For the case when the number (M) of channels is 2, the impulse responses are $h_0(0) = 1 + j$, $h_0(1) = 1-j$, $h_1(0) = -1-j$, $h_1(1) = 1-j$, which demonstrate the conjugate symmetric and conjugate anti-symmetric properties $h_0(0) = h_0^*(1)$, $h_1(0) = h_1^*(1)$.

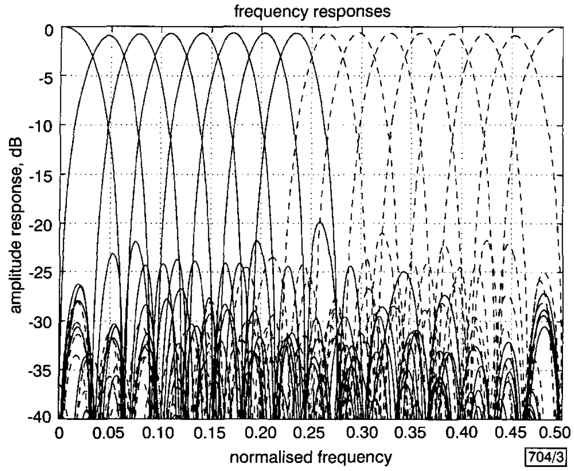

Fig. 3 *Amplitude responses of 16-channel complex-valued linear-phase filter bank*

Table 1: Impulse responses of eight-channel filter bank (real component)

n	$h_0(n)$	$h_1(n)$	$h_2(n)$	$h_3(n)$	$h_4(n)$	$h_5(n)$	$h_6(n)$	$h_7(n)$
0	0.08157	–0.08157	0.03339	–0.03339	0.03297	–0.03297	0.08447	–0.08447
1	0.06452	–0.06452	–0.15516	0.15516	0.15419	–0.15419	–0.06633	0.06633
2	–0.02638	0.02638	–0.21406	0.21406	–0.21800	0.21800	–0.02790	0.02790
3	–0.14391	0.14391	0.08052	–0.08052	–0.07991	0.07991	0.14413	–0.14413
4	–0.28185	0.28185	0.37903	–0.37903	0.37623	–0.37623	–0.28449	0.28449
5	–0.37044	0.37044	0.25553	–0.25553	–0.25367	0.25367	0.37167	–0.37167
6	–0.40201	0.40201	–0.24071	0.24071	–0.24757	0.24757	–0.40332	0.40332
7	–0.29908	0.29908	–0.39435	0.39435	0.39226	–0.39226	0.29148	–0.29148
8	–0.29908	–0.29908	–0.39435	–0.39435	0.39226	0.39226	0.29148	0.29148
9	–0.40201	–0.40201	–0.24071	–0.24071	–0.24757	–0.24757	–0.40332	–0.40332
10	–0.37044	–0.37044	0.25553	0.25553	–0.25367	–0.25367	0.37167	0.37167
11	–0.28185	–0.28185	0.37903	0.37903	0.37623	0.37623	–0.28449	–0.28449
12	–0.14391	–0.14391	0.08052	0.08052	–0.07991	–0.07991	0.14413	0.14413
13	–0.02638	–0.02638	–0.21406	–0.21406	–0.21800	-0.21800	–0.02790	–0.02790
14	0.06452	0.06452	–0.15516	–0.15516	0.15419	0.15419	–0.06633	–0.06633
15	0.08157	0.08157	0.03339	0.03339	0.03297	0.03297	0.08447	0.08447

Table 2 Impulse responses of eight-channel filter bank (imaginary component)

n	$h_0(n)$	$h_1(n)$	$h_2(n)$	$h_3(n)$	$h_4(n)$	$h_5(n)$	$h_6(n)$	$h_7(n)$
0	0.00023	0.00023	0.00041	0.00041	–0.00113	–0.00113	0.00005	0.00005
1	–0.00181	–0.00181	–0.00069	–0.00069	0.00178	0.00178	0.00401	0.00401
2	–0.00040	–0.00040	–0.00064	–0.00064	0.00086	0.00086	–0.00147	–0.00147
3	0.00116	0.00116	0.00215	0.00215	–0.00328	–0.00328	–0.00185	–0.00185
4	0.00006	0.00006	–0.00016	–0.00016	0.00257	0.00257	0.00311	0.00311
5	0.00353	0.00353	0.00543	0.00543	–0.00857	–0.00857	–0.00606	–0.00606
6	–0.00120	–0.00120	–0.00212	–0.00212	0.00553	0.00553	–0.00092	–0.00092
7	–0.00366	–0.00366	0.00575	0.00575	–0.00531	–0.00531	0.01117	0.01117
8	0.00366	–0.00366	–0.00575	0.00575	0.00531	–0.00531	–0.01117	0.01117
9	0.00120	–0.00120	0.00212	–0.00212	–0.00553	0.00553	0.00092	–0.00092
10	–0.00353	0.00353	–0.00543	0.00543	0.00857	–0.00857	0.00606	–0.00606
11	–0.00006	0.00006	0.00016	–0.00016	–0.00257	0.00257	–0.00311	0.00311
12	–0.00116	0.00116	–0.00215	0.00215	0.00328	–0.00328	0.00185	–0.00185
13	0.00040	–0.00040	0.00064	–0.00064	–0.00086	0.00086	0.00147	–0.00147
14	0.00181	–0.00181	0.00069	–0.00069	–0.00178	0.00178	–0.00401	0.00401
15	–0.00023	0.00023	–0.00041	0.00041	0.00113	–0.00113	–0.00005	0.00005

An eight-channel complex-valued filter bank was designed by employing eqn. 5 – 7 and optimising the stopband energy of the analysis filters $H_k(z)$, i.e. $\Omega = \Sigma_{k=0}^{M-1}\int_k th_{stopband}|H_k(e^{j\omega})|^2 d\omega$. The amplitude and phase responses are shown in Figs. 1 and 2. The impulse response data are given in Tables 1 and 2 where the coefficients show that the filters are conjugate symmetric and conjugate anti-symmetric. A 16-channel filter bank of length 64 is shown in Fig. 3.

Conclusion: In this Letter, we have presented an algorithm for the design of complex-valued linear-phase paraunitary filter banks. The designed filter banks have, for the first time in the design of sub-band filter banks, complex-valued filter responses. The difference between a real-valued and complex-valued linear-phase paraunitary filter bank is that conjugate-centro-symmtric matrices are employed. Filter banks with 8 and 16 channels have been presented to validate the algorithm.

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Modified Kay's method with improved frequency estimation

S.H. Leung, Y. Xiong and W.H. Lau

A modified frequency estimation method for use in the presence of additive complex white Gaussian noise is proposed. This method is basically an extension of Kay's method, but includes the use of autocorrelation functions. It retrieves the frequency value from the optimal linear combination of the differenced phases of the autocorrelation functions. It yields a considerably lower variance threshold than Kay's method while remaining unbiased and retaining the same frequency range.

Introduction: The estimation of the frequency of a single complex sinusoid in the presence of white Gaussian noise is a basic problem in many applications. Without loss of generality, the signal model of the problem can be described as

$$x(i) = \exp(j[\omega i + \phi]) + \nu(i) \qquad i = 0, 1, ..., N-1 \quad (1)$$

where the amplitude of the sinusoid is normalised to one. The frequency ω and the initial phase ϕ are deterministic and unknown constants. The noise $\nu(i)$ is an independent complex white Gaussian process with zero mean and variance σ^2.

It is well known that determination of the frequency location of the peak of a periodogram is equivalent to the function of a maximum likelihood estimator (MLE) [1]. It yields an unbiased estimate and has the best estimation performance for a single sinusoid. However, the periodogram method requires an intensive search in the frequency domain to achieve the desired accuracy.

Kay introduced a fast and simple method based on the differenced phase of the received data [2]. The estimator can be considered to be either a weighted phase averager or weighted linear predictor. The shortcoming of this method is the large variance threshold in comparison with that of other frequency estimation methods. Use of the autocorrelation functions of the received data has been proposed for the estimation of frequency by Fitz [3] and Luise and Reggiannini (L&R) [4]. We can consider the Fitz estimator and L&R estimator as a weighted phase averager and weighted linear predictor, respectively. Since the phase detection based on $\hat{R}(m)$ is so 'hard', these approaches will cause unrecoverable phase wrapping for high frequencies. Consequently, their operating ranges will be reduced and limited by M, the number of autocorrelation terms in the estimators.

In this Letter we propose the use of the differenced phase of the autocorrelation functions to estimate the frequency. The new estimator will increase the number of computations in the calculation of the autocorrelation functions but can achieve the Cramer-Rao bound (CRB) for high SNR and maintain a wide frequency range and obtain an improved variance threshold over Kay's method.

Derivation of new frequency estimator: We can express the received data $x(i)$ in eqn. 1 as follows:

$$x(i) = e^{j(\omega i+\phi)}[1+u(i)] \qquad i = 0, 1, ..., N-1 \tag{2}$$

where $u(i)$ can be shown to be zero mean independent complex white Gaussian noise with variance $\sigma_u^2 = \sigma^2$. The input SNR is given by $SNR_x = \sigma^{-2}$. The autocorrelation function of $x(i)$ is given by

$$R(k) = \sum_{i=0}^{N-k-1} x^*(i)x(i+k) = e^{j\omega k}[(N-k)+\eta(k)] \qquad k = 1, ..., N-1 \tag{3}$$

where

$$\eta(k) = \sum_{i=0}^{N-k-1} [u^*(i) + u(i+k) + u^*(i)u(i+k)] \tag{4}$$

Noting that $R(k)$ is proportional to $e^{j\omega k}$ plus noise, we can obtain the frequency without the need for phase unwrapping by constructing differenced phase variable $z(k)$ defined as follows:

$$z(k) = R^*(k)R(k+1) = e^{j\omega}[(N-k)(N-k-1)+\zeta(k)] \qquad k = 1, ..., N-2 \tag{5}$$

where

$$\zeta(k) = (N-k)\eta(k+1) + (N-k-1)\eta^*(k) + \eta^*(k)\eta(k+1) \tag{6}$$

For high SNR_x, the signal-noise-ratio in the argument of $z(k)$, denoted by $SNR_{\angle z}$, is given by

$$SNR_{\angle z} = (N-k)^2(N-k-1)^2/E\{\text{Im}[\zeta(k)]^2\} \tag{7}$$

where

$$E\{\text{Im}[\zeta(k)]^2\} = (N-k-1)(N-k)+N\max\{N-2k-1,0\} - k^2\delta(k-0.5(N-1))$$

Based on eqn. 7, it can be shown that $SNR_{\angle z}(k)$ is considerably improved by a factor of order $(N-k)^2$ over $SNR_{\angle z}(k)$.

The frequency can be estimated by taking the argument of the linear combination of the differenced phase variables as given by

$$\hat{\omega} = \arg(\mathbf{H}^T\mathbf{Z}) = \arg[\exp(j\omega)\mathbf{H}^T(\mathbf{G}+\Xi)] = \omega + \arg[\mathbf{H}^T(\mathbf{G}+\Xi)] \tag{8}$$

where $\mathbf{H}$ is a positive real weight vector, and the vectors $\mathbf{Z}$, $\mathbf{G}$, and Ξ are defined as follows:

$$\mathbf{Z}^T = [z(1), z(2), ..., z(N-2)] \tag{9a}$$

$$\mathbf{G}^T = [(N-1)(N-2), ..., 2] \tag{9b}$$

$$\Xi^T = [\zeta(1), ..., \zeta(N-2)] \tag{9c}$$

Without loss of generality, we set the weights with the following constraint $\mathbf{H}^T\mathbf{G} = 1$. Assuming high SNR, i.e. $\mathbf{H}^T\mathbf{G} >> |\mathbf{H}^T\Xi|$, we can approximate eqn. 8 as

$$\hat{\omega} = \omega + \mathbf{H}^T\Xi_I \tag{10}$$

where Ξ_I denotes the imaginary part of Ξ.

The minimum variance solution of eqn. 10 is given by

$$\mathbf{H}_{opt} = \frac{\mathbf{C}^{-1}\mathbf{G}}{\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G}} \tag{11}$$

where $\mathbf{C}$ is the correlation matrix of Ξ_I given by

$$\mathbf{C} = \mathbf{C}_1\sigma^2 + \mathbf{C}_2\sigma^4 + \mathbf{C}_3\sigma^6 + \mathbf{C}_4\sigma^8 \tag{12}$$

where $\mathbf{C}_1$ is given by (the expressions of other matrices $\{\mathbf{C}_i\}_{i=2,3,4}$ are omitted for brevity)

$$\mathbf{C}_1 = \mathbf{A}(\mathbf{I}-\mathbf{J})\mathbf{A} + \mathbf{A} + N\mathbf{K} \tag{13}$$

where

$$A = diag\{N-2, N-3, 1\} \tag{14a}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix} \quad \mathbf{J} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \cdot^{\cdot^{\cdot}} & 1 & 0 \\ 0 & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 1 & \cdots & 1 & 0 \\ \vdots & \cdot^{\cdot^{\cdot}} & 0 & 0 \\ 1 & \cdot^{\cdot^{\cdot}} & \cdot^{\cdot^{\cdot}} & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{14b}$$

Since the correlation matrix $\mathbf{C}$ is a function of the noise variance, the optimal weight vector $\mathbf{H}_{opt}$ is thus dependent on the input SNR_x. We can obtain a solution for the weight vector $\mathbf{H}$ independent of the noise variance by considering high SNR_x (i.e. the noise variance approaches zero). We refer to this solution as $\mathbf{H}_0$, which is given by

$$\mathbf{H}_0 = \frac{\mathbf{C}_1^{-1}\mathbf{G}}{\mathbf{G}^T\mathbf{C}_1^{-1}\mathbf{G}} \tag{15}$$

It is noted that $\mathbf{C}_1$ is a singular matrix for data length $N > 4$. Therefore $\mathbf{C}_1^{-1}$ in eqn. 15 is generally the pseudo-inverse of $\mathbf{C}$. If the noise variance is known, then the solution in eqn. 11 provides a better performance than the solution $\mathbf{H}_0$ in eqn. 15. Nevertheless the performance of the weight vector $\mathbf{H}_0$ is still considerably better than that of Kay's method.

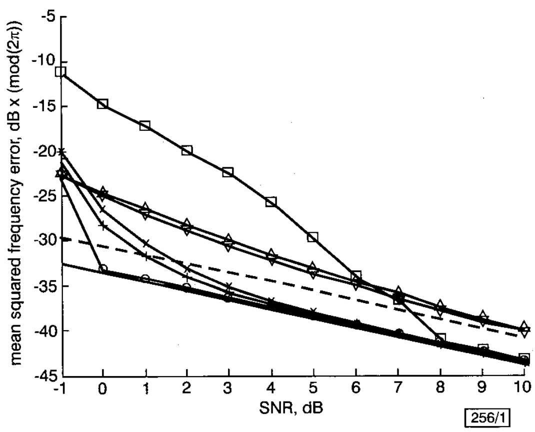

Fig. 1 *Performance comparison of different esimators for* $\omega = 0.0400\pi$

—□— Kay
—△— L&R (M = 3)
—▽— Fitz (M = 3)
—✳— $\mathbf{H}_0$
—+— $\mathbf{H}_{opt}$
—○— periodogram
- - - - 3dB above CRB
——— CRB

Simulation results: Computer simulations were performed to compare the performance of $\mathbf{H}_{opt}$ in eqn. 11, $\mathbf{H}_0$ in eqn. 15, and the Kay [2], Fitz [3], and L&R [4] estimators. A data record of $N = 24$ was used. The initial phase ϕ was randomly selected from within the range $[0, 2\pi)$. The mean squared frequency errors (MSFEs) were averaged over 5000 independent trials. Two signal frequencies $\omega = 0.04\pi$, 0.7617π were considered in the experiments and their mean squared frequency errors for the estimators against input SNR_x are, respectively, shown in Figs. 1 and 2. As benchmarks for comparison, the CRBs and the MSFE obtained using a periodogram are plotted in the two Figures.

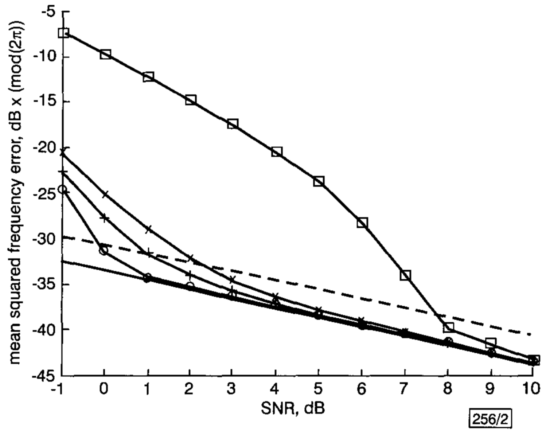

Fig. 2 *Performance comparison of different estimators for* $\omega = 0.7617\pi$

—□— Kay
—✳— $\mathbf{H}_0$
—+— $\mathbf{H}_{opt}$
—○— periodogram
- - - - 3dB above CRB
——— CRB

For ease of performance comparison, we define the variance threshold as the input SNR_x that yields an MSFE 3dB above the CRB (a line 3dB above the CRB is plotted in the Figures). In Figs. 1 and 2, it is shown that the proposed method ($\mathbf{H}_{opt}$ and $\mathbf{H}_0$) and the Kay estimator can achieve the CRB at a moderate level of SNR_x, while the Fitz and L&R estimators require much larger SNR values. The results show that the variance threshold of $\mathbf{H}_0$ is improved by > 5dB compared with that for the Kay estimator and ~2dB above that of a periodogram. Both the proposed method and the Kay method have a wide frequency range. For $\omega = 0.7617\pi$, the Fitz and L&R estimates can no longer operate owing to their limited frequency range.

Conclusion: A new frequency estimator has been discussed. It has been shown that the estimator can give much better frequency estimates than Kay's method with considerably lower variance threshold while remaining unbiased and having a wide frequency range.

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Nonlinear decorrelator for multiuser detection in non-Gaussian impulsive environments

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A nonlinear decorrelator is proposed for robust multiuser detection in CDMA non-Gaussian channels. The new detector involves the use of a nonlinear front-end followed by a linear decorrelating stage. Simulation results show that the new detector exhibits good near-far resistance under both Gaussian and non-Gaussian impulsive noise environments.

Introduction: A substantial amount of research has been recently devoted to developing sub-optimal multiuser detectors for direct sequence code division multiple access (DS/CDMA) systems. One of the main approaches involves the use of a linear decorrelating detector, which is a least squares estimator that provides a zero-forcing solution for multiaccess interference (MAI) rejection in the noiseless case [1]. The ambient noise is usually assumed to be Gaussian distributed primarily because of its simplicity and is justified by the central limit theorem. However, many wireless communications channels are primarily impulsive and exhibit non-Gaussian statistics [2], therefore the linear decorrelator is no longer appropriate due to its lack of robustness against outliers.

System and noise model: We consider this problem in the context of a coherent, synchronous DS/CDMA system. After conventional chip-matched filtering and sampling at the chip rate $1/T_c$, the discrete time version of the received signal within the ith symbol interval T can be written in vector form as

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n} \qquad (1)$$

where the bold variables denote column vectors in R^N, N is the system processing gain and $NT_c = T$, K is the number of users, A_k is the received amplitude of user k, and $b_k \in \{-1, 1\}$ is the symbol transmitted by user k. The normalised signature sequence of the kth user is denoted by

$$\mathbf{s}_k = \frac{1}{\sqrt{N}}[\beta_1^k \cdots b_N^k]^T$$

where $(\beta_1^k, ..., \beta_N^k)$ is a signature sequence of ±1's assigned to user k. $\mathbf{n}$ is the ambient noise vector and is assumed to be a sequence of independent and identically distributed (i.i.d.) random variables. In this Letter, we model $\mathbf{n}$ with symmetric α-stable (SαS) random variables, which have been shown to accurately model impulsive noise processes [3]. The characteristic function of a zero mean SαS random variable is

$$\varphi(t) = e^{-\gamma|t|^\alpha} \qquad (2)$$

where α ($0 < \alpha \le 2$) is called the characteristic exponent; a smaller α signifies more impulsive behaviour and *vice versa*. γ ($\gamma > 0$) is a scale parameter known as the dispersion. An α-stable distribution is justified by the generalised central limit theorem: if the sum of an infinite number of i.i.d. random variables (with finite or infinite variance) converges in a distribution, then the limiting distribution is α-stable. This suggests that ambient noise with α-stable distribution can arise in the physical world as the result of a large number of i.i.d. effects in the same way as Gaussian noise does. Since SαS noise with $\alpha < 2$ has no finite variance, the standard signal-to-noise ratio (SNR) measure becomes inconsistent. Therefore, a new scale parameter, geometric power, is used to indicate the strength of the α-stable noise [4]:

$$S_0 = \frac{(C_g \gamma)^{1/\alpha}}{C_g} \qquad (3)$$

where $C_g \simeq 1.78$ is the exponential of the Euler constant. The geometric signal-to-noise ratio (G-SNR), which provides a mathemat-