MS-E2191 HW 15 Model Solution

Jussi Leppinen

November 2020

Question 1

Explain what $\mathbf{q}_1^{1,3}(t{+}1)$ is. Calculate the $\mathbf{q}_1^{1,3}(t{+}1)$ when $\mathbf{q}(t) = (0.85, 0.10, 0.05, 0, 0)$.

Solution

The vector $\mathbf{q}_1^{1,3}(t+1)$ tells what the information of the state of the system is at time $t+1$ when we decided to use control action 1 and inspection strategy 3 at time t and the outcome of the inspection is 1 at $t+1$. This state information also depends on the previous information. Now $S = \{1, 2, 3, 4, 5\}$ and we can calculate the information vector based on $\mathbf{q}(t)$, $\mathbf{P}(1)$ and $\mathbf{R}(3)$ as follows:

$$\begin{aligned}\boldsymbol{q}_1^{1,3}(t+1) &= \left(\frac{\sum_{l=1}^{5} q_l(t) p_{l1}(1) r_{11}(3)}{\sum_{i=1}^{5} \left[\sum_{l=1}^{5} q_l(t) p_{li}(1) \right] r_{i1}(3)}, ..., \frac{\sum_{l=1}^{5} q_l(t) p_{l5}(1) r_{51}(3)}{\sum_{i=1}^{5} \left[\sum_{l=1}^{5} q_l(t) p_{li}(1) \right] r_{i1}(3)} \right) \\ &= (0.2906, 0.6942, 0.0153, 0, 0)\end{aligned}$$

Example of calculations with Matlab is found in *hw_15_solution_leppinen.m*.

Question 2

Explain what $\boldsymbol{\alpha}_1^{3,2}$ is. Calculate the $\boldsymbol{\alpha}_1^{3,2}$ when $\beta = 0.99$.

Solution

The vector $\boldsymbol{\alpha}_1^{3,2}$ is a vector including the costs related to the choosing control action 3 and inspection strategy 2 one time period before terminating the system. When we know the current information vector $\mathbf{q}(t)$ we can calculate the value of the decision with $\mathbf{q}(t)\boldsymbol{\alpha}_1^{3,2}$.

The calculation of $\boldsymbol{\alpha}_1^{3,2}$ is based on the value function. When looking at the presentation slide 9 and doing things like with decisions $k = 1$ and $l = 1$ we end up having

$$\begin{aligned}\boldsymbol{\alpha}_1^{3,2} &= \mathbf{C}_3 + \beta \mathbf{P}(3) \mathbf{C}_2^I \\ &= (28.96, 83.96, 103.96, 453.96, 2503.96)^T\end{aligned}$$

The values of $\mathbf{C}_3$, $\mathbf{P}(3)$ and $\mathbf{C}_2^I$ are found from presentation or from article. Example of calculations with Matlab is found in *hw_15_solution_leppinen.m*.