Lecture 5 : Basic ship hydrostatics and stability

After defining the ship's main dimensions, form coefficients, hull form the next logical step of preliminary design is to calculate hydrostatics and some parameters beneficial for stability assessment. The concept of hydrostatics and stability can be deemed as one of the most important areas of focus in ship design and operation. With this in mind this lecture outlines basic concepts for use in basic ship design.

1. Ship flotation and stability – the basics

Hydrostatics is a branch of mechanics assuming that fluids lie at rest (i.e. fluid velocity effects are neglected) and the pressure in a fluid or the pressure exerted by a fluid on a floating or immersed body is investigated within the context of static equilibrium. This is fundamental in terms of understanding the concept of flotation namely: *''If a solid body is immersed in a liquid, there is an apparent loss in weight. This loss in weight is the up - thrust (known as* ***buoyancy****) exerted by the liquid on the body and is equal to the weight of the volume of liquid which the body displaces''*. So, if a solid body is suspended in fresh water, completely immersed, this up - thrust represents the weight of the fresh water having the same volume as the body. ***This principle is the base of hydrostatics and stability and is known as*** **Archimedes' Principle[4].**

The centroid of the operating waterplane is the point about which the ship will list and trim. This point is called the **center of flotation (F)** and it acts as a fulcrum or pivot point for a floating ship. The distance of the center of flotation from the centerline of the ship is called the **transverse center of flotation (TCF)**. When the ship is upright, the center of flotation is located on the centerline so that the TCF = 0. The distance of the center of flotation from amidships (or the forward or after perpendicular) is called the **longitudinal center of flotation (LCF)**. LCF is referenced from amidships or from one of the perpendiculars. If the reference is amidships, one must also indicate if the distance is toward the forward or aft end of the ship. By convention, a negative sign is used to indicate distances aft of midships. The center of flotation is always located at the centroid of the current waterplane, meaning its vertical location is always on the plane of the water. When the ship lists to port or starboard, trims down by the bow or stern, or changes draft, the shape of the waterplane will change, thus the location of the centroid will move, leading to a change in the location of the center of flotation.

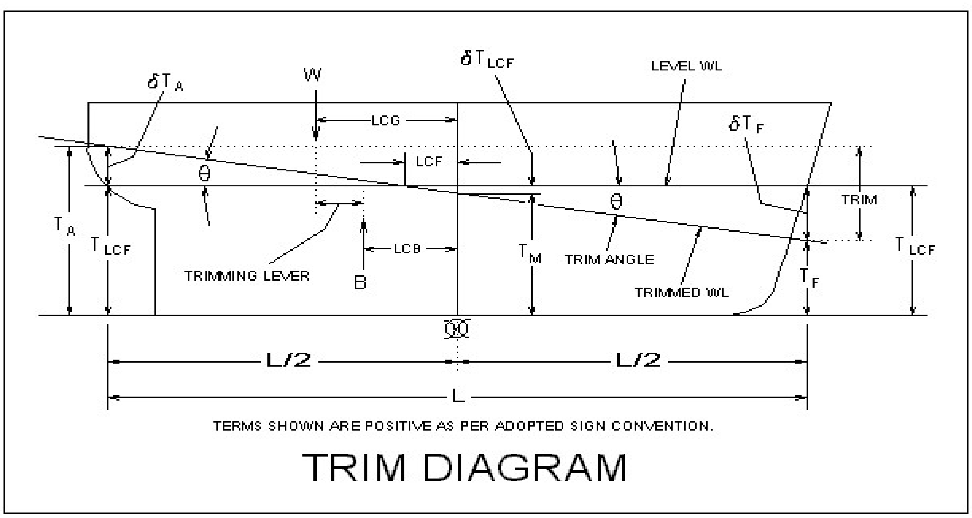
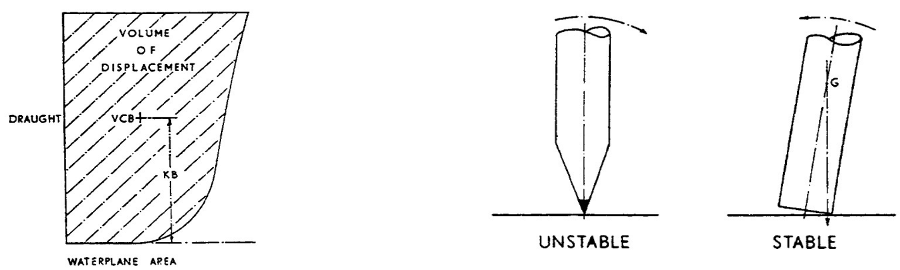


Figure 5.1 Ship trim diagram (image credit http://hawaii-marine.com)

[4] For a vessel to float freely in water the weight of the vessel must be equal to the weight of the volume of water she displaces (see https://www.britannica.com/biography/Archimedes)

The centroid of the underwater volume of the ship is the location where the resultant buoyant force acts. This point is called the **center of buoyancy (B)** and is extremely important in static stability calculations (see Figures 5.1,5.2). The center of buoyancy is always located at the centroid of the submerged volume of the ship. When the ships lists to port or starboard, or trims down by the bow or stern, or changes draft, the shape of the submerged volume will change, thus the location of the centroid will move and alter the center of buoyancy. As opposed to center of flotation, the center of buoyancy is always located below the plane of the water. The distance of the center of buoyancy from the centerline of the ship is called the **transverse center of buoyancy (TCB)**. When the ship is upright the center of buoyancy is located on the centerline so that the TCB = 0. The vertical location of the center of buoyancy from the keel (or baseplane) is written as **KB**. The distance of the center of buoyancy from amidships (or the forward or aft perpendicular) is called the **longitudinal center of buoyancy (LCB)**. Similar to LCF, LCB is referenced from forward or aft perpendiculars and if referenced from amidships, the distance forward or aft of must be annotated (+ve FWD and -ve AFT).



(a) (b)

Figure 5.2 Concepts of floatation and stability (a) displacement curve.; The concept of static stability[5]. Image Credits: (Stokoe, 2012)

A ship displaces her own weight when afloat. This **body weight (w)** acts downward through the **center of gravity of the body (G)** and is resisted by an **upward buoyant force** (equal to w), which acts through the center of buoyancy (**B**) (see Figure 5.3). B is the geometric center of the submerged volume displaced by the ship. The **metacenter (M)** is the point through which all vertical forces are said to act. According to Archimedes the actual "all up weight" of a ship and its contents is equal to the weight of water displaced by the hull. This is referred to as **displacement volume**. A vessel's displacement varies over a range of conditions from extreme lightship to a deep, heavy-loaded condition. For example, the displacement alters as cargo or ballast is loaded or discharged or as fuel is consumed.

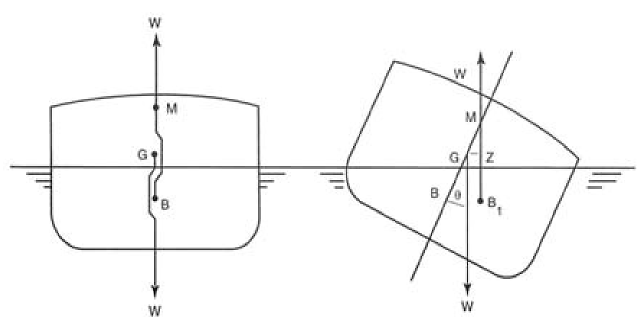


Figure 5.3 Righting lever

[5] Theoretically, it is possible to balance a pencil on its sharp end if the center of gravity is vertically above this end, but in practice, this is impossible as any disturbance will cause a loss of stability and the pencil will fall. Contrarily, it is possible to balance a pencil on its flat end even if there is a slight disturbance, the center of gravity will still lie within the limits of the flat base and the pencil may return upright again.

If the ship is stable, the centers of buoyancy and gravity strive at all times to remain vertically aligned. When a stable ship is caused to heel by an external force, such as wind, wave, or turning motion (not weight shift), the consequent change in underwater hull shape will result in (B) moving to one side while (G) does not move. The horizontal separation of (B) and (G) is referred to as the righting lever, GZ (see Figure 5.3), and the resulting righting moment, (w x GZ), will cause the vessel to oscillate from side to side as it attempts to realign (B) and (G). The measure of a ship's initial stability, when she is upright or nearly upright, is indicated by the height of the metacenter (M) above (G), which is referred to as the **metacentric height (GM).** The horizontal distance (GZ), more accurately indicates the measure of stability at **angles of heel (*θ*)** in excess of 5 degrees from the vertical plane ordinate. **GZ is referred to as the measured of static stability**. Should the ship's center of gravity (G) coincide with (M) and an external force is applied, the ship will assume an ''**angle of loll**''. In real conditions the ship will maintain the assumed angle until a further force is applied. Should another external force be applied, the ship may assume an angle of loll to the other side or may worsen the existing loll condition if there is no righting lever, GZ, to correct the assumed heel angle. In this condition the ship is said to have zero or no GM (see Figure 5.4).

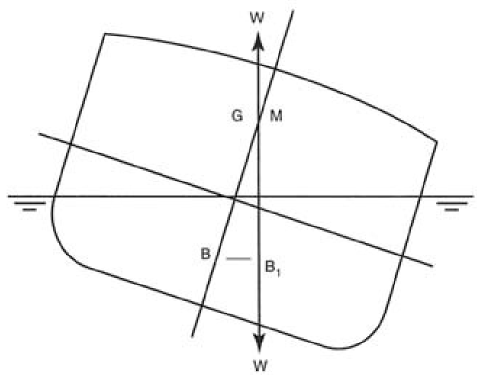


Figure 5.4 Condition of zero GM

As shown in Figure 5.3, when the vessel is upright, the center of buoyancy, (B), and the center of gravity of the ship, (G), remain in line and the ship is transversely stable, even with slack; that is, even with the ballast tanks partially full. However, if an external force is applied (because of wind, wave action or the ship turning) and the ship is caused to heel, the center of gravity of the water in the ballast tanks (G_1) and (G_2) will move to positions (G_3) and (G_4) ; thus resulting in the center of gravity of the ship moving from (G) to (G_5) (see Figure 5.5).

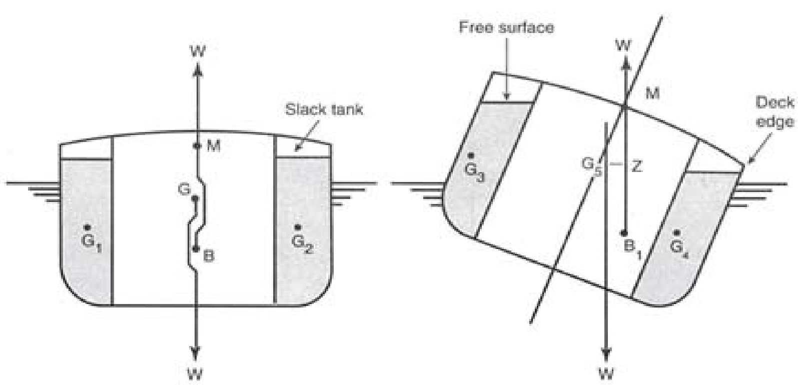


Figure 5.5 Reduced GM with slack tanks (free surface effect).

This movement reduces the original righting lever, GZ, and the resultant height of the ship's GM. The ship therefore becomes less transversely stable. Should the ship, for whatever reason, continue to heel, the position of (G_5) will continue to move due to the movement of the COG of the affected ballast tanks, thereby reducing the vessel's GM and the resulting GZ. If the deck edge becomes immersed, then B will move inboard, the effect of which will again reduce GM and the resultant righting moment, GZ. This change will make the vessel become very tender and she will possibly flop from side to side.

If more tanks are made slack or there is a cargo shift due to the excessive angle of heel, it is possible for (B) and (G_5) to reverse positions. Should this occur, GZ will be converted from a righting lever into a coupled turning force, which could cause the ship to capsize. This condition is known as negative GM (see Figure 5.6).

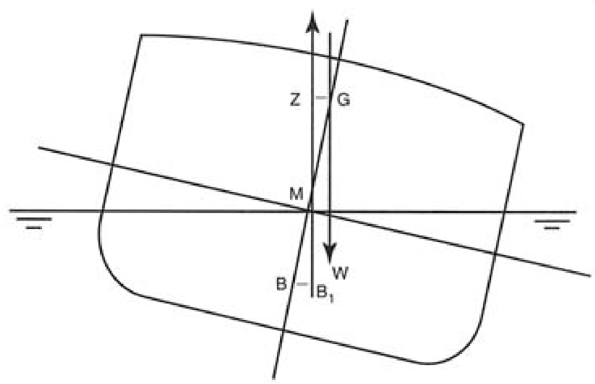


Figure 5.6 Condition of negative GM that may lead to ship capsize

2. Basic hydrostatic calculations

After defining the hull form lines plan ship designers calculate areas, volumes and their centers that are enclosed by the hull lines (see Lecture 4). To get the center of volume of those, we have to calculate the first moment of areas about a chosen reference axis (aft perpendicular - AP, amidships or forward perpendicular - FP). The second moment of area (moment of inertia about a reference axis) is also needed in some of these calculations. Properties could be calculated analytically for uniform shapes. However, it is very difficult to define hull mathematical expressions as the form has three-dimensional curvatures.

In practice, numerical integration methods may be used. In these methods non-uniform curves are approximated using a series of uniform curves; a series of straight lines or higher order curves. Areas are usually defined by dividing a space into equal sections using parallel lines representing stations. Results are obtained by different numerical integration methods. In the **trapezoidal rule method**, the points of intersection of the parallel lines with the area perimeter are joined by straight lines, so that each section could be represented by a trapezoid. The final area is a summation of this set of trapezoids (Figure 7). When hand based or spreadsheet numerical calculations are used Simpson's 1st Rule is the preferred numerical integration technique used to calculate geometric properties. This is because it is less tedious than higher order integration methods (e.g. Simpson's 2nd Rule, Integration over Bonjean curves) and gives good enough accuracy when using a small number of points. Further details are given in the following section.

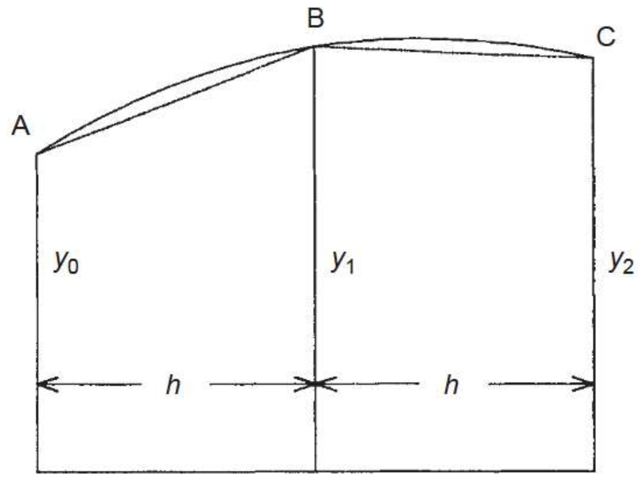


Figure 5.7 Trapezoidal rule. Image Credits: (Tupper, 2013)

2.1 Simpson's 1st Rule

Simpson's rule approximates the ship curves by 3rd order polynomial functions, i.e. $y = a_0 + a_1x + a_2x^2 + a_3x^3$. Vertical divisions are represented by three equally spaced ordinates y_0, y_1 and y_2 (see Figure 135.8). The derivation finally leads to the formula :

$$A = \frac{h}{3}(y_0 + 4y_1 + y_2) \quad (5.1)$$

It may be generalized for any odd number of equally spaced ordinates with distance h as follows :

$$A = \frac{h}{3}(y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4 + y_4 + ..) = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_{n-1} + y_n) \quad (5.2)$$

For classic merchant ship designs, 10 divisions of the ship's hull with 11 ordinates may be considered sufficiently accurate. Calculating areas in a tabular format results to better accuracy. The distance h is termed the common interval and the numbers 1, 4, 2, 4, etc. are termed Simpson's multipliers.

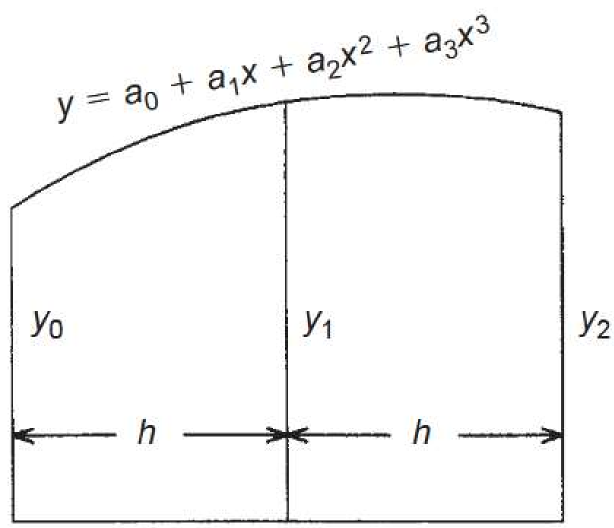


Figure 138 Simpson's first rule. Image Credits: (Tupper, 2013)

Problem statement 1 – area integration: Let us assume equally spaced half-ordinates of a waterplane section for a symmetric mono - hull vessel that is 27m long. Ship ordinates are positioned at 1.1, 2.7, 4, 5.1, 6.1, 6.9 and 7.7 m respectively. *How can we calculate the waterplane area ?*

Solution : The ship is symmetric about her centerline. So, half-ordinates may be used and then the area calculated can be multiplied by 2. The Simpson's integration ordinates, multipliers and area products are given on Table 5.1.

½ ordinates	Simpson's multiplier	Product for area
1.1	1	1.1
2.7	4	10.8
4	2	8
5.1	4	20.4
6.1	2	12.2
6.9	4	27.6
7.7	1	7.7
		$\sum_A = 87.8$

Table 4.1 Simpson's integration table

There are seven ordinates with 6 spaces, therefore,

$$h = \frac{27}{6} = 4.5\ m \text{ and } A = \frac{h}{3}\Sigma_A \quad \times 2 = \frac{4.5}{3} \times 87.8 \times 2 = 263.4\ m^2$$

As seen in the example, once you have ordinates of any waterplane along the ship's vertical position, you can calculate its area using the same rule. The center of area and the moment of inertia about it are also required to calculate the 1st and 2nd moments of area. As we explained before the centroid of the waterplane area at the waterline is the center of floatation and its longitudinal position is denoted by LCF.

The next step is to calculate the centroid of the waterplane area and the second moment of area about the aft perpendicular and about the centroid. Since the centroid is near to amidships, the reference axis is usually taken at amidships. In this example, for simplicity, let us assume the axis will be in way of the aft perpendicular. The centroid of an area is defined as :

$$C = \frac{\text{First moment of area @ reference axis}}{Area} \qquad (5.3)$$

Thus the first moments of area may be calculated as shown in Table 5.2. The area according to Simpson's rule is given by $A = \frac{h}{3}(\Sigma_A \quad)$,and the first moment of area is $\frac{h}{3}(\sum MA \times h) = \frac{h^2}{3}\sum MA$ .The center of area is $\frac{\frac{h^2}{3}\sum MA}{\frac{h}{3}\Sigma_A} = h\frac{\sum MA}{\Sigma_A}$ and the 2nd moment of area about aft perpendicular is $I_{AP} = \frac{h}{3}\sum i \times h^2 = \frac{h^3}{3}\sum i$. Using parallel axis theorem, the second moment of area about the center of floatation is obtained as follows: $I_F = I_{AP} - Ax^2$ where, x is the distance between the aft perpendicular and the center of floatation. Results are summarized in Tables 5.2, 5.3.

½ ordinates	Simpson's multiplier	Product for area	Lever@ AP	Product for 1st moment	Lever@ AP	Product for 2nd moment
y_0	1	$1y_0$	$0h$	0	$0h$	0
y_1	4	$4y_1$	$1h$	y_1h	$1h$	y_1h^2
y_2	2	$2y_2$	$2h$	$2y_2h$	$2h$	$4y_2h^2$
y_3	4	$4y_3$	$3h$	$3y_3h$	$3h$	$9y_3h^2$
y_4	2	$2y_4$	$4h$	$4y_4h$	$4h$	$16y_4h^2$
y_5	4	$4y_5$	$5h$	$5y_5h$	$5h$	$25y_5h^2$
y_6	1	$1y_6$	$6h$	$6y_6h$	$6h$	$36y_6h^2$
		$\sum_A$		$\sum MA \times h$		$\sum i \times h^2$

Table 5.2 First moment of area

½ ordinates	Simpson's multiplier	Product for area	Lever@ AP	Product for 1st moment	Lever@ AP	Product for 2nd moment
1.1	1	1.1	0	0	0	0
2.7	4	10.8	1	10.8	1	10.8
4	2	8	2	16	2	32
5.1	4	20.4	3	61.2	3	183.6
6.1	2	12.2	4	48.8	4	195.2
6.9	4	27.6	5	138	5	690
7.7	1	7.7	6	46.2	6	277.2
		$\sum_A = 87.8$		$\sum MA = 321$		1388.8

Table 5 Area centroid and second moment of area calculation

The centroid of area LCF= $4.5 \times \frac{321}{87.8} = 16.452\ m$ from the AP. The same rule can be applied to midship section or any section along the ship's longitudinal position. The second moment of area of the waterplane is evaluated as :$2 \times (\frac{h^3}{3} \sum i) = 2 \times \frac{(4.5)^3}{3} \times 1388.8 = 84369.6\ m^4$. Thus, the second moment of area about the centroid becomes : $I_F = I_{AP} - Ax^2 = 84369.6 - 263.4(16.452)^2 = 13075.57\ m^4$.

Problem statement 2 – volume intergration: Assume that the immersed cross-sectional areas of a ship that is 180m long, are positioned at equal intervals namely 5, 118, 233, 291, 303, 304, 304, 302, 283, 171, and 0 m^2. Follow similar overall methodology to problem statement 1 to *calculate the displacement and LCB in sea water of 1.025* $tonne / m^3$.

Station	Cross-sectional areas	SM	Product for volume	Level @AP	Product for 1st moment
AP	5	1	5	0	0
1	118	4	472	1	472
2	233	2	466	2	932
3	291	4	1164	3	3492
4	303	2	606	4	2424
5	304	4	1216	5	6080
6	304	2	608	6	3648
7	302	4	1208	7	8456
8	283	2	566	8	4528
9	171	4	684	9	6156
FP	0	1	0	10	0
			$\sum_V = 6{,}995$		$\sum MV = 36{,}188$

Table 6 Displaced volume and LCB calculation table

$h = \frac{180}{10} = 18\,m$

Volume of displacement, $\nabla = \frac{h}{3}\sum_V \quad = \frac{18}{3} \times 6995 = 41970\,m^3$; this displacement $\Delta = \rho \times \nabla = 1.025 \times 41970 = 43019$ tonnes and the $LCB = h\frac{\sum MV}{\sum_V} = 18 \times \frac{36188}{6995} = 93.12m$ from AP

2.2 Simpson's 2nd order Rule

Simpson's 2nd order rule may be used with even number of equally spaced ordinates and the area given as a function of 4 ordinates as follows:

$$A = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_4) \qquad (5.4)$$

The above mathematical expression can be extended to cover 7, 10, 13 and so on ordinates, becoming:

$$A = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + \ldots + 3y_{n-1} + y_n) \qquad (5.5)$$

The method can be applied in the same fashion to 1st rule as describe in section 5.2.2 and 5.2.4. It is noted that as an alternative Gauss rules may be used when uneven ordinate spacing and multipliers are useful or simply for greater accuracy. The Gauss approach is not often used in manual calculations as it is mathematically complex. In computer programs this is not such a problem and they may be preferred to obtain a greater accuracy.

3. Curves of form

Bonjean curves are used to idealise and estimate underwater volume. They are immersed cross-sectional areas plotted against **T** for each transverse section. They are represented on a ship's profile as shown in Figure 4.1,4.2 and 5.9. The immersed areas for such a waterline are obtained by drawing a horizontal line that intersects horizontally (cuts across) all transverse sections. The immersed sectional area value for such e section is the distance from the section position to the intersection point of a Bonjean curve with the waterline. Using Bonjeans displacement and **B** can be calculated for inclined waterlines. This may be useful in trim analysis and ship launching calculations.

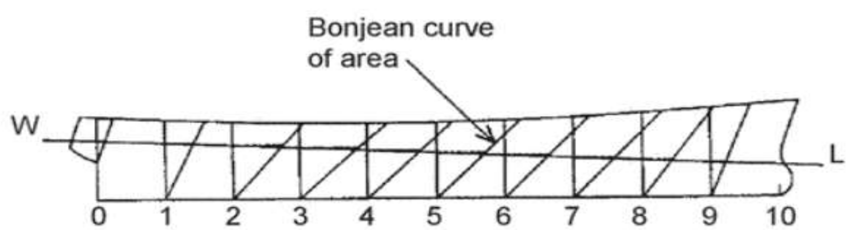


Figure 5.9 Bonjean Curves. Image Credits: (Tupper, 2013)

Consider a symmetric ship that heels by a very small angle $\delta\phi$ (Figure 14.5); **B** moved off the ship's centerline but w remains unchanged (see Figure 5.10). The distance between the lines of action of weight and buoyancy is the known by the **righting arm (GZ)**. A vertical line that passes through the new position of the center of buoyancy will intersect the centerline of the ship at metacenter **M**. Transverse metacenter (**M_T**) relates to heeling; Longitudinal metacenter (**M_L**) relates to trim. The location of this point will vary with the ship's displacement and trim, but, for any given drafts, it will always be in the same place. Unless there is an abrupt change in the shape of the ship in the vicinity of the waterline, point M will remain practically stationary with respect to the ship. This is because the ship is inclined to small angles (7-10 degrees). The position of M - especially its height - is important indication of ship's stability. In practical ship design it is conventional to obtain the displacement, the position of the center of buoyancy B, the metacenter M and the center of floatation F for a set of waterlines parallel to the design waterline and plot them against the draught on the y-axis. This set of curves is called hydrostatic set curve (Figure 5.10). Each curve has her own scale along horizontal axis.

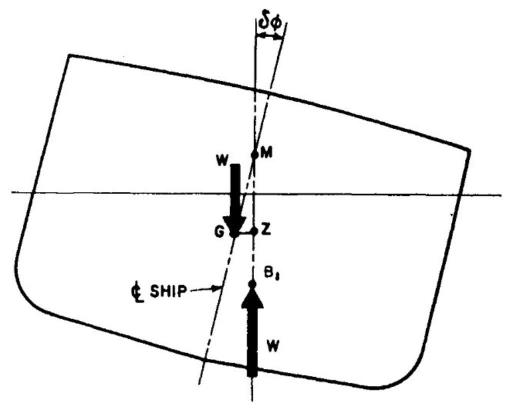


Figure 14.9 Metacenter. Image Credits: (Lewis, 1988)

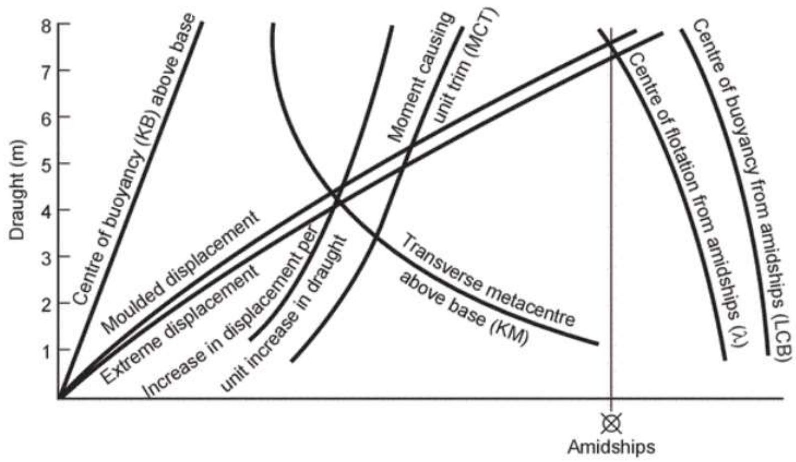


Figure 5.10 Hydrostatic curves: (Tupper, 2013)

4. IMO Intact stability requirements

The International Maritime Organisation (IMO) has formulated rules under the Safety of Life at Sea (SOLAS) Convention for ensuring adequate intact stability for merchant ships during operation. The main requirements of properties of the GZ curve are as follows.

- The initial metacentric height G M0 should not be less than 0.15 m.
- The righting lever GZ should be at least 0.20 m at an angle of heel equal to or greater than 30°.
- The maximum righting lever should occur at an angle of heel preferably exceeding 30° but not less than 25°.
- The area under the righting lever curve (GZ curve) should not be less than 0.055 m·rad up to $\phi = 30°$ angle of heel.

- The area under the righting lever curve should not be less than 0.09 m·rad up to $\phi = 40°$ or the angle of flooding θi if this angle is less than 40°.
- The area under the righting lever curve between the angles of heel of 30° and 40° or between 30° and ϕi, if this angle is less than 40°, should not be less than 0.03 m·rad.

The regulations also specify methods of estimation of loss of stability due to free surface in the tanks of the ship, amplitude and period of roll due to wind heeling moment and heel due to ship taking a turn. The regulations also specify satisfaction of stability criteria in all possible loading conditions – fully loaded, partially loaded, ballast and lightship conditions – both in departure and arrival states of the ship. The intact stability requirements of passenger vessels specify that heel due to overcrowding of passengers to one side should not exceed 10°. Similarly, severe wind conditions and turning at speed should ensure roll or heel within specified limits. Grain-carrying ships should ensure limitation of heel due to grain shifting moment. IMO also specifies additional regulations for different ship types other than conventional merchant ships (e.g. container ships. timber cargo vessels, fishing vessels etc.). This is because the classic IMO SOLAS intact stability conditions can be satisfied only up to a certain value of KG (VCG) and if the KG value crosses this limit, this condition would be violated.

5. The inclining experiment

As the position of the center of gravity is so important for initial stability it is necessary to establish it accurately. It is determined initially by calculation by considering all weights making up the ship (e.g. steel, outfit, fittings, machinery and systems) and by assessing their individual centers of gravity. From these data can be calculated the displacement and center of gravity of the light ship.

It is difficult to account for all items accurately in such calculations and it is for this reason that the lightship weight and center of gravity are measured experimentally. The experiment is called the inclining experiment and involves causing the ship to heel to small angles by moving known weights known distances transversely across the deck and observing the angles of inclination (see Figure 5.11). The draughts at which the ship floats are noted together with the water density. Ideally the experiment is conducted before ship launch. To obtain accurate results a number of precautions are taken. First the number of people onboard should be minimum and the experiment should be conducted in calm water with little wind and the ship must be floating freely when records are taken so any mooring lines must be slack and the brow must be lifted clear. All weights must be secure and tanks must be empty or pressed full to avoid free surface effects. If the ship does not return to her original position when the inclining weights are restored this is an indication that a weight has moved in the ship, or that fluid has moved from one tank to another, possibly through a leaking valve.

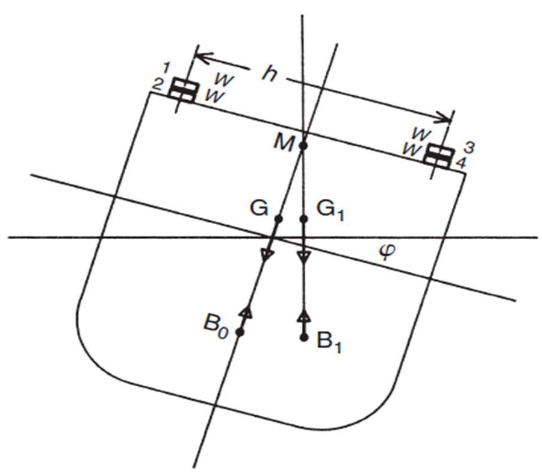


Figure 5.11 Set up of the inclining experiment (Tupper, 2013)