Solution to homework 19

November 25, 2020

Recap slide 8 and read subsections B. Decomposition and D. Grouping maintenance activities from section II. Model of the Urbani paper. Briefly explain what economic profit of a grouping is comprised of and what is the intuition behind the two parts of the penalty function for shifting maintenance times.

The economic profit of grouping G^k is

$$EP(G^k) = U_{G^k} - \Delta H^*_{G^k} \tag{1}$$

Like mentioned in the slides, U_{G^k} is represents the amount of set-up costs S saved, meaning

$$U_{G^k} = (|G^k| - 1)S. \tag{2}$$

The penalty function is composed of two parts:

1. The change of the expected cost with regard to the components current schedule cycle

$$E(x_i^* + \Delta t_{ij}) - E(x_i^*) \tag{3}$$

2. And the changing cost due to the deferments of future activities executed after t_i^j

$$\Delta t_{ij}\, \phi_i^* \tag{4}$$

Intuition of the first part is that if we deviate from the optimal schedule of the component's current renewal cycle (=change the schedule of a maintenance activity of the component in group G^k), we either take unnecessary risk that the component breaks if the maintenance is postponed, or we actually take less risk of breakage.

The intuition behind the second part is that by deviating from the optimal schedule now, we also affect future schedules which may lead to either too many maintenances done on the long run (if maintenance time is advanced) or a fewer amount of maintenances (if maintenance time is postponed). This is represented through the minimal long-run average cost of maintenance per unit of time ϕ_i^* and the deviation time:

$$\Delta t_{ij}\,\phi_i^* = \Delta t_{ij}\,\frac{C_i^p S\beta_i}{x_i^*(\beta_i - 1)} \tag{5}$$

The total penalty function for a single component is expressed as

$$h_i = E(x_i^* + \Delta t_{ij}) - E(x_i^*) - \Delta t_{ij}\,\phi_i^* \tag{6}$$