OPTIMIZATION OF DUAL-MIXER TIME-DIFFERENCE MULTIPLIER

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Abstract

The dual-mixer time-difference (DMTD) multiplication technique used in frequency stability measurement of precision frequency sources shows potential for achieving a frequency stability measurement floor as low as a few parts in 10^{15} at 1 s in terms of Allan deviation (ADEV) at 5 MHz at ambient temperature. The paper discusses the optimization of the DMTD multiplier using the frequency stability floor as a criterion for its efficiency. The instability sources of concern are all the noise sources as well as non linear effects and deterministic effects, such as phase noise in the common oscillator, noise in the mixers and amplifiers, inter-modulation products including the power-line pickup, electrical asymmetry (impairs the common-mode rejection of the phase and amplitude variations), phase dependence on environmental fluctuations, disturbing electromagnetic fields, and inappropriate construction (connectors, cabling, shielding, grounding). The optimization is demonstrated on the second version of DMTD multiplier designed at the Institute of Radio Engineering and Electronics (IREE). The IREE system has the "classical" DMTD structure, i.e. with a time-interval counter measuring the time difference between the zero crossings of the two beat-note signals. The modular construction allows modifications of the DMTD critical elements. The common oscillator as well as the time-interval counter are located outside the mixer-amplifier unit. The system can be operated at 5 and 10 MHz with the beat frequencies of 5 and 10 Hz, respectively, giving the multiplication factor of 10^6. The frequency stability measurement floor achieved through the optimization shows ADEV(τ = 1 s) = 6.9×10^{-15} at 5 MHz. At the basic sampling interval, τ_0 = 0.2s, the time deviation TDEV is 3.5 fs with the TDEV floor of 2 fs. This outstanding performance makes it possible to measure the frequency stability of future 5MHz quartz crystal oscillators of a few parts in 10^{-14} at 1s.

1 Introduction

Precise time and frequency applications require ultra-sensitive frequency/phase stability measurement of low-noise frequency sources, frequency conversion systems, time/frequency transfer elements etc. Since the measurement of short-term frequency/phase stability of the above systems is an integral part of time and frequency metrology, appropriate attention should be paid to it by metrology laboratories.

The IREE has recently decided to establish a special laboratory, within its Department of Standard Time and Frequency, that would allow high-precision short-term frequency/phase stability measurements in both the time and the frequency domains. The laboratory has been equipped with two ultra-stable 5 MHz Oscilloquartz 8600-BC5GE BVA oscillators each with ADEV(τ = 1 s) $\approx 8\times10^{-14}$ and with a frequency-domain measurement system based on the FSS 1000E Femtosecond Systems assembly which makes use of the phase detection method and shows the background noise at 5 MHz: L(f)= -145dBc/Hz at f=1Hz and L(f)= -177dBc/Hz at f=100kHz [11].

Concerning the time domain measurement, it was decided to develop an ultra-low noise system based on the DMTD multiplier of IREE's own design. The first version of the multiplier achieved the stability floor of ADEV(τ = 1 s) = 4.2×10^{-14} at 5 MHz with 100 Hz low-pass cutoff, and it showed successful in comparison with similar time or frequency difference multiplication systems that have been tested at IREE [6].

The relative success of the first version was motivation to continue the work on the IREE DMTD system, which has resulted in its second version whose design and performance are discussed in this paper.

2 DMTD multiplication

2.1 Principle

The DMTD multiplication for the time-domain frequency-stability measurement has been known and used for years [1-8]. The principle is apparent from the block diagram in Fig. 1. Assume that the phases of the two sine-wave signals from oscillators 1 and 2 evolve as

$$\Phi_1(t) = 2\pi\nu t, \quad (1)$$
$$\Phi_2(t) = 2\pi\nu\, t + \varphi(t), \quad (2)$$

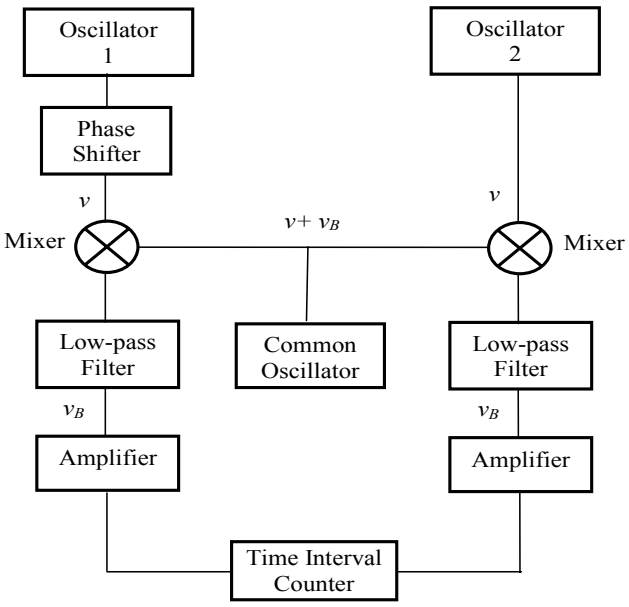

Fig.1: Block diagram of the DMTD multiplication

where ν is the frequency and φ(t) is a time-dependent phase difference. Assume further that the signals can be made quasi-synchronous during the measurement, i.e. $|\varphi(t)| << 1$. The two signals are mixed with the sine-wave signal from the common oscillator, whose phase evolves as

$$\Phi_C(t) = 2\pi(\nu + \nu_B)t + \varphi_C(t), \qquad (3)$$

to provide two beat-note signals at frequency ν_B with the phases

$$\Phi_{B1}(t) = 2\pi\, \nu_B t + \varphi_C(t), \qquad (4)$$

$$\Phi_{B2}(t) = 2\pi\, \nu_B t + \varphi_C(t) - \varphi(t). \qquad (5)$$

The beat time difference between the zero crossings at instants t_0 and $t_0 + x_B(t_0)$ can be written in terms of phase differences as

$$x_B(t_0) = [\varphi(t_0) + \varphi_C(t_0 + x_B(t_0)) - \varphi_C(t_0)]/(2\pi\nu_B). \qquad (6)$$

Assuming that $\varphi_C(t_0 + x_B(t_0)) \approx \varphi_C(t_0)$, which obviously holds better for small x_B and stable common oscillator, we obtain

$$x_B(t_0) \approx \varphi(t_0)/(2\pi\nu_B). \qquad (7)$$

Since the "input" time difference at frequency ν is

$$x(t_0) = \varphi(t_0)/(2\pi\nu) \qquad (8)$$

we have

$$x_B(t_0) \approx m\, x(t_0) \qquad (9)$$

where $m = \nu/\nu_B$ is the multiplication factor.

2.2 IREE Design

The IREE DMTD design is in the main based on the "classical" architecture depicted in Fig.1. The block diagram of one channel is shown in Fig. 2.

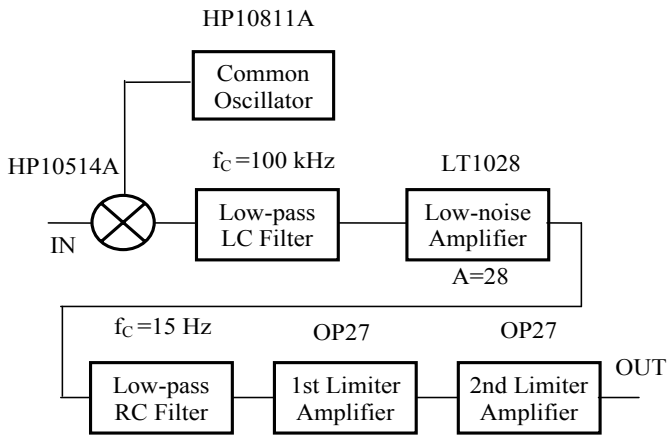

Fig.2: Block diagram of the IREE multiplier (one channel)

The common frequency source is a HP10811A quartz oscillator that provides 10 MHz at +7 dBm power. Part of the oscillator block is a built-in modulo-2 divider that provides 5 MHz at +11 dBm level. The oscillator frequency is offset by +10 Hz which gives $\nu_B = 5$ Hz at $\nu = 5$ MHz and 10 Hz at 10 MHz. Thus in both cases the multiplication factor is 10^6. The coefficient h_2 characterizing the white phase noise modulation in the frequency domain is 4×10^{-29} calculated from the L(f) floor of -153 dBc/Hz at 5MHz measured with the Femtosecond Systems equipment.

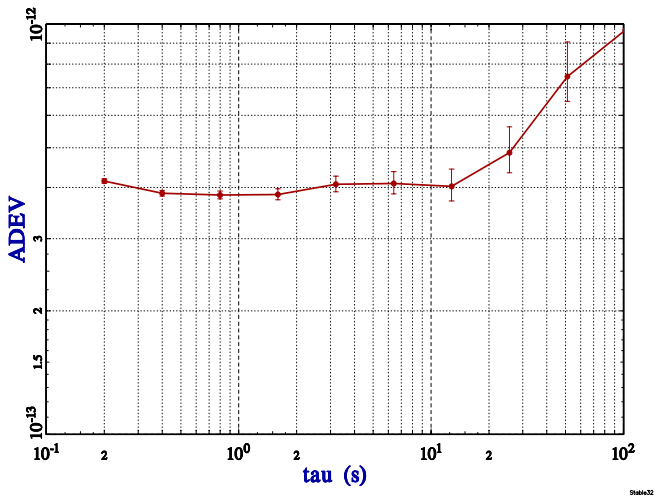

Fig.3: Frequency stability of the common oscillator.

The frequency stability of the HP10811A at 5 MHz is shown in Fig. 3. In both cases the reference was the 8600-BC5GE BVA oscillator whose contribution can be neglected.

The mixer is a double-balanced HP10514A. To our knowledge this model exhibits the lowest flicker phase modulation out of the mixers available at the market, most likely because it is assembled from separate diodes. The mixer output has a capacitive loading (22 nF) that increases the zero-crossing slope [2]. The low-pass filter following the mixer is a two-stage LC structure with the cutoff frequency around 100 kHz. It is used to efficiently attenuate the upper-band products. A single-pole RC filter (not shown in Fig. 2) with a cut-off frequency of 2.2 kHz is added right after the LC to compensate for imperfection of the L,C components.

The low-noise (LN) amplifier is a LT1028 connected in non-inverting mode (voltage gain A=28). The LT1028 amplifier exhibits a voltage noise of less than 1nV/√Hz, the 1/f corner frequency around 3 Hz, and the gain-bandwidth product (GBW) of 75 MHz. It is followed by a RC single-pole low-pass filter with a cut-off frequency of 15 Hz. In the version 2 the cut-off frequency can be switched to 140 Hz for experimental reasons.

The limiter amplifiers are the OP27 (Analog Devices). The 1st OP27 limiter amplifier is connected in non-inverting mode while the 2nd is in inverting mode. The OP27 exhibits the voltage noise of 3nV/√Hz, the 1/f corner frequency of 3Hz and GBW= 8 MHz . The DC voltage gains are set to 130 and 100, respectively. The 1st limiter amplifier contains a single-pole RC filter in the feed-back with a bandwidth of 160 Hz in the zero-crossing area. The 2nd limiter amplifier provides a trapezoidal waveform with a slope of $\pm 7\times10^5$V/s at the zero crossings.

3 Sources of instability

3.1 Noise from common oscillator

A comprehensive analysis of the rejection of the phase variations $\varphi_C(t)$ from the common oscillator has been made in [9]. It has been shown that the residual Allan deviation of the relative frequency deviation

$$y_C(t) = \frac{1}{2\pi(\nu+\nu_B)}\dot{\varphi}_C(t) \qquad (10)$$

takes the form

$$ADEV_{CR}(T_B,x_B) = [\int_0^\infty S_{yC}(f)\, G_C(f,T_B)\, G_R(f,x_B)\, df]^{1/2} \qquad (11)$$

where S_{yC} is the power spectral density of $y_C(t)$ and

$$G_C(f,T_B) = \frac{2\sin^4 \pi T_B f}{(\pi T_B f)^2} \qquad (12)$$

is the well-known power transfer function of the equivalent "filter" of $y_C(t)$ which corresponds to the algorithm for computing the Allan deviation. The rejection is achieved through the multiplication by the transfer function

$$G_R(f, x_B) = 2\,(1 - \cos 2\pi x_B f). \qquad (13)$$

Apparently, for x_B approaching zero also $ADEV_{CR}$ approaches zero.

Solving the integral (11) for the case of white phase modulation where $S_{yC}(f)=h_2 f^2$, and for an ideal low-pass filter with the cutoff frequency f_C, and considering $x_B \ll T_B$, we obtain

$$ADEV_{CR} = [\frac{3h_2 f_C}{2(\pi T_B)^2}(1-\frac{\sin 2\pi x_B f_C}{2\pi x_B f_C})]^{1/2}. \qquad (14)$$

Again for $x_B \to 0$ also $ADEV_{CR} \to 0$.

The theoretical values of $ADEV_{CR}$ calculated from equation (14) as a function of x_B for various f_c are shown in Fig. 4. The parameters h_2 and T_B are those used in the IREE DMTD system, i.e. $h_2 = 4\times10^{-29}$ (HP10811A oscillator) and T_B = 0.2 Hz. It should be noted that for large f_C and x_B, the value of $ADEV_{CR}$ becomes independent of x_B while the condition $x_B \ll T_B$ still holds.

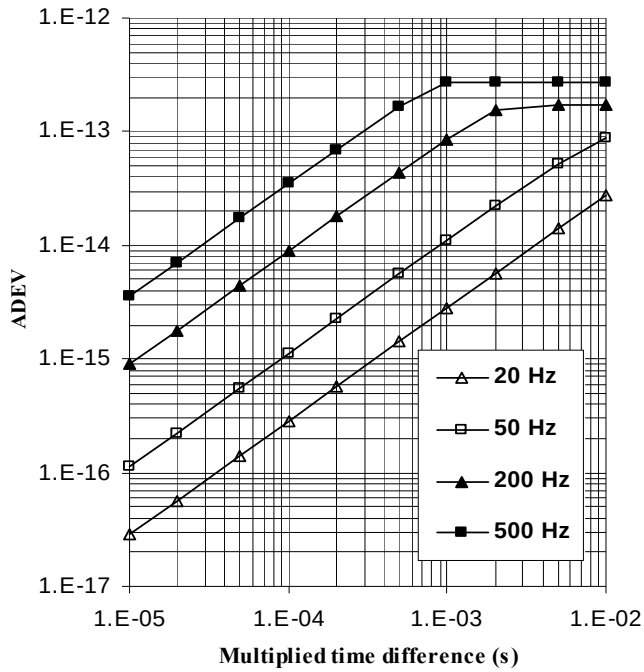

Fig.4: Theoretical plots of $ADEV_{CR}(x_B,f_C)$.

For low and very low Fourier frequencies, where flicker frequency noise and random walk of frequency noise dominate the fluctuations, a limited expansion of the right hand side in (11) shows that the corresponding contributions in ADEV remain extremely low and do not affect the noise floor of the DMTD system.

Due to the system symmetry, the noise impact of the test oscillator is described by the same formulas as that of the common oscillator. Since the Oscilloquartz 8600-BC5GE used as the test oscillator has $h_2 = 1.3\times10^{-29}$ (about three times less than the common oscillator), its noise contribution can be neglected.

3.2 Noise from time interval counter

The multiplied time difference x_B is eventually measured with a time interval counter whose noise impact (primarily the trigger noise) on the DMTD multiplication must be evaluated. Experimental evaluation can be made by applying one of the beat signals to both start and stop counter inputs. A suitable delay in the stop input can be ensured by a small voltage offset in the triggering level.

The result in terms of TDEV(τ) obtained for the $7x10^5$V/s slope and $T_B = \tau_0 = 0.2$ s with the SR620 counter used in the IREE DMTD system is shown in Fig. 5.

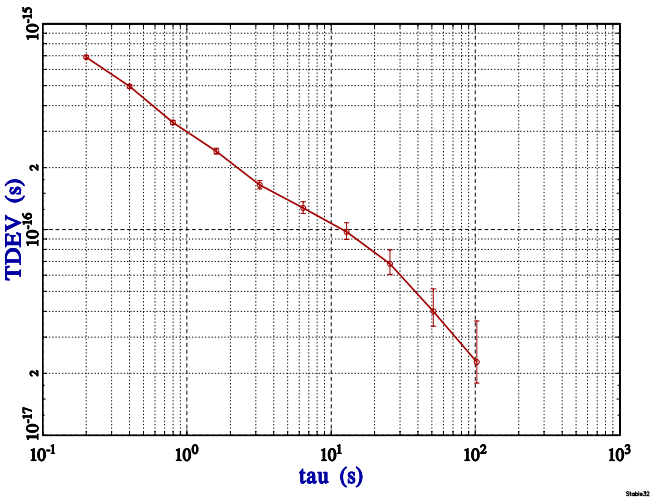

Fig.5: Time stability of the SR620 counter.

It is apparent from the TDEV(τ) slope that in the short run the counter shows mainly the white phase modulation. The corresponding frequency stability is ADEV(τ) = $1.2x10^{-15}\,\tau^{-1}$. Thus the noise contribution of the counter is negligible.

3.3 Noise in mixers and amplifiers

In the configuration shown in Fig. 2, the overall noise includes two main components: the noise in the first elements of the channel with a narrow bandwidth, and the noise in the limiter amplifiers with a wide bandwidth. The first case concerns the proper noise of the mixers (specially the flicker phase modulation), the noise of the lossy elements used to filter the high frequency components at the output of the mixer, the noise in the resistors around the LN amplifier, and the proper (voltage) noise of the LN amplifier.

The wideband noise contribution is provided by the elements located after the low pass filter following the low-noise amplifier. Generally speaking, the noise contribution of all the elements located between the mixer and the counter is reduced by the gain K_d (in V/rad) of the mixer, which must be as high as possible.

The 1st limiter amplifier increases the zero-crossing slope and therefore it requires a higher bandwidth (in the linear zone). Experimental optimization of the bandwidth (B) can be made at this stage: narrowing the bandwidth without affecting the amplitude of the harmonics that allow a steep slope makes the time noise decrease. However, excessive narrowing causes a less steep slope and an overall increase in time noise.
The same applies to the 2nd limiter amplifier.

If the slope of the output signal of the 2nd limiter amplifier reaches the amplifier's slew rate ($2.8x10^6$ V/s in OP27), an additional non-linear effect may take place which can reduce the system background noise but also the fluctuations of interest that are to be measured. Currently, the slope at the output of the last op amp is $7x10^5$V/s and this effect need not be considered.

3.4 Interference

Interference signals belong to important degradation factors of the multiplier performance. At this extremely-low noise level one should expect to encounter a variety of mechanisms for interference many of which are difficult to identify and quantify. Apparently the more perturbing is the interference that takes place only in one channel and thus cannot be rejected by the symmetry of the setup. Difficult to identify is the interference involving unstable sources since their effect is not stationary.

One of the known mechanisms is intermodulation in the mixers due to their imperfect (i.e. not quadratic) mixing characteristics. Intermodulation may also take place in the amplifiers. Obviously, of concern are all the intermodulation products

$$\nu_S = r_1\,\nu_1 + r_2\,\nu_2 + \ldots\,, \quad r_i = 0, \pm1, \pm2, \ldots \qquad (15)$$

that fall in the system bandwidth. In fact their first contribution improves the performance since they increase the gain of the mixer. They do not directly provide spurious fluctuations, but they may provide an unexpected beat note with a disturbing signal. The frequencies ν_1, ν_2, ... may come not only from the oscillators involved in the DMTD system, but also from other frequency sources in the laboratory including the power-line frequency.

Fig. 6 shows an example of interference with a stable source we have experienced in one of our measurements. It turned out that the source of interference was another 5 MHz BVA oscillator which happened to be offset by $3.8x10^{-9}$ or 19mHz, and which was located close to the test BVA oscillator. The interfering oscillator had an unterminated coaxial cable of about 1 m length connected to its output and the interference disappeared only after the cable had been removed.

Interference may cause an undesired phase lock by an injection locking process between the measured oscillators. Fortunately these undesired phase locks can be easily detected. The more dangerous are near-false phase locks where the phase is only disturbed.

The use of battery supplies for all DMTD blocks (test oscillator, common oscillator and the mixer-amplifier unit) is

a natural remedy against power-line pickup but there remains always a portion of power-line pickup that comes from the radiated low frequency magnetic fields, that can be shielded only by magnetic packages.

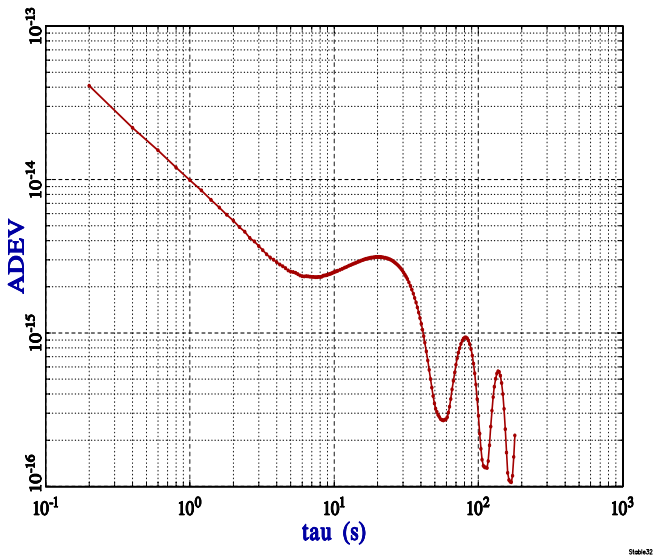

Fig.6: Interference with a nearby oscillator.

3.5 Environment

It is apparent that such a sensitive electronic system is affected by the environment. One would expect that vibrations and electromagnetic disturbances will have an impact on short-term stability while changes in temperature will rather affect the long-term performance. The problem is that the short-term manifestation of the environment can hardly be discerned from other short-term variations and the long-term effects can be masked by large-period spurious signals. Also the question arises about what kind of fluctuations we obtain when translating the temperature variations into DMTD.

In Fig. 7 we can see the background frequency stability (ADEV) and the corresponding time stability (TDEV) of the IREE DMTD system calculated from the longest available measurement. The right-hand slopes are roughly similar to the effect of a random-walk frequency noise. With possible deterministic effects and with the limited data length, however, we can just speculate about the cause of the long-term impairment: most likely it is not a random-walk flicker frequency modulation but rather a manifestation of some spurious signal with a period larger than 2000 s (compare the character of the ADEV plot with that in Fig. 6), or a spurious thermal effect not identical in the two channels.

3.6 Construction

Shielding, grounding, connectors, cables, printed circuit design etc. all may have an impact on the multiplier noise performance. Here in particular the result depends on the designer experience and know-how. Because of the optimization, the version 2 is constructed so as to allow an access to critical elements of the system. This, on the other hand, remains a weak point of version 2 since we cannot yet use semi-rigid cables and, consequently, neither SMA connectors (we use common BNCs). In a number of measurements a low performance has been observed because of mechanical effects.

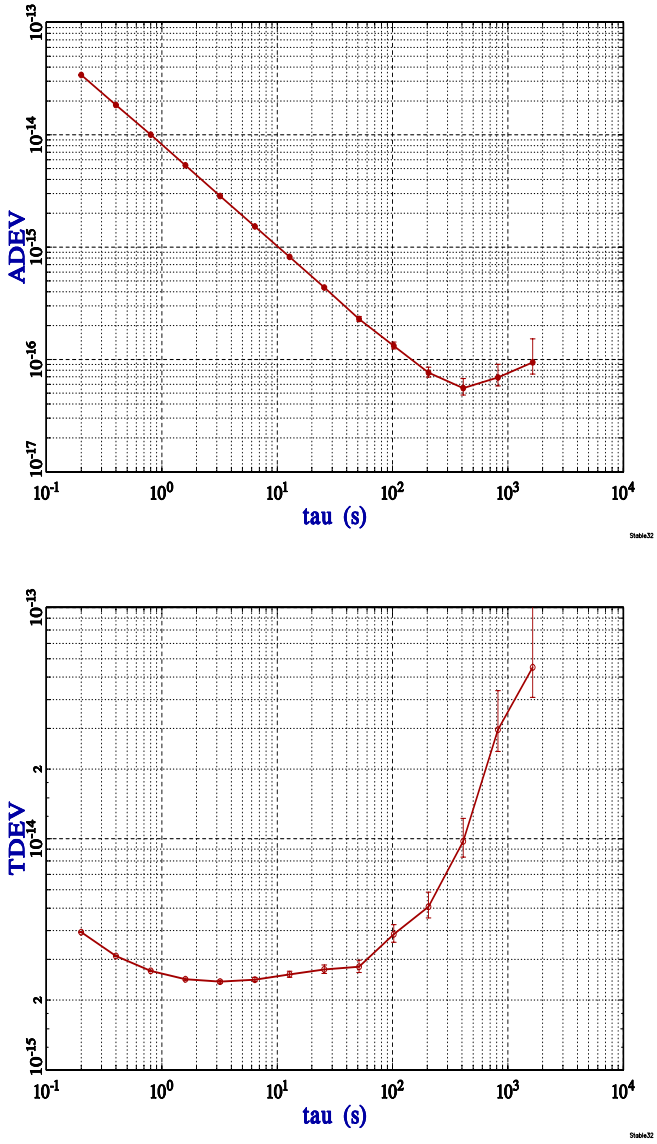

Fig. 7: Long-term performance of IREE DMTD system.

In designing the amplifier chain we have followed this principle: though a very low frequency signal is treated the construction is made as if it were a low-noise HF apparatus (unilateral signal path, short connections, appropriate grounding and shielding). Careful attention has been paid to avoiding parasitic feedbacks and the leakage of RF signals from the mixer. Also, the two DMTD channels must be prevented from cross-talks (separate mounting, separate power supply).

4 Optimization

4.1 Methodology

The term "optimization" reflects a dual approach: the conceptual approach which is not based on the classical overall noise figure since the optimum way to operate the DMTD multiplier involves non-linearity, but on considering several noise processes (with various bandwidths) and components whose overall impact must be understood and minimized [2]. A similar problem exists in heterodyne receivers, where the noise performance does not depend only on the noise figure of the first amplifier.

The engineering approach complements the conceptual approach and is based on gradual adjustment of the system in order to practically achieve the best noise performance. This is assessed by the magnitude (and character) of the fluctuations in the multiplied time difference $x_B(t)$ in the time domain by using the standard statistical measures: Allan deviation (ADEV) for frequency stability and time deviation (TDEV) for time stability. The criterion for whether the optimization step was good or bad was the "standard" value of ADEV at 1 s. By employing robust data the uncertainties obtained are very small. For calculations and simulations we have made use of Stable32 [10].

Throughout the optimization the test signal was generated from the same output of the 8600-BC5GE (sn 315) oscillator via a T-connector to both multiplier inputs.

4.2 Results

All the measurements mentioned below have been made at $\nu = 5$ MHz , $T_B = 0.2$ s (i.e. $m = 10^6$) and $f_C = 15$ Hz (- 3dB cut-off frequency of the dominant single-pole low-pass RC filter after the LN amplifier – see Fig. 2). The value of T_B is about the minimum obtainable due to the limited tuning range of the common oscillator.

In what follows only reproducible steps that lead to smaller ADEV(1s) are reported. The steps intended to improve the performance but which have a negative impact (e.g. implementation of separation amplifiers in the common signal) are not mentioned. The value of the TDEV floor is about an average within the flicker-phase interval. Taking the minimum TDEV would give a misleading picture because of the wavy character of the flicker floor (see Fig. 8)

- The starting point was the first version described in [6] with

  $$\text{ADEV(1s)} = 4.2\text{x}10^{-14}.$$

- In the first optimization step a number of changes have been made in the amplifying channel: different regime of LT1028 (linear, no band limitation, RC low-pass filter after it), decrease of low-pass cut-off frequency (from ≈ 100 Hz to 15 Hz), simplification of the output architecture (two OP27 used), adn separation of power supplies for the two channels. The result was:

  $$\text{ADEV(1s)} = 2.6\text{x}10^{-14}, \text{ TDEV floor} \approx 8 \text{ fs}$$

- Test signals fed through a simple T connector instead of (-3 dB) power splitter:

  $$\text{ADEV} = 2.2\text{x}10^{-14}, \text{ TDEV floor} \approx 7 \text{ fs}$$

- Capacitive loading (C=22 nf) used at mixer outputs:

  $$\text{ADEV} = 1.9\text{x}10^{-14}, \text{ TDEV floor} \approx 5 \text{ fs}$$

- Replacement of the SRA1 integrated mixers with separate-diode HP10514A mixers:

  $$\text{ADEV} = 1.1\text{x}10^{-14}, \text{ TDEV floor} \approx 3 \text{ fs}$$

- Optimization of the bandwidth of the 1st limiter amplifier (f_C=160 Hz):

  $$\text{ADEV} = 8.5\text{x}10^{-15}, \text{ TDEV floor} \approx 2.5 \text{ fs}$$

- Replacement of the 10 MHz Milliren MTI 250-0502A common oscillator with the 10 MHz HP10811 oscillator. The improvement, however, was not because of better oscillator noise performance (which is about the same) but because of the higher power level at 5 MHz obtained from the modulo-2 dividers, which gives a larger gain Kd of the mixers. Namely, the low-noise regenerative divider (RD) connected to the Milliren oscillator provides +7 dBm (which shows not sufficient) while the built-in digital divider (DD) in the HP10811 oscillator block provides +11 dBm. Thus since RD is by 7 dB less noisier than DD, a good solution would be to low-noise amplify the signal from RD. This could not be done in this version because of its construction. The result of the replacement change was:

  $$\text{ADEV} = 7.5\text{x}10^{-15}, \text{ TDEV floor} \approx 2 \text{ fs}$$

- Mechanical revision (fixing the connectors, cables, covers):

  $$\text{ADEV} = 6.9\text{x}10^{-15}, \text{ TDEV floor} \approx 2 \text{ fs}.$$

The frequency and time instability corresponding to the best result is shown in Fig. 8.

We have to realize that we are treating a very sensitive system which makes it possible to measure very tiny effects including those (unwanted) from environment. Therefore the performance is always slightly environment-dependent. Therofore it should be pointed out that the above results have been obtained in very favorable environmental conditions that the IREE's special metrology room for frequency stability measurement offers (located under ground, partly Faraday cage, no electronics active besides the measurement system).

At the same time, it should be noted that no additional precautions were taken : the measurements were performed during the working hours and common AC-DC power sources were used.

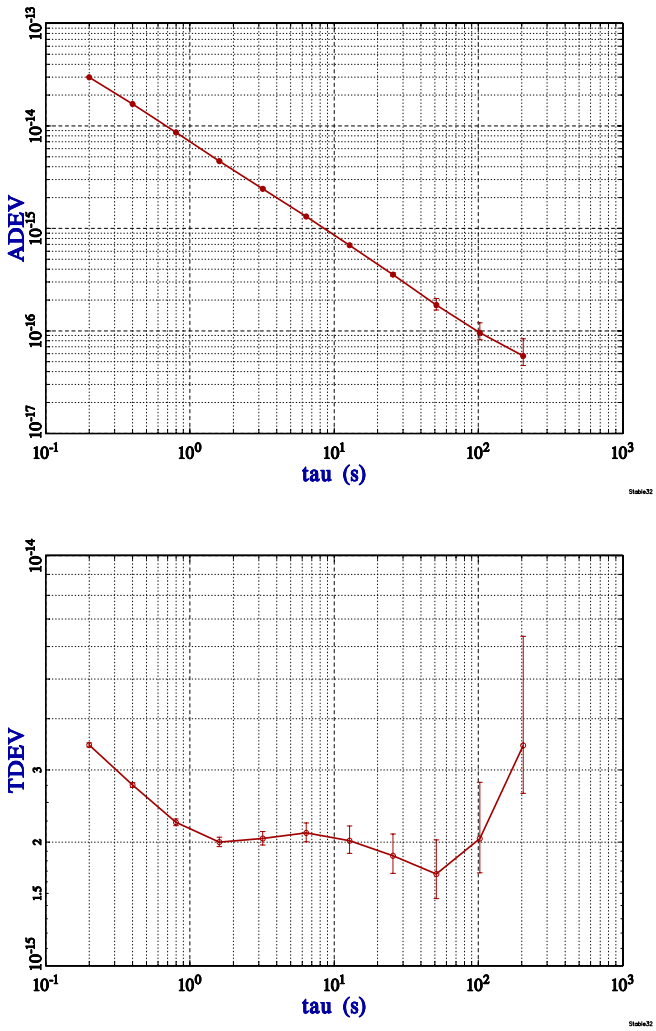

Fig. 8: Best measurement capability of IREE DMTD system.

5 Conclusions

The results have confirmed the excellent potential of the DMTD multiplication. The optimization has shown that an outstanding noise performance can be obtained from a relatively simple apparatus.

Concerning further investigation, it is of interest to determine the instability limits of this DMTD system, both long-term and short-term. It is naturally appealing to make use of the extreme sensitivity achievable with the DMTD multiplication also in other types of measurement (i.e. not only for frequency stability). This concerns primarily its possible use in accurate measurement of small time delays in electronic elements (connectors, cables etc.). A challenge would be to employ the DMTD system in the cases where the measured magnitude (temperature, vibrations) can be translated via an appropriate sensor into the change in time difference between the zero crossings of the input signals.

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References

[1] D.W. Allan, H. Daams. "Picosecond time difference measurement system", *Proc. 29th Annu. Symp. Frequency Contr.,* Atlantic City, USA, pp. 404-411, (1975).

[2] R. Barillet. "Comparateur de phase ultra faible bruit pour les futurs étalons de frequence", *Proc 3th European Time and Frequency Forum*, Besancon, France, pp. 249-254, (1989).

[3] G. Brida. "High resolution frequency stability measurement", *Rev. Sci. Instrum.*, **volume** 73, pp. 2171-2174, (2002).

[4] L.S. Cutler, C.L. Searle. "Some aspects of the theory and measurement of frequency fluctuations in frequency standards. *Proc. IEEE*, **volume** 54, pp. 136-154, (1966).

[5] C.A. Greenhall. "Common-source phase error of a dual-mixer stability analyzer", *TMO Progress Report 42-143*, Jet Propulsion Laboratory, Pasadena, XA, (2000).

[6] L. Sojdr, J. Cermak, G. Brida. "Comparison of high-precision frequency-stability measurement systems. *Proc. IEEE International Frequency Control Symposium and 17th European Frequency and Time Forum*, Tampa, USA, pp. 317-325, (2003).

[7] S.R. Stein. "Frequency and time – their measurement and characterization", in *Precision Frequency Control*, **volume** II, E.A. Gerber and A. Ballato, Eds. New York: Academic Press, pp.229-231, (1985)

[8] S. Stein, D. Glaze, J. Levine, J. Gray, D. Hilliard, D. Howe, L.A. Erb. "Automated high-accuracy phase measurement system", *IEEE Trans. Instrum. Meas.* **volume** IM-32, pp. 227-231, (1983)

[9] L. Sze-Ming, "Influence of noise of common oscillator in dual-mixer time-difference measurement system", *IEEE Trans. on Instr. and Meas.*, **volume** IM-35, pp. 648-651 (1986).

[10] "Stable32 version 1.35: frequency stability analysis," *Hamilton Technical Services*, S. Hamilton, MA 01982 USA, (2002).

[11] "FSS1000E Phase noise detector", Operation Manual Rev 3.2, Femtosecond Systems, Inc., p.4-6 (1999)