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A Semi-Markov decision problem for proactive and reactive transshipments between multiple warehouses

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Overview

- Set of **warehouses** &
  set of **markets**
- Warehouses sell a single **specialized**, low demand rate product in small quantities
  - Expensive spare parts, aerospace equipment, military equipment, …

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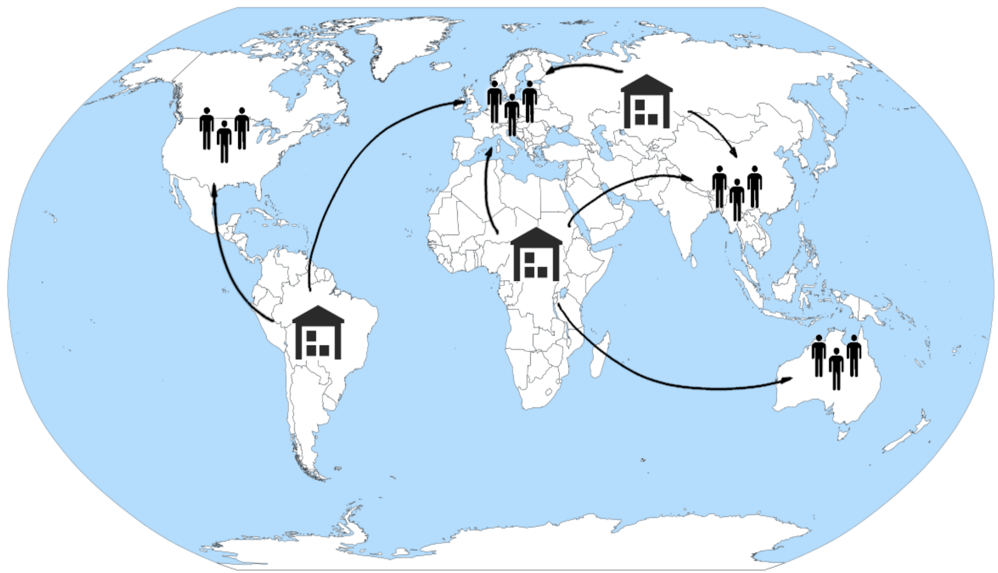
Overview

- Set of **warehouses** & set of **markets**
- Warehouses sell a single **specialized**, low demand rate product in small quantities
  - Expensive spare parts, aerospace equipment, military equipment, …
- Markets can be understood as market areas, i.e. the locations where the warehouses ship the products



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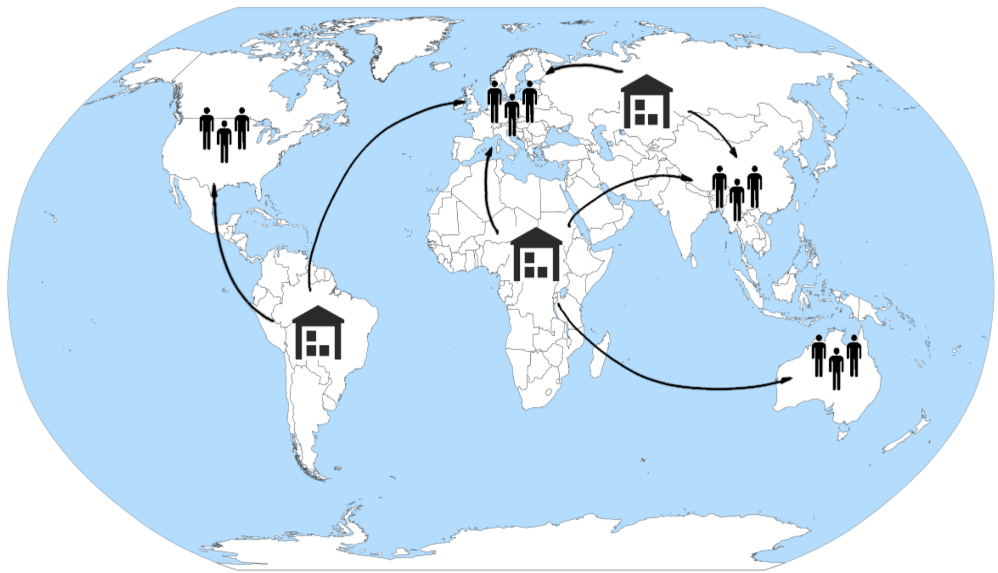
Problem statement

- Depending on the **inventory** levels of each warehouse and the current **demand** at each market, **where should we ship the products from?**

- Goal: **minimize expected costs** in the long run



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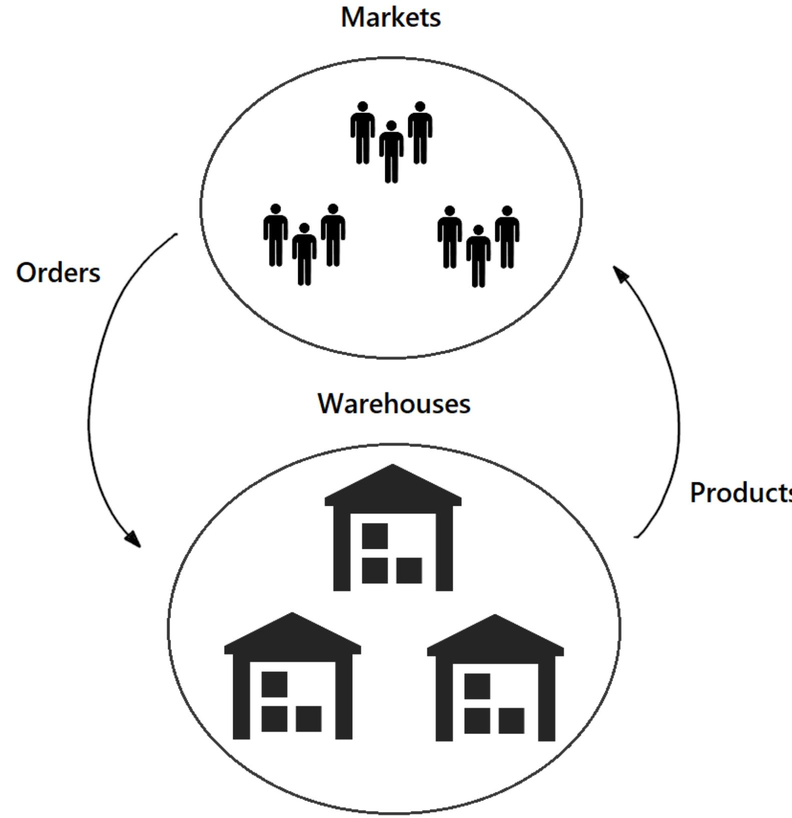
Markets & warehouses: Details

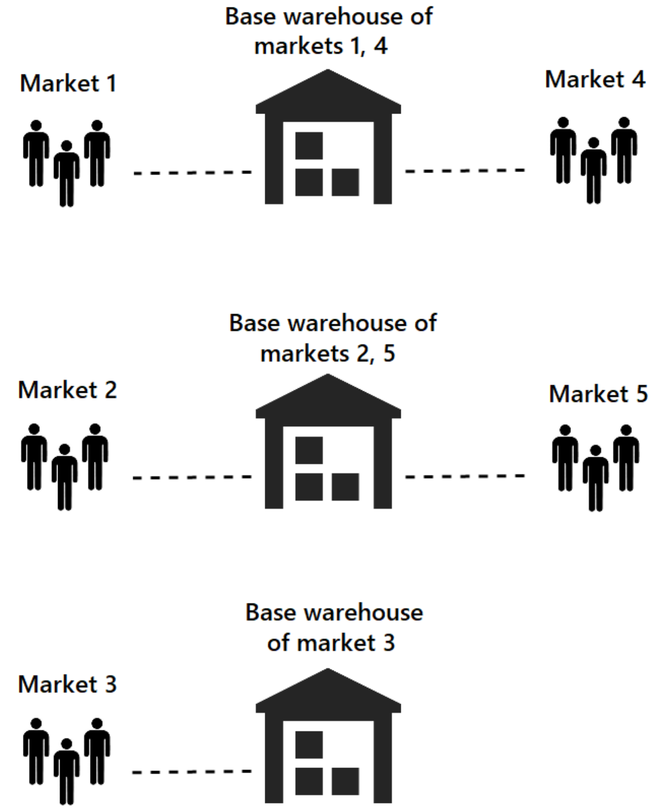
Setting

- Set of markets $m \in M$,
  set of warehouses $r \in R$
- Markets place orders for the product, warehouses ship the product to the markets
- Time is continuous
  - Orders arrive at random time instants
  - Shipments take time



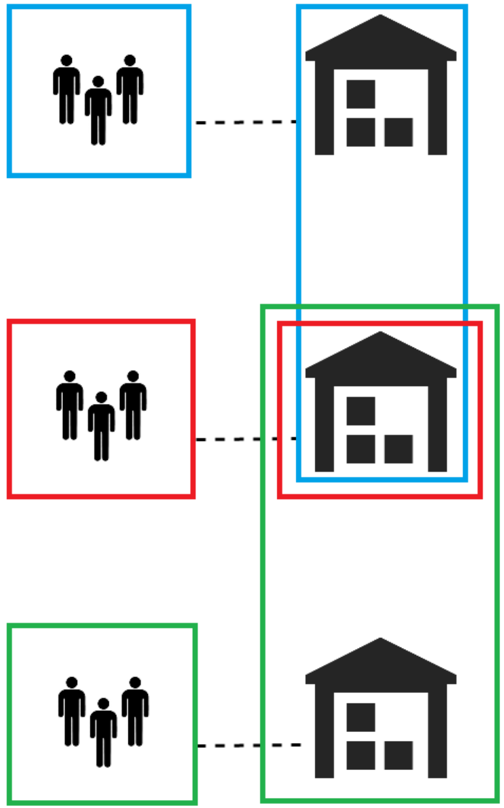
Base warehouses

- Each market m has a **base warehouse** $w(m)$ associated with it
- The base warehouse serves the market **without additional costs**
    - Delivery costs are included in the unit replenishment cost



Available warehouses

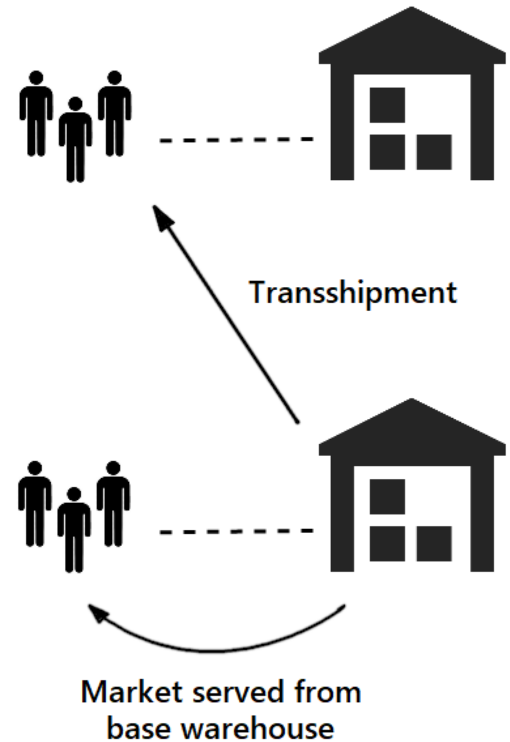
- Each market m also has a set of **available warehouses** W_m from which deliveries can be made
- This is because some warehouses do not meet service time requirements



Transshipments

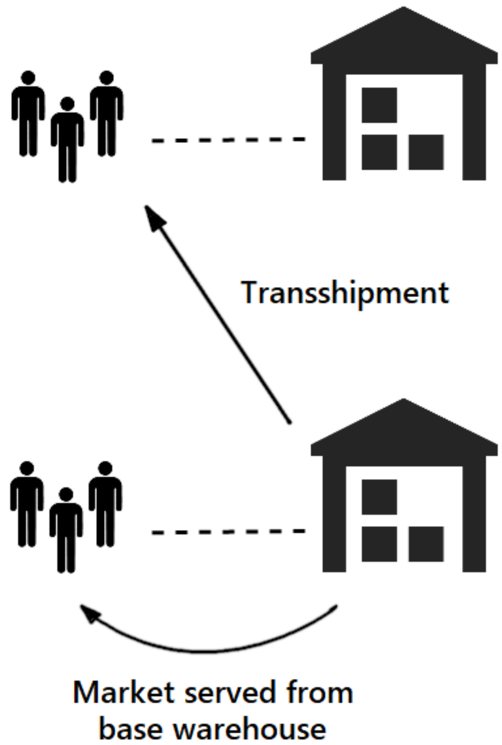
- If a market is served (item is shipped to meet demand) from a warehouse which is not the base warehouse, this is considered a **transshipment**

- A transshipment from warehouse r to market m has an **additional shipment cost** t_{mr}



Transshipments

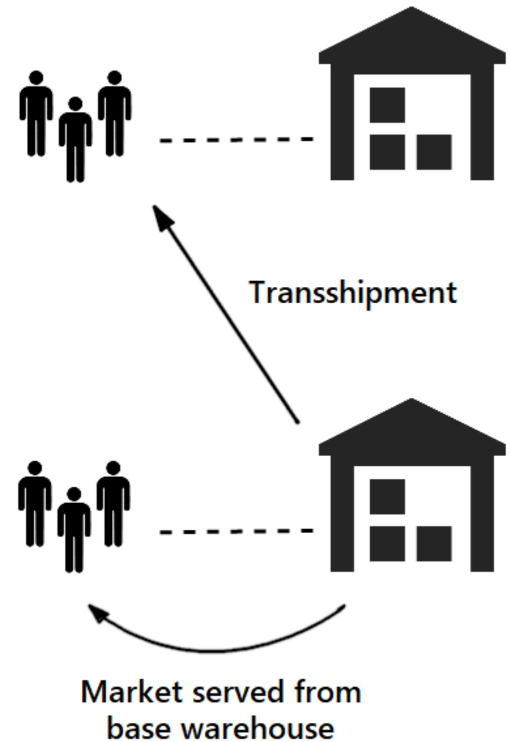
- Transshipment policies:
  - **Reactive**
    - TS only when base warehouse empty
  - **Proactive**
    - TS any time
  - No transshipments



Transshipments

- Which warehouse should we do the transshipment from?

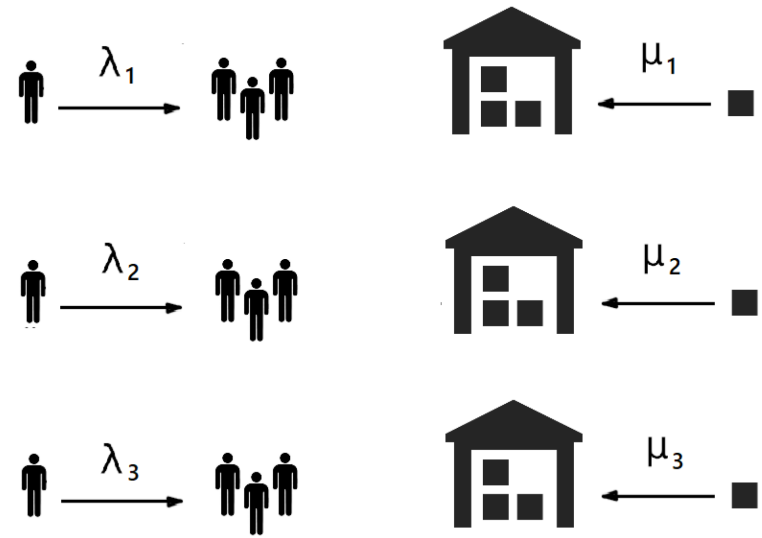
- Options:
  - Random
  - Highest inventory
  - Minimal TS cost
  - Highest run-out time
  - Minimum expected cost



Demand & Replenishment

- Demand at market m is a **Poisson process** with parameter λ_m
  - Customer arrival =
    demand arrival =
    order placement for 1 item

- Warehouses replenished with **one-for-one** policy
  - As soon as one item is sold, an order is placed for another
  - Item replacement time for warehouse r follows exponential distribution with parameter μ_r



Formulating into a Semi-Markov Decision Process (SMDP)

MDP vs. SMDP

Markov Decision Process

- Discrete states
- Actions / decisions
- Rewards / costs
- Random state transitions
- Discrete time
- Time between state transitions constant
- Markov property satisfied at each time point

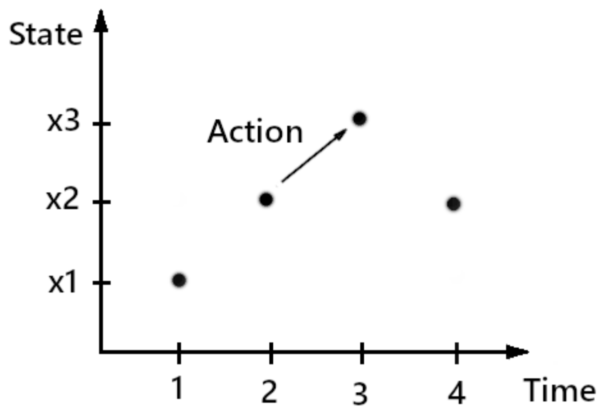
Semi-Markov Decision Process

- Discrete states
- Actions / decisions
- Rewards / costs
- Random state transitions
- Continuous time
- Time between state transitions random
- Markov property satisfied only at jump points between states
    - -> “Semi”-Markov

MDP vs. SMDP

Markov Decision Process

- Discrete time
- Time between state transitions constant
- Markov property satisfied at each time point



Semi-Markov Decision Process

- Continuous time
- Time between state transitions random
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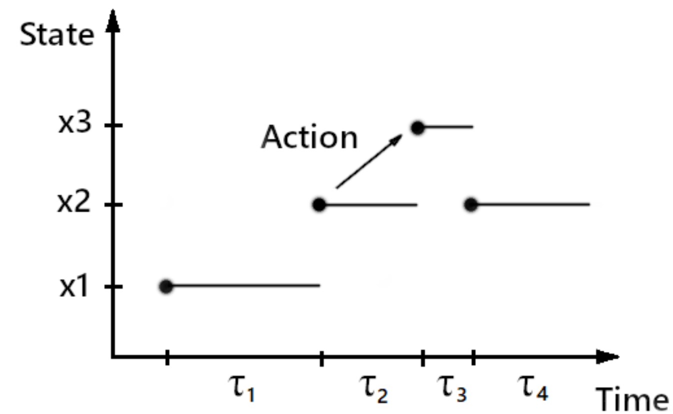
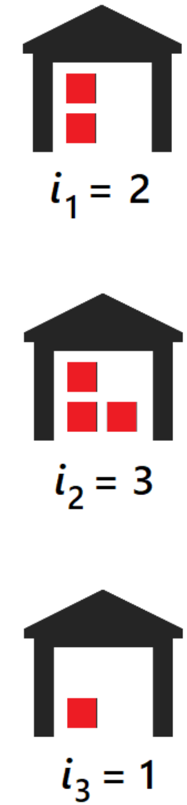


Formulating into SMDP

- **State** $i \in I$: current **inventory levels** of each warehouse



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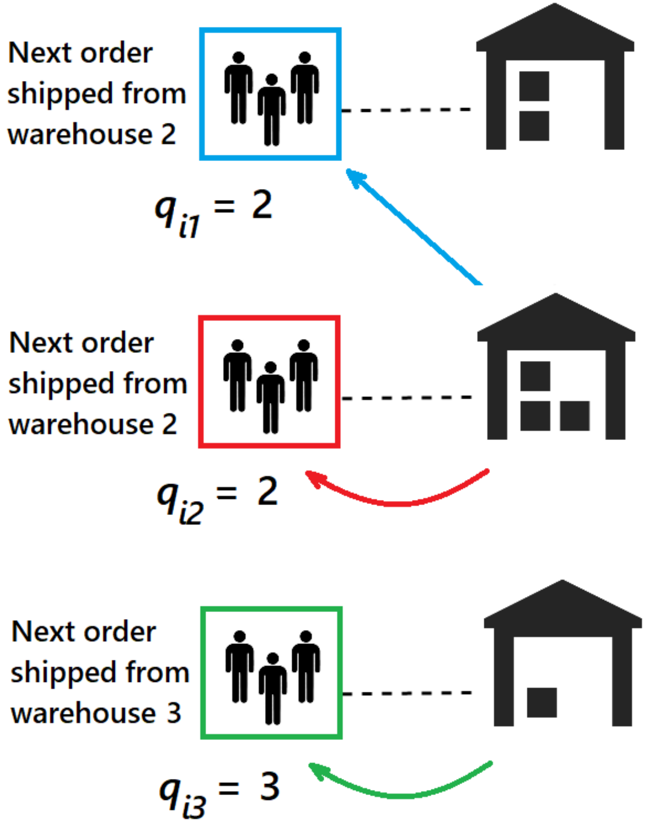
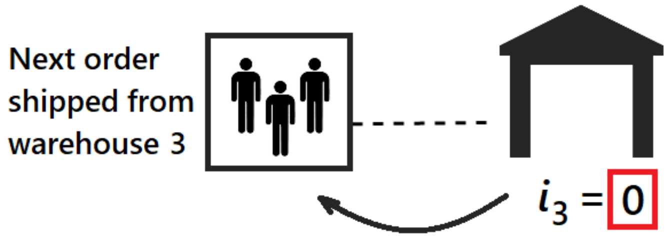
Formulating into SMDP

- **State** $i \in I$: current inventory levels of each warehouse
- **Decisions** for each state i: sending warehouse q_{im} for each market m



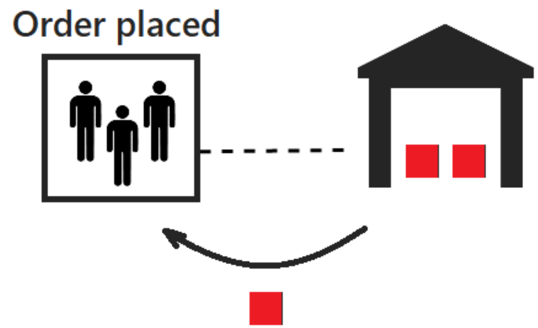
Formulating into SMDP

- **State** $i \in I$: current inventory levels of each warehouse
- **Decisions** for each state i: sending warehouse q_{im} for each market m
    - Demand lost if order arrives and sending warehouse is out of stock – cost l_m

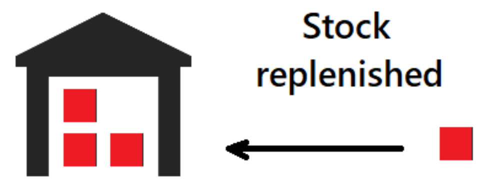


Formulating into SMDP

- **State transitions** triggered by
  - **Order** arrival → stock decreases by 1 in some warehouse
  - **Replenishment** arrival → stock increases by 1 in some warehouse



- Time spent in each state is random
- State transition probabilities depend on decisions



Formulating into SMDP

- **State** $i \in I$: current inventory levels of each warehouse
- **Decisions** for each state i: sending warehouse q_{im} for each market m
- **Costs:** expected cost of being in state i under decisions Q_i consists of
  - Holding cost
  - Lost demand cost
  - Replenishment cost
  - Transshipment cost

Formulating into SMDP

- **State** $i \in I$: current inventory levels of each warehouse
- **Decisions** for each state i: sending warehouse q_{im} for each market m
- **Costs:** holding, lost demand, replenishment, transshipment
- **State transition probabilities** obtainable from parameters of Poisson processes

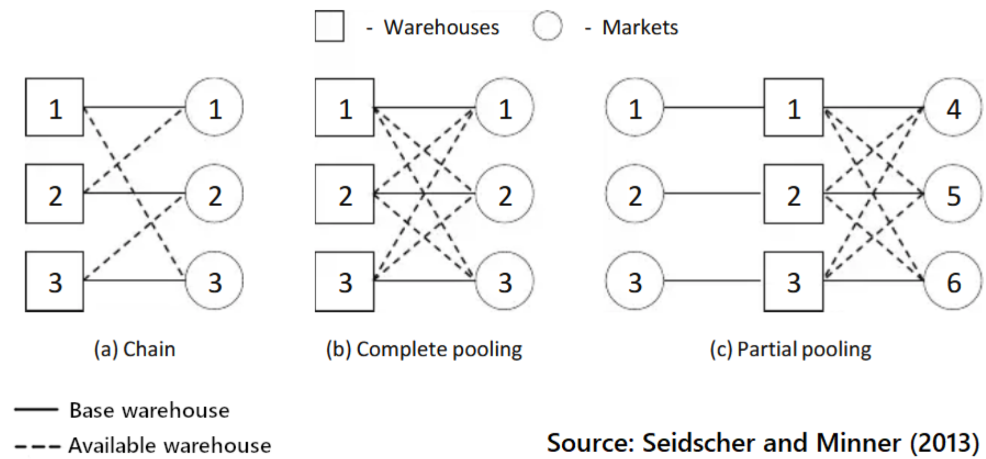
- Find decisions for each state such that expected cost is minimized in the long run
- Solved by linear programming, value iteration or policy iteration

Simulated case example

Simulated case example

- Goal: compare **reactive** and **proactive** transshipment policies and the policy of no transshipments

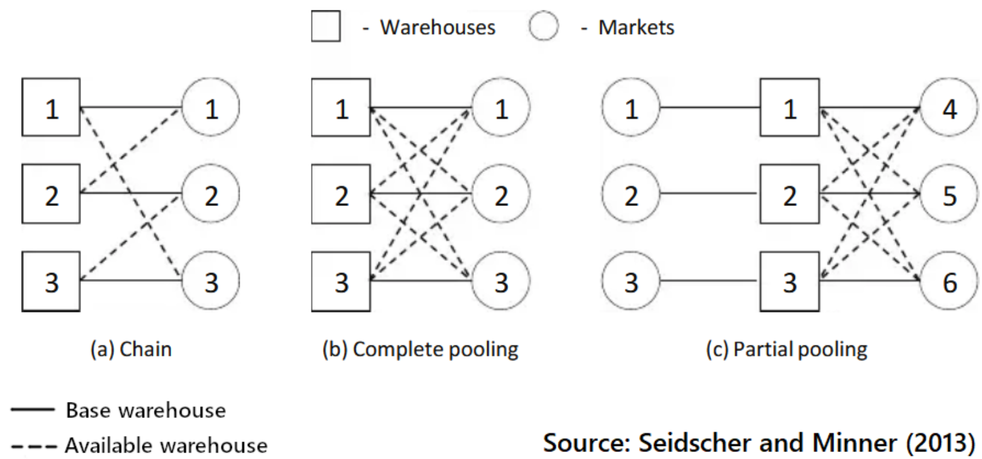
- Simulated systems small



Source: Seidscher and Minner (2013)

Simulated case example: results

- Proactive vs. reactive
  - For intermediate pooling networks, proactive TS can lead to significant savings over reactive TS
  - For complete pooling networks, no benefit of proactive over reactive
- TS better than no TS
- For reactive TS, difference between TS rules negligible

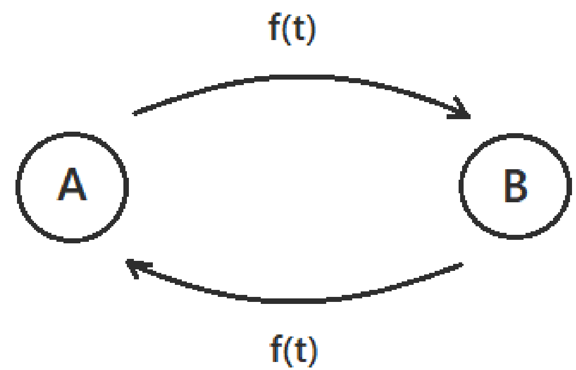


Source: Seidscher and Minner (2013)

References

Seidscher, Arkadi, and Stefan Minner. "A Semi-Markov decision problem for proactive and reactive transshipments between multiple warehouses." *European Journal of Operational Research* 230.1 (2013): 42-52.

Homework



1. What is the difference between a Semi-Markov Process and a Continuous-time Markov Chain?

2. Consider the (continuous-time) stochastic process of the above figure where the transition time T from state A to B (and B to A) has the probability density $f(t) = \frac{1}{\mu},\ 0 \leq t \leq \mu$ and 0 otherwise. Is the process memoryless? Prove your answer.

DL 4.12. klo 09

Homework

2. *Hint:* compute $P(T > s + t \mid T > s)$, where

T : time spent in the state (random variable),

t : how much longer we will spend in the state,

s : how long we have already spent in the state.

Is the above equal to $P(T > t)$?

Thank you for listening!