0	0101	0101	
750	0100	0100	0100	0100	0101	0101		
825	0100	0100	0100	0100				
$\rho = 0.95$								
75	0	0	0	0	0	0	0001	
150	0	0	0	0	0	0	0001	
225	0	0	0	0	0	0	0001	
300	0	0	0	0	0	0	0001	
375	0	0	0	0	0	0	0101	
450	0	0	0	0	0	0	0101	
525	0	0	0	0	0101	0101		
600	0	0	0	0100	0101			
675	0100	0100	0100	0100				



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# Conclusions

# Summary

## Maintenance scheduling problem

- A **discrete time Markov decision process** where the state depends on the components **ages** and the **failure state**
- Apply **policy-iteration** to find a stationary policy
- Optimal in terms of average cost over a very long time period



# Summary

## Design decisions

- **Component level:** distributions of failure probabilities
- **System level:** structure as a directed graph
- **Environmental level:** discretization period

# Source

Leppinen, J. (2020). A Dynamic Optimization Model for Maintenance Scheduling of a Multi-Component System (Master's thesis, Aalto University).

# Homework

Consider the case example (Chapter 5) in Leppinen, J. (2020) available in course material.

Briefly explain why the policy-iteration algorithm outperforms the simple and heuristic opportunistic policy. Why, in some cases should you still consider the simple policy over the presented policy-iteration algorithm?

Return your solution to [kalle.alaluusua@aalto.fi](mailto:kalle.alaluusua@aalto.fi) by 13.11. 09:15.