Aalto University
School of Science

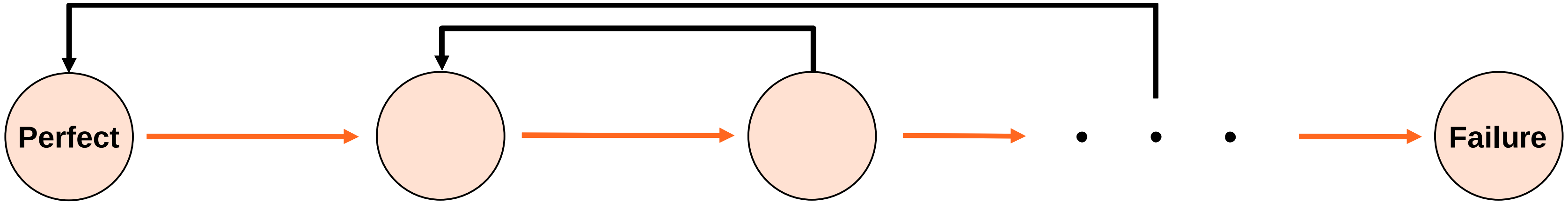
Semi-Markov decision processes - an application in maintenance

Jessica Norrbäck
Presentation *22*
27.11.2020

MS-E2191 Graduate Seminar on Operations Research
Fall 2020

Today we continue with maintenance

Condition based preventive maintenance



Action is taken after each inspection based on the state of the system

1. **No action**
2. **Minimal maintenance:** recover the system to its previous state
3. **Major maintenance:** bring the system to a state as good as new

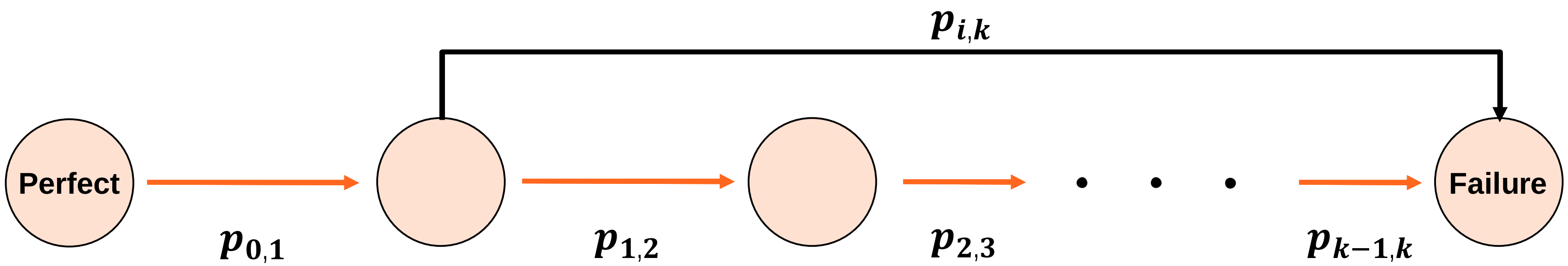
Problem setting

We formulate a semi-Markov decision process (SMDP)

Assumptions

- System can be modeled by a Markov model
- Poisson failures possible

Objective: Optimize system inspection rate and optimal maintenance policy for maximum steady-state availability



SMDP Notation

$\boldsymbol{k}$	Number of deterioration stages
$\boldsymbol{i}$	Index of deterioration stage
$\boldsymbol{g}$	Minimal preventive maintenance threshold
$\boldsymbol{b}$	Major preventive maintenance threshold
$\boldsymbol{\lambda_i}$	Failure rate at stage i
$\boldsymbol{F_{\mathrm{in}}}$	Time to inspection trigger interval distribution
$\boldsymbol{F_{\mathrm{d}}}$	Distribution of inspection duration
$\boldsymbol{F_{\mathrm{m}}}$	Distribution of minimal maintenance duration
$\boldsymbol{F_{\mathrm{M}}}$	Distribution of major maintenance duration
$\boldsymbol{F_{\mathrm{R}}}$	Distribution of failure repair time
$\boldsymbol{F}$	System failure state ($i = k + 1$)
$\boldsymbol{A_n}$	Maintenance action at the n^{th} decision epoch
$\boldsymbol{\theta}$	Next inspection time
$\boldsymbol{Y_n}$	System state at n^{th} decision epoch

Inspection states
$0 \leq i \leq k$

$$A_n = \begin{cases} 0, \text{no action} \\ 1, \text{minimal action} \\ 2, \text{major action} \end{cases}$$

Decision at n^{th} decision epoch

SMDP Formulation (1/4)

Transition probability from inspection state with deterioiration state i to the next inspection state with deterioration state j:

$$P(Y_{n+1} = j | Y_n = i, A_n = a, \theta_n = \theta) = \begin{cases} \int_0^{\infty} \tilde{P}_i^j(t) \mathrm{d}F_{in}(t, \theta), & a = 0 \\ \int_0^{\infty} \tilde{P}_{i-1}^j(t) \mathrm{d}F_{in}(t, \theta), & a = 1 \\ \int_0^{\infty} \tilde{P}_0^j(t) \mathrm{d}F_{in}(t, \theta), & a = 2 \end{cases}$$

Probability that the system changes from state i to j without any inspection event

SMDP Formulation (2/4)

Transition probability from inspection state/failure state to failure state F:

$$P(Y_{n+1} = k + 1 | Y_n = i, A_n = a, \theta_n = \theta) = \int_0^{\infty} \left(1 - F_{in}(t, \theta)\right) \mathrm{d}(\tilde{P}_i^{k+1}(t))$$

SMDP Formulation (3/4)

Expected time to the occurence of next decision epoch, given current deterioration state i and chosen action a:

$$\tau(i,a) = \begin{cases} \int_0^\infty \left(1 - \tilde{P}_i^{k+1}(t)\right)\left(1 - F_{in}(t)\right)\mathrm{d}t + \int_0^\infty F_d(t)\mathrm{d}t, & 0 \le i \le k, a = 0 \\ \int_0^\infty \left(1 - \tilde{P}_{i-1}^{k+1}(t)\right)\left(1 - F_{in}(t)\right)\mathrm{d}t + \int_0^\infty (F_d(t) + F_m(t))\mathrm{d}t, & 0 \le i \le k, a = 1 \\ \int_0^\infty \left(1 - \tilde{P}_0^{k+1}(t)\right)\left(1 - F_{in}(t)\right)\mathrm{d}t + \int_0^\infty (F_d(t) + F_M(t))\mathrm{d}t, & 0 \le i \le k, a = 2 \\ \int_0^\infty \left(1 - \tilde{P}_0^{k+1}(t)\right)\left(1 - F_{in}(t)\right)\mathrm{d}t + \int_0^\infty (F_R(t)\mathrm{d}t, & i = k+1 \end{cases}$$

SMDP Formulation (4/4)

c_m	cost per unit time of downtime due to maintenance
c_d	cost per unit time of downtime due to inspection
c_R	cost per unit time of downtime due to repair
c'_m	cost of minimal maintenance
c'_M	cost of major maintenance
c'_R	cost of repair

Cost function in state x with action a:

$$c(i,a) = \begin{cases} c_d \int_0^\infty \big(1 - F_d(t)\big)\mathrm{d}t, & i = 0 \\ c_d \int_0^\infty \big(1 - F_d(t)\big)\mathrm{d}t, & 1 \le i \le k, a = 0 \\ c_d \int_0^\infty \big(1 - F_d(t)\big)\mathrm{d}t + c_m \int_0^\infty \big(1 - F_m(t)\big)\mathrm{d}t + c'_m & 1 \le i \le k, a = 1 \\ c_d \int_0^\infty \big(1 - F_d(t)\big)\mathrm{d}t + c_M \int_0^\infty \big(1 - F_M(t)\big)\mathrm{d}t + c'_M & 1 \le i \le k, a = 2 \\ c_d \int_0^\infty \big(1 - F_R(t)\big)\mathrm{d}t + c'_R 1 \le i \le k, & i = k+1 \end{cases}$$

Value iteration algorithm

Step 0. Choose $V_0(x)$ such that $0 \le V_0(x) \le \min_a \frac{c(x,a)}{\tau(x,a)}, \quad \forall x$. Let $n = 1$.	
Step 1. Find stationary policy $R(n)$ that minimizes $V_n(x) = \min_{a \in A(x), \theta} \left[\frac{c(x,a)}{\tau(x,a)} + \frac{\tau}{\tau(x,a)} \sum_{y \epsilon I} P_{xay} V_{n-1}(y) + \left\{ 1 - \frac{\tau}{\tau(x,a)} \right\} V_{n-1}(x) \right], \quad x \in I$	
Step 2. Compute bounds $m_n = \min_{x \in I}(V_n(x) - V_{n-1}(x))$ and $M_n = \max_{x \in I}(V_n(x) - V_{n-1}(x))$ **If** $0 \le (M_n - m_n) \le \varepsilon m_n$, stop algorithm with optimal policy $R(n)$ **Else** go to step 3.	
Step 3. Return to step 1 with $n = n + 1$.	

Example with numerical values

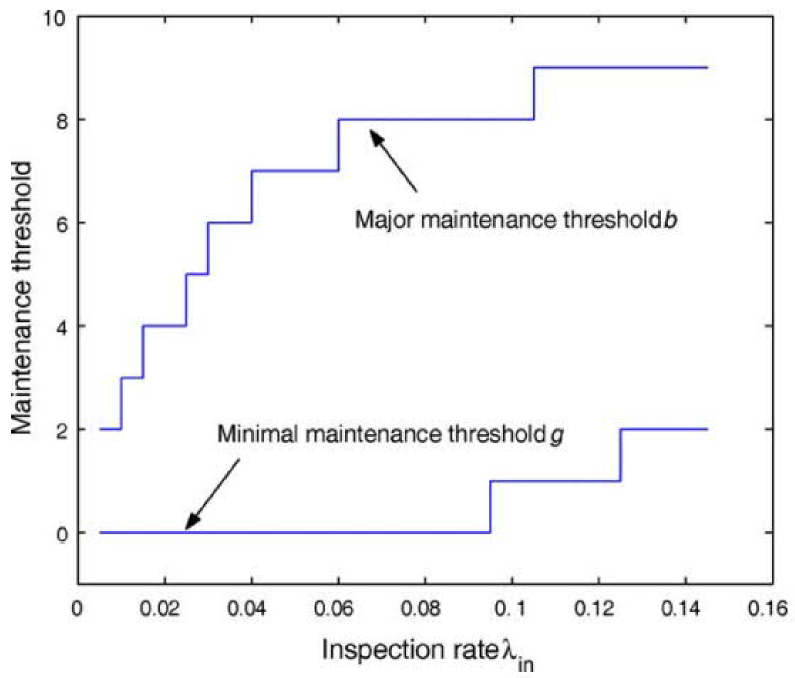
- All times ($F_{in}, F_d, F_m, F_M, F_R$) are deterministically distributed
- Parameter values

$$k = 10, \quad t_d = 0.5, \quad t_m = 0.5, \quad t_M = 0.5, \quad t_R = 100, \quad \lambda_i = 0.03$$

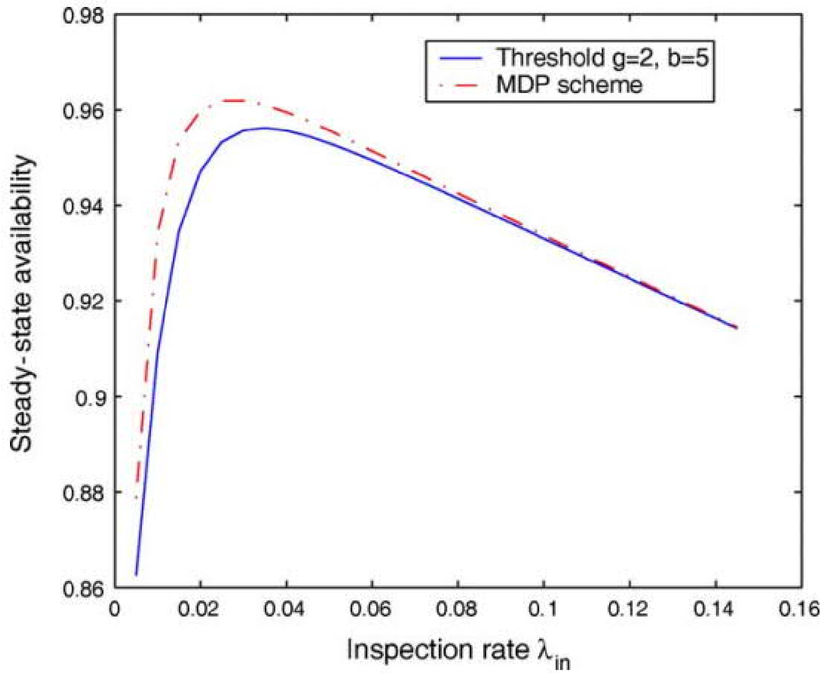
- System costs

$$c_d = c_m = 1, \qquad c_R = c'_m = c'_M = c'_R = 0$$

Results – Thresholds for optimal maintenance policy



Results – MDP vs Fixed threshold policy



Summary

- Preventive maintenance can be employed to avoid expensive system failure costs
- Joint optimization of system inspection rate and optimal maintenance policy can give better results than optimization of only inspection rate
- With steady-state availability and same deterioration rate at each failure stage, the optimal policy is a threshold-type policy

Aalto University
School of Science

Thank you!

References

Chen, D., & Trivedi, K. S. (2005). Optimization for condition-based maintenance with semi-Markov decision process. Reliability engineering & system safety, 90(1), 25-29

Homework

In which real-world situations would you apply the following maintenance policies?

a) Repair
b) Condition based preventive maintenance
c) Time based preventive maintenance

One example is enough in each case, but please motivate why you think the chosen maintenance policy is the best.

Example
We could choose a time-based preventive maintenance for oral health, visiting the dentist every year even if we do not have any problems. Thus, cavities are noticed in time and plaque and tartar is removed regularly.

(Please come up with an example of your own for c))