The optimization problem is

$$
\begin{array}{rrcl}
J^*(x_0) = \min\limits_{\pi\in\Pi} & \mathbb{E}_w\big[\sum_{t=0}^{2}(u_t + 2(x_t+u_t-w_t)^2)\big] & & \\
\text{s.t.} & x_{t+1} & = & \max(0, x_t+u_t-w_t) \\
 & x_t + u_t & \leq & 2 \ \forall t = 0,\dots,3 \\
 & x_t & \geq & 0 \ \forall t = 0,\dots,3 \\
 & u_t & \geq & 0 \ \forall t = 0,\dots,3 \\
 & x_0 & = & 1
\end{array}
$$

Starting from the final time $t = 3$, $J_3(x_3) = g_3(x_3) = 0$ as it was assumed that there is no terminal cost.

At period $t = 2$, we compute the cost-to-go as

$$
\begin{aligned}
J_2(x_2) &= \min_{u_2=0,1,2} \mathbb{E}_{w_2}[u_2 + 2(x_2+u_2-w_2)^2] \\
&= \min_{u_2=0,1,2} \big(u_2 + 0.1\cdot 2(x_2+u_2)^2 + 0.6\cdot 2(x_2+u_2-1)^2 + 0.3\cdot 2(x_2+u_2-2)^2\big)
\end{aligned}
$$

for all possible values of x_2 and u_2. From the given storage capacity constraint, it follows that $x_2 \in 0,1,2$ and $u_2 \in 0,1,2$ so both the state and the control are constrained. The values of $J_2(x_2)$ are

x_2/u_2	0	1	2
0	3.6	1.8	4
1	0.8	3	
2	2		

in which we have taken into account the constraint of storage size. Hence, the optimal costs for each state x_2 are

$$J_2^*(0) = 1.8,\ J_2^*(1) = 0.8,\ J_2^*(2) = 2,$$

so that the optimal controls are the values of u_2 corresponding to the optimal cost, that is

$$\mu_2^*(0) = 1,\ \mu_2^*(1) = \mu_2^*(2) = 0.$$

Now we have solved the subproblem of length one.

Traversing one time step back at period $t = 1$, we compute

$$
\begin{aligned}
J_1(x_1) &= \min_{u_1=0,1,2} \mathbb{E}_{w_1}[u_1 + 2(x_1+u_1-w_1)^2 + J_2(x_2)] \\
&= \min_{u_1=0,1,2} \mathbb{E}_{w_1}[u_1 + 2(x_1+u_1-w_1)^2 + J_2(\max(0, x_1+u_1-w_1))] \\
&= \min_{u_1=0,1,2} \big(u_1 + 0.1\cdot(2(x_1+u_1)^2 + J_2(\max(0,x_1+u_1))) \\
&\qquad + 0.6\cdot(2(x_1+u_1-1)^2 + J_2(\max(0,x_1+u_1-1))) \\
&\qquad + 0.3\cdot(2(x_1+u_1-2)^2 + J_2(\max(0,x_1+u_1-2)))\big)
\end{aligned}
$$

where on the second line the state equation is substituted so that we get rid of terms regarding the future state. Now when computing the values of $J_1(x_1)$, we take into account the obtained results for state $t = 2$ by picking up the optimal value for $J_2(\max(0, x_1 + u_1 - w_1))$. The values of $J_1(x_1)$ are

x_1/u_1	0	1	2
0	5.4	3.5	5.2
1	2.5	4.2	
2	3.2		

which results in the following optimal costs

$$J_1^*(0) = 3.5,\ \ J_1^*(1) = 2.5,\ \ J_1^*(2) = 3.2,$$

and thus, the optimal controls are

$$\mu_1^*(0) = 1,\ \ \mu_1^*(1) = \mu_1^*(2) = 0.$$

Repeating the same steps $t = 0$, the cost-to-go, which is now the optimal cost for the problem, is

$$J_0(x_0) = \min_{u_0=0,1,2} \mathbb{E}_{w_0}[u_0 + 2(x_0 + u_0 - w_0)^2 + J_1(\max(0, x_0 + u_0 - w_0))].$$

It attains the following values

x_0/u_0	0	1	2
0	7.1	5.2	6.9
1	4.2	5.9	
2	4.9		

so that the optimal costs are

$$J_0^*(0) = 5.2,\ \ J_0^*(1) = 4.2,\ \ J_0^*(2) = 4.9.$$

Hence, the optimal controls are

$$\mu_0^*(0) = 1,\ \ \mu_0^*(1) = \mu_0^*(2) = 0.$$

Noting that the optimal controls are the same in every state, the optimal control can be stated as

$$\mu^*(0) = 1,\ \ \mu^*(1) = \mu^*(2) = 0\ \forall t = 0, \ldots, 2,$$

and this function alone constitutes the optimal policy $\pi^* = \mu^*$.