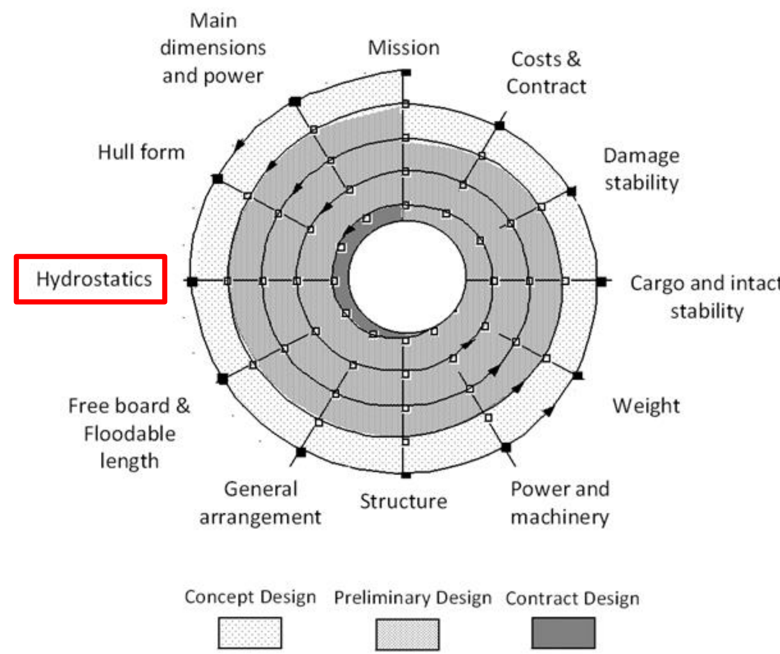
Aalto University
School of Engineering

MEC-E1004 Principles of Naval Architecture

Lecture 5 –Hydrostatics

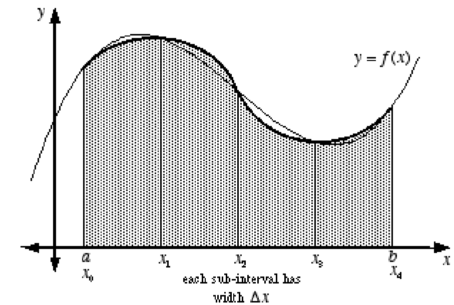
Learning points !

- Explain the purpose of numerical integration in ship design
- Explain and apply basic hydrostatic formulas and methods



Assignment 5 – Hydrostatics

- Create draft line drawings of your ship's hull
- Estimate the hull volume and verify the correctness of your calculations using numerical integration methods (e.g. Simpson Rules on frame area or hull volume)



The shaded area bounded by the parabolas (the thicker curves) is approximately equal to the area bounded by $y = f(x)$.

$$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

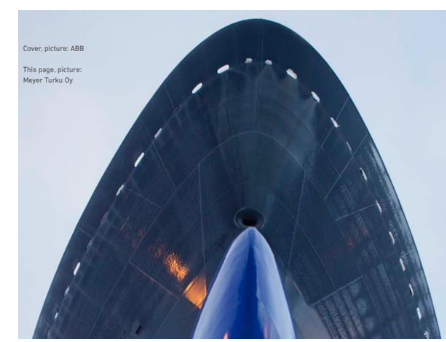


Image credit Meyer Turku

Hydrostatics

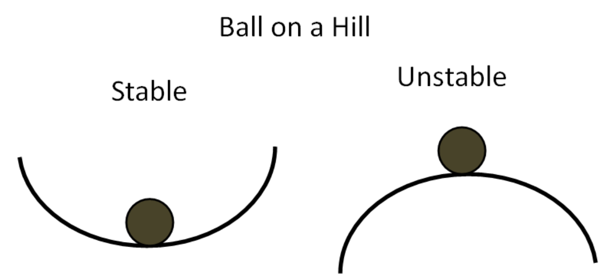
Question: What is hydrostatics and why they are important in ship design?

What is Hydrostatics ?

- A branch of physics that deals with the characteristics of fluids at rest. The fluid can be gas or liquid exerting pressure on an immersed body.

- Through hydrostatics we investigate the floating position

- Applied in ship design to assess a ship’s
  - *Floatation*
  - *Stability*



Ship Geometry

- The determination of a ship’s geometry is a complex task that requires the consideration of multiple factors

  - *Volumes, dimensions of cargo holds*
  - *Seakeeping, i.e safety in waves*
  - *Resistance, i.e. hull performance and energy efficiency*
  - *Aesthetics*
  - *...*

- Accurate representation of geometry is important for

  - *Safety and Performance assessment*
  - *Manufacturing*

- Challenge

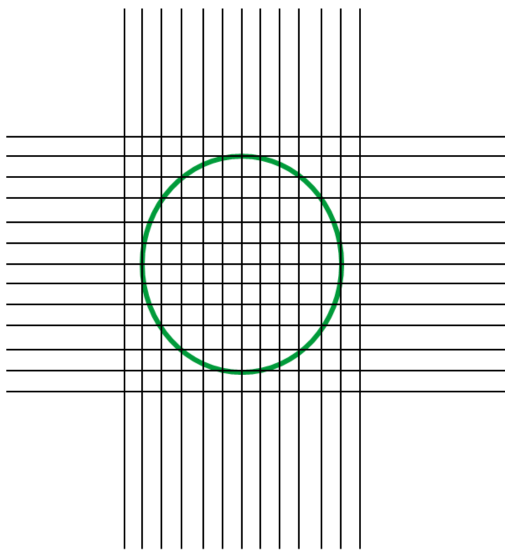
  - *Complex hull shapes*
    - Double curvature in many places
      - Bended beams, plates,..

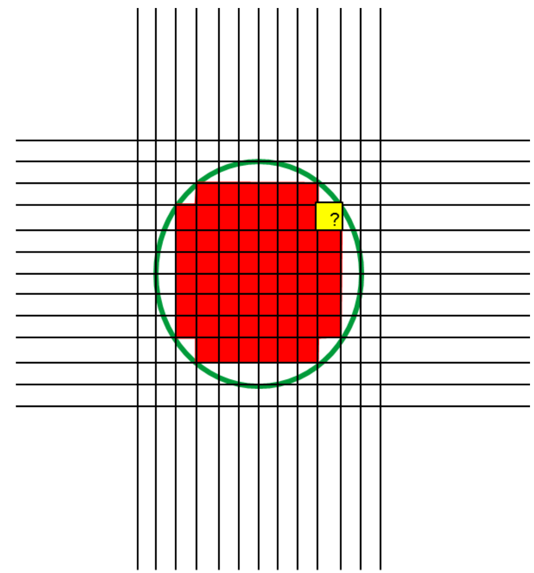
Exact methods vs numerical aprox.

$A = pr^2 = p(5)^2 = 25p^2$ » 78.5

$A \approx whole\ \ squares + \frac{1}{2}\ part\ \ squares = 59 + \frac{29}{2} = 73.5$

A » $60 + \frac{28}{2} = 74$





Numerical integration

- Integration is nowadays carried out using computers, e.g.
  - *AutoCAD*
  - *SolidEdge*
  - *Plot Digitizer*
  - *NAPA*
  - *etc…*

- Numerical estimations do not always represent reality

- Useful for any geometry (even geometries which do not have analytical solutions)

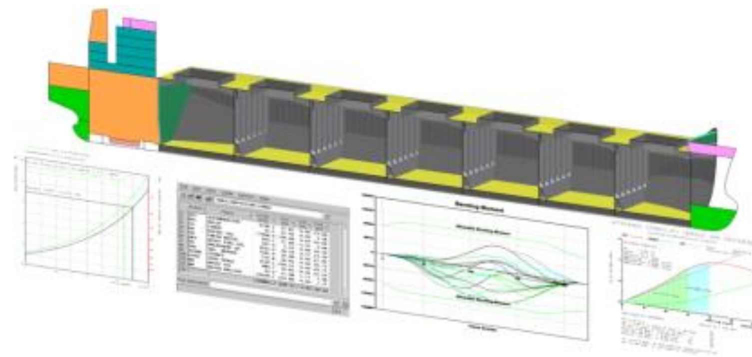
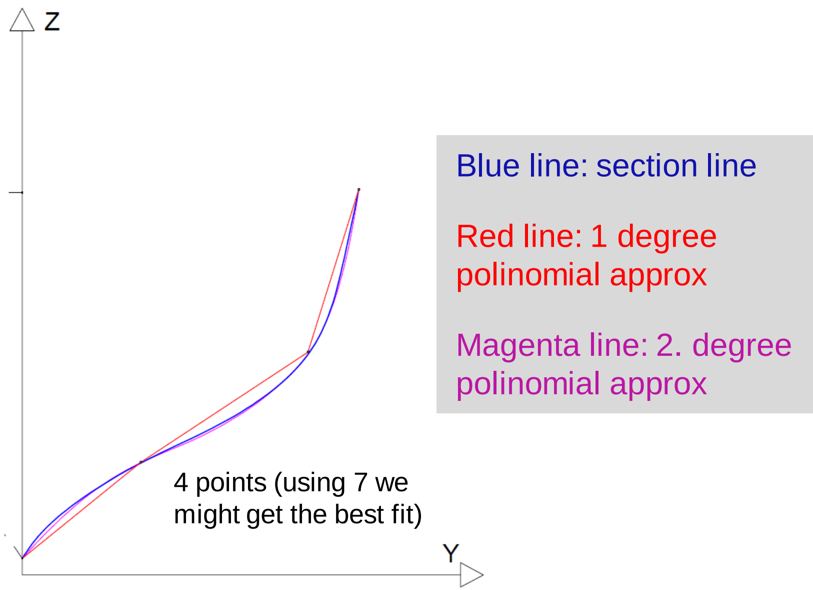
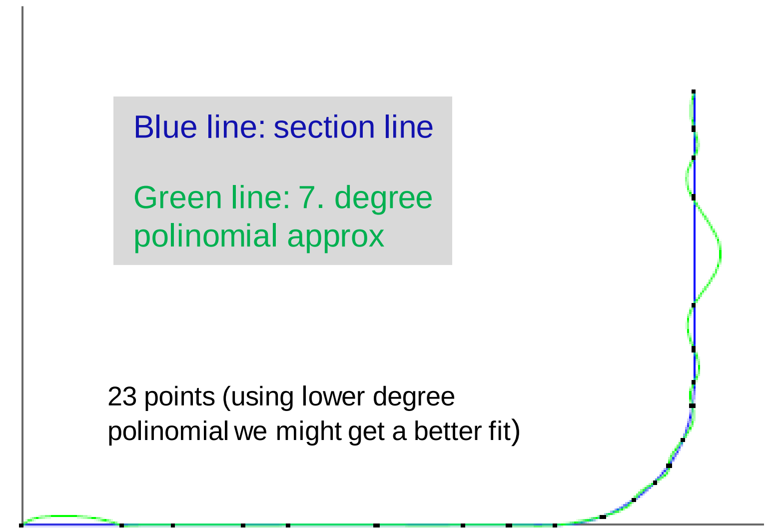


Image credit Napa

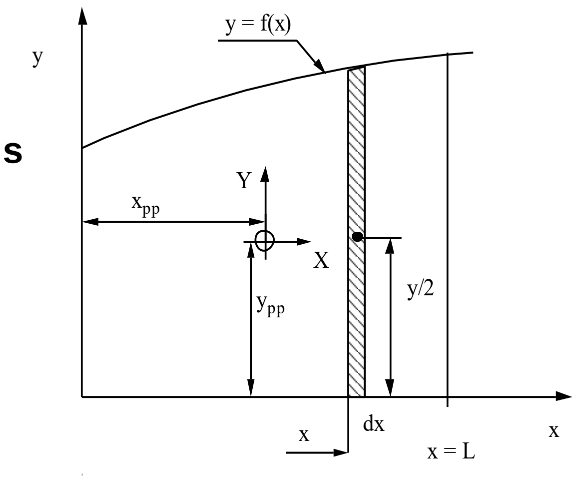
Numerical integration

How many points?...too many is not necessarily good !





Numerical integration



Area

$$A = \int_0^A dA = \int_0^L y dx$$

Distance from y-axis

$$x_{pp} = \frac{M_y}{A}$$

Distance from x-axis

$$y_{pp} = \frac{M_x}{A}$$

Moment of area A with respect to y-axis M_y

$$M_y = \int_0^A \left(x + \frac{dx}{2} \right) dA = \int_0^L \left(x + \frac{dx}{2} \right) y dx = \int_0^L \left(xydx + \underbrace{\frac{y(dx)^2}{2}}_{\approx 0} \right) \approx \int_0^L xydx$$

Moment with respect to x-axis

$$M_x = \int_0^A \frac{y}{2} dA = \int_0^L \frac{y}{2} y dx = \frac{1}{2} \int_0^L y^2 dx$$

Parallel axis theorem

$$I_Y = I_y + x_{pp}^2 A$$

Second moment of area (A) with respect to x-axis

$$I_x = \int_0^A y^2 dA = \int_0^L \int_0^y y^2 dydx = \frac{1}{3} \int_0^L y^3 dx$$

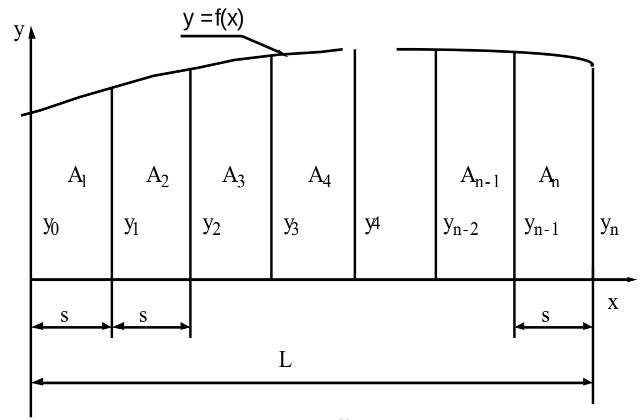
Second moment of area (A) with respect to y-axis

$$I_y = \int_0^A x^2 dA = \int_0^L x^2 ydx$$

Numerical integration

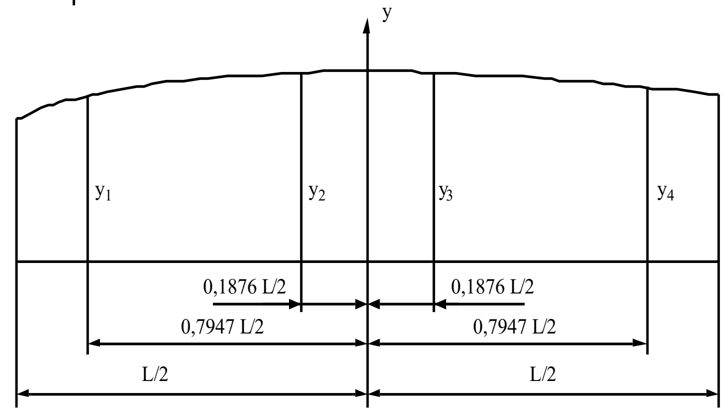
If evenly spaced ordinates

- Rectangle rule (Piecewise constant estimation of curve)
- Trapezoidal rule (Piecewise linear estimation of curve)
- Simpson I rule
- Simpson II rule



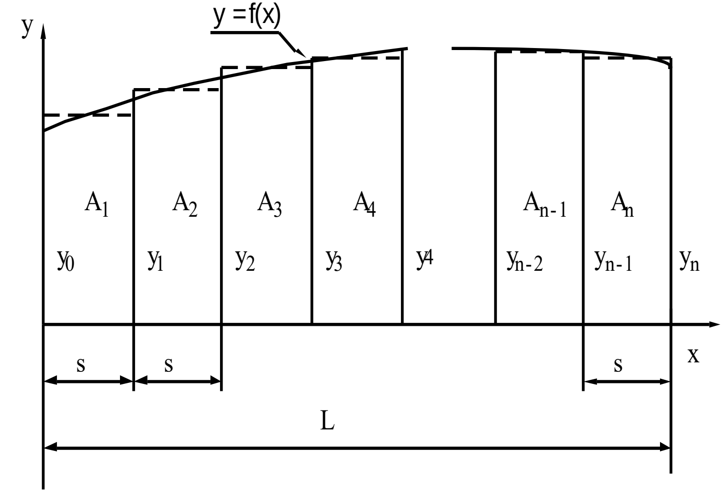
If unevenly spaced ordinates

- Tsebysev rule
- Gauss rule (used in FEM)



https://link.springer.com/bookseries/10523

Numerical integration



$$A_i = \frac{1}{2} s\left(y_{i-1} + y_i\right)$$

$$A = \sum_{i=1}^{n} A_i = A_1 + A_2 + A_3 + \ldots + A_n$$

$$= \frac{1}{2} s\left(y_{1-1} + y_1\right) + \frac{1}{2} s\left(y_{2-1} + y_2\right) + \frac{1}{2} s\left(y_{3-1} + y_3\right) + \ldots + \frac{1}{2} s\left(y_{n-2} + y_{n-1}\right) + \frac{1}{2} s\left(y_{n-1} + y_n\right)$$

$$= \frac{1}{2} s\left(y_0 + y_1\right) + \frac{1}{2} s\left(y_1 + y_2\right) + \frac{1}{2} s\left(y_2 + y_3\right) + \ldots + \frac{1}{2} s\left(y_{n-2} + y_{n-1}\right) + \frac{1}{2} s\left(y_{n-1} + y_n\right)$$

$$= s\left[\frac{1}{2}\left(y_0\right) + \frac{1}{2}\left(y_1 + y_1\right) + \frac{1}{2}\left(y_2 + y_2\right) + \frac{1}{2} y_3 + \ldots + \frac{1}{2}\left(y_{n-2}\right) + \frac{1}{2}\left(y_{n-1} + y_{n-1}\right) + \frac{1}{2} y_n\right]$$

$$= s\left[\frac{1}{2}\left(y_0\right) + y_1 + y_2 + \ldots + y_{n-1} + \frac{1}{2} y_n\right]$$

Numerical integration – Simpson I

The spacing needs to be even, because the solution of parabolic approximation requires solution of coefficients a, b and c based on three consecutive coordinates

$$\begin{cases} y_0 = a\cdot 0 + b\cdot 0 + c \\ y_1 = as^2 + bs + c \\ y_2 = 4as^2 + 2bs + c \end{cases} \Leftrightarrow \begin{cases} c = y_0 \\ b = \dfrac{4y_1 - y_2 - 3y_0}{2s} \\ a = \dfrac{y_2 - 2y_1 + y_0}{2s^2} \end{cases}$$

Equation for parabola

$$y = ax^2 + bx + c$$

Known values

x=0

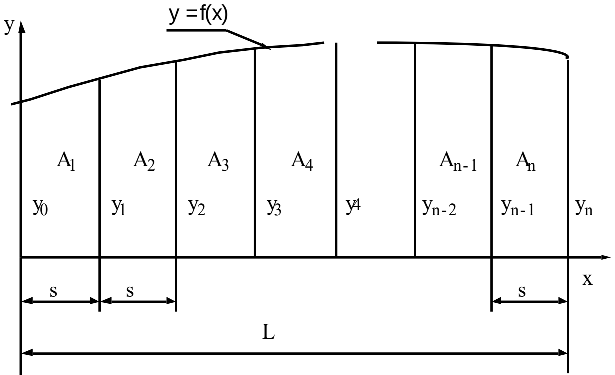
x=s

x=2s

So area for two areas is

$$A = A_1 + A_2 = \int_0^{2s} y\,dx = \int_0^{2s} \left(ax^2 + bx + c\right)dx = \frac{8}{3}as^3 + 2bs^2 + 2sc$$

$$= \frac{4}{3}\left(y_2 - 2y_1 + y_0\right)s + \left(4y_1 - y_2 - 3y_0\right)s + 2s\left(y_0\right) = \frac{s}{3}\left(y_0 + 4y_1 + y_2\right)$$



And the whole area investigated

$$A = \left(A_1 + A_2\right) + \left(A_3 + A_4\right) + \ldots + \left(A_{n-1} + A_n\right)$$

$$= \left[\frac{s}{3}\left(y_0 + 4y_1 + y_2\right)\right] + \left[\frac{s}{3}\left(y_2 + 4y_3 + y_4\right)\right] + \ldots + \left[\frac{s}{3}\left(y_{n-2} + 4y_{n-1} + y_n\right)\right]$$

$$= \frac{s}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

Reference : Kreyszig, Erwin. Advanced Engineering Mathematics. New York :Wiley, 10th Edition., see p. 832

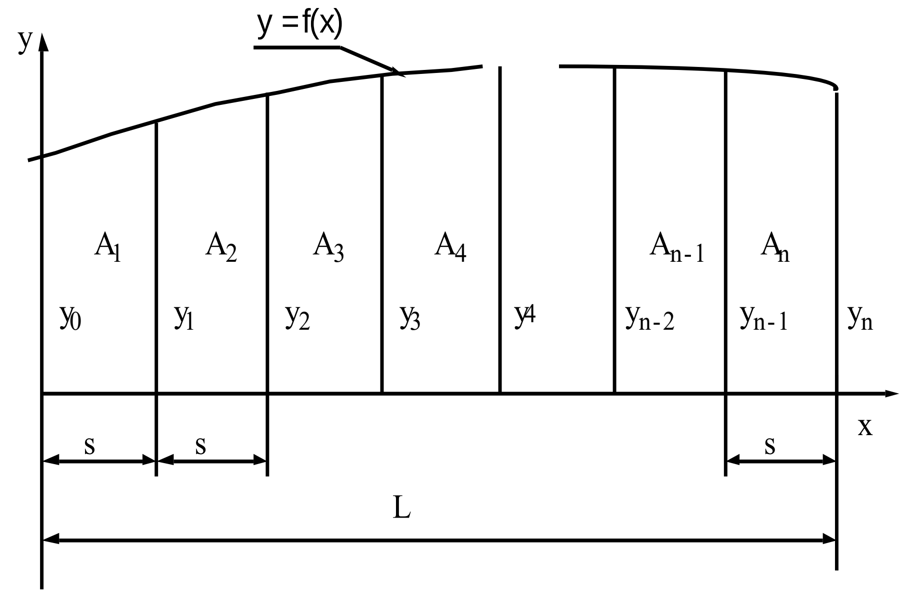
Numerical integration – Simpson II

- Third order polynomial calls for 4 ordinates
- Spacing n should be multiple of 3

$$A_1 + A_2 + A_3 = \frac{3}{8} s\left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

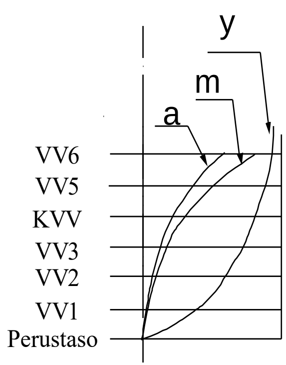
$$A = \frac{3}{8} s\left(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + \ldots + 3y_{n-1} + y_n\right)$$

$$\underbrace{\qquad\qquad}_{A_1 + A_2 + A_3} \quad \underbrace{\qquad\qquad}_{A_4 + A_5 + A_6}$$



Kreyszig, Erwin. Advanced Engineering Mathematics. New York :Wiley, 10th Edition., see p. 832

Numerical integration – use of tables



Frame area

kaari no	puoli-leveys y_n (m)	1/2S Mkerroin	A-tulo	M-varsi	M-tulo	varsi 2	I_L-tulo	y_n^3	I_T-tulo
1	**2**	**3**	**4 =2*3**	**5**	**6= 2*3*5**	**7**	**8= 2*3*7**	**9**	**10= 3*9**
0		1/4		-5,0		25,0			
1/2		1		-4,5		20,25			
1		1/2		-4,0		16,0			
1 1/2		1		-3,5		12,25			
2		3/4		-3,0		9,0			
3		2		-2,0		4,0			
4		1		-1,0		1,0			
5		2		0		0			
6		1		1,0		1,0			
7		2		2,0		4,0			
8		3/4		3,0		9,0			
8 1/2		1		3,5		12,25			
9		1/2		4,0		16,0			
9 1/2		1		4,5		20,25			
10		1/4		5,0		25,0			
			Σ_1		Σ_2		Σ_3		Σ_4

Volume of the Ship

kaari no	kaaren a (m^2)	1/2S Mkerroin	∇-tulo	M-varsi	M_L-tulo	kaaren m (m^3)	M_V-tulo
1	**2**	**3**	**4 =2*3**	**5**	**6= 2*3*5**	**7**	**8=3*7**
0		1/2		-5,0			
1/2		2		-4,5			
1		1		-4,0			
1 1/2		2		-3,5			
2		3/2		-3,0			
3		4		-2,0			
4		2		-1,0			
5		4		0			
6		2		1,0			
7		4		2,0			
8		3/2		3,0			
8 1/2		2		3,5			
9		1		4,0			
9 1/2		2		4,5			
10		1/2		5,0			
			Σ_1		Σ_2		Σ_3

Hydrostatic curves

A series of graphs that give values such as the center of buoyancy, displacement, moment causing unit trim, and center of flotation

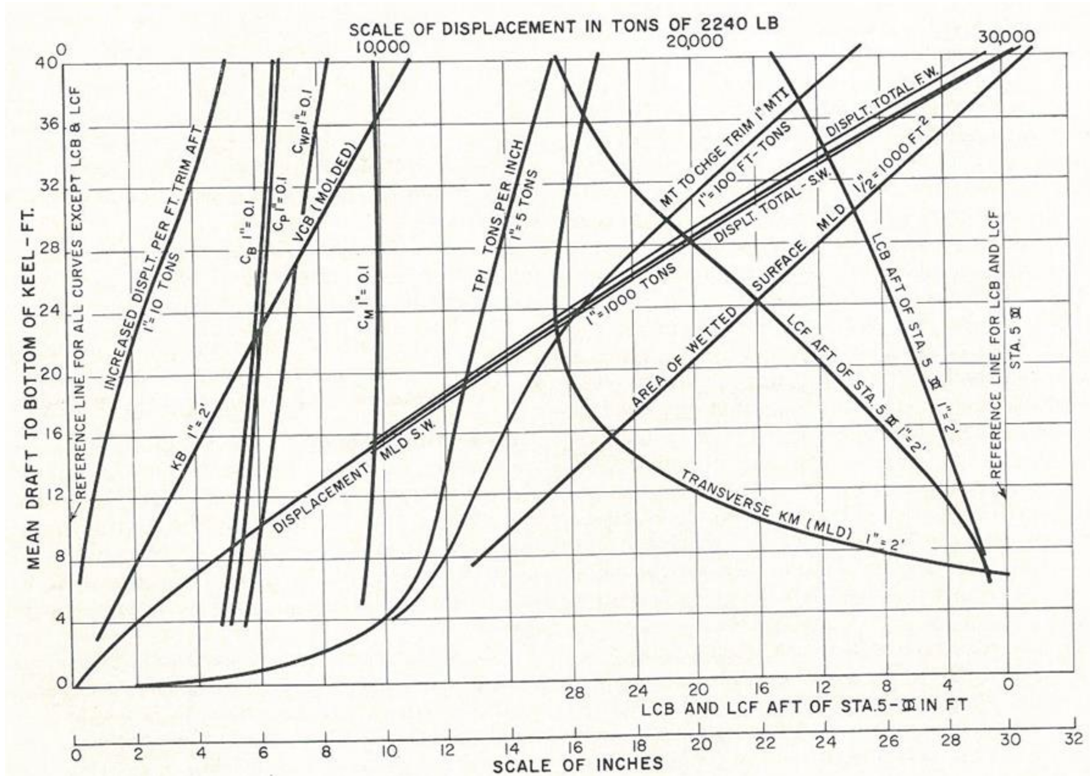


Image credit Principles of Naval Architecture

Bonjean curves

They are used for the purpose of obtaining, for any given waterline, the areas of the immersed portion of each transverse section throughout the ship's length

Example

- Path KCL_1W_1K equals frame area A at point draught Q
- This can be used to estimate the buoyancy V=A*L of this section having length L

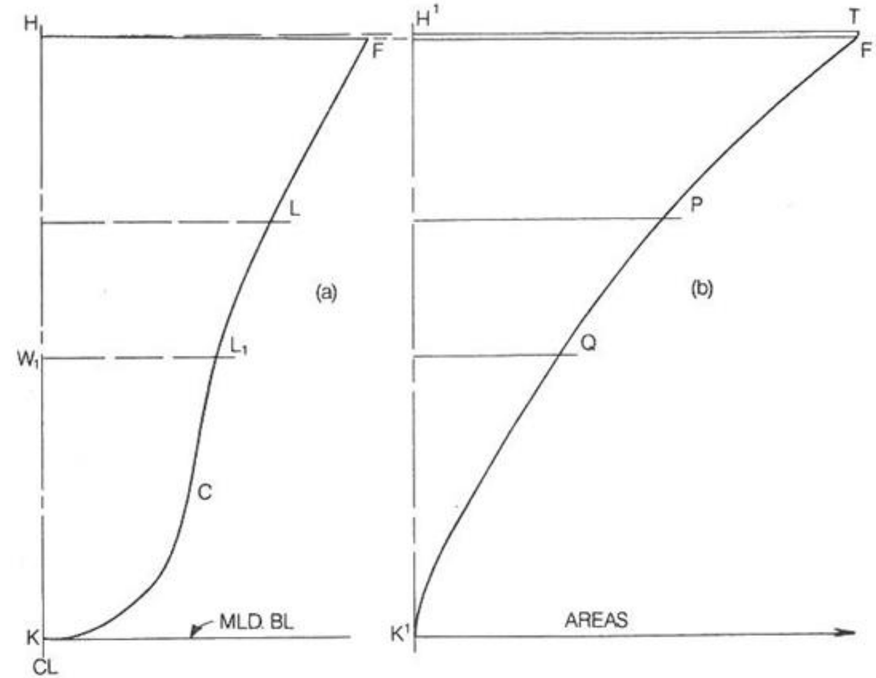
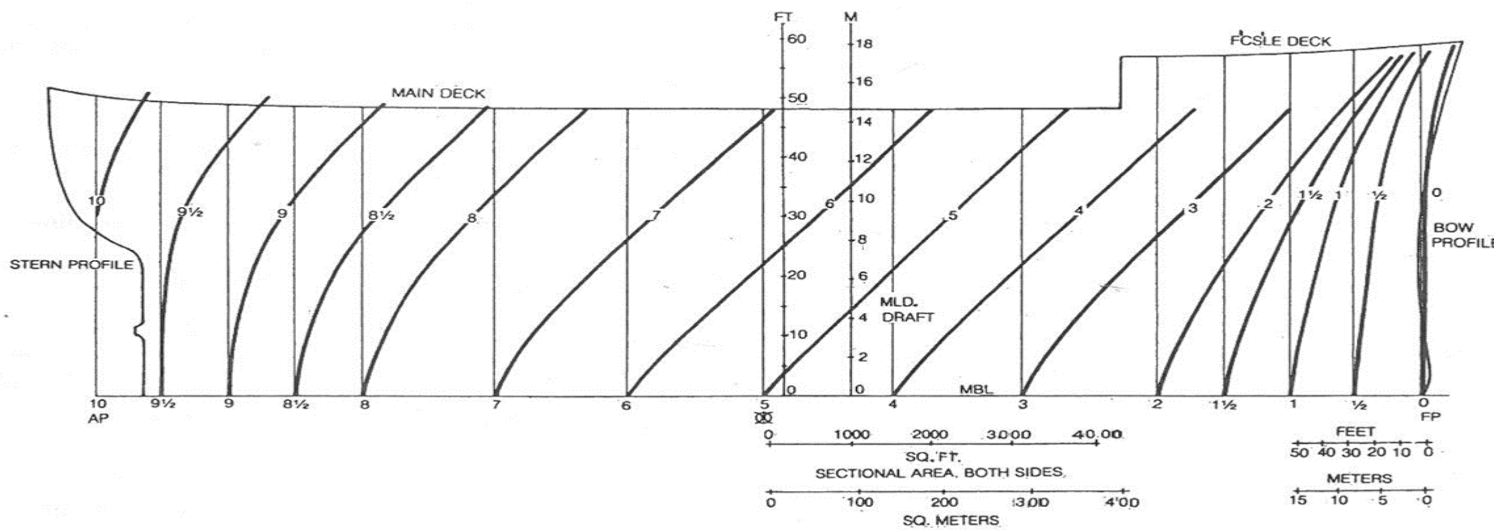


Fig. 32 Body plan section (a) and Bonjean Curve (b)

Bonjean curves, example



Strip theory

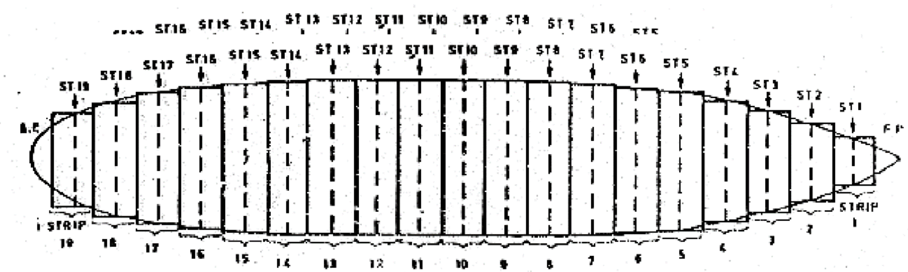
A classic method by Korvin-Kroukovsky (1955, 1957) to evaluate the loads on a hull girder in regular waves

$$\sum_{k=1}^{6}\left[\left(M_{jk}+A_{jk}\right)\eta_k + B_{jk}\eta_k + C_{jk}\eta_k\right] = F_j e^{-i\omega t},\ j=1,2,\ldots,6$$

$$\left|H(\omega)\right|^2 = \left[\frac{Y(\omega)}{A(\omega)}\right]^2$$

Assumption:

- The flow in length direction is negligible → the hull can be divided into strips describing 2D phenomena
- Straight hull sides



Summary

- **Ship Hydrostatics relate with complex hull shapes.**

- **Numerical intergration methods help with the determination of areas, volumes, moments etc.** These are used to assess ship characteristics such as **:**
  - Buoyancy
  - Wetted surface
  - Center of buoyancy
  - …etc.