Sectional Modulus Tutorial

In the lecture notes, we explain how the sectional modulus is important in the hull girder analysis, here we recall the equations relate the hull bending to the sectional modulus:

$$\sigma = \frac{Mz}{I}$$

The sectional modulus: $Z_{deck} = \frac{I}{z_{deck}}$, and $Z_{keel} = \frac{I}{z_{keel}}$ where z_{deck} and z_{keel} are the distance from the neutral axis of the ship section considered to the deck and the bottom as they are the farthest from the neutral axis and they are likely to have the highest bending stresses. In order to calculate the sectional modulus, we shall calculate the first and second moment of area of each structural component (only those contribute in the longitudinal stiffness) about its centroid and the neutral axis of the section.

The second moment of area of such a structural component will be at first calculated at the component's centroid then at the neutral axis using the parallel axis theorem.

$$I_x = I_o + A \cdot h^2$$

Where I_x is the moment of inertia about the neutral axis of the section, I_o is the moment of inertia about the centroid of the component, A is the cross-sectional area of the structural component, and h is the distance from the centroid to the neutral axis.

The main procedure to be followed to calculate the sectional modulus is shown below:

1. Calculate the area of each component (a_j).
2. Take the baseline as your first reference line and calculate the height of each component's area centroid above the baseline (h_j).
3. Calculate the 1st moment of area about the baseline ($a_j h_j$).
4. Calculate the 2nd moment of area of each component about the baseline ($a_j h_j^2$).
5. Calculate the moment of inertia of each component about its own horizontal axis passing through the centroid "NA" (i).

   **The moment of inertia of the plate or any rectangular cross-section is equal to: $i = \frac{bd^3}{12}$

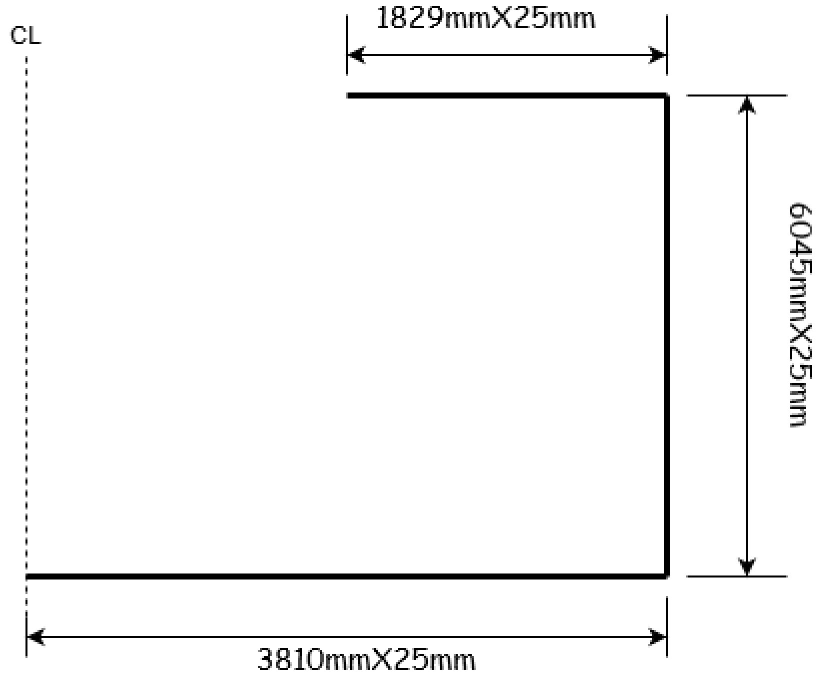
   where b is the breadth of the plate (in other words, the dimension parallel to the neutral axis) and d is the depth of the plate (in other words, the dimension perpendicular to the neutral axis).

6. Calculate the distance of the neutral axis above the baseline ($\frac{\sum a_j h_j}{\sum a_j}$).
7. Calculate the moment of inertia of the total section about the baseline ($I_{BL} = \sum a_j h_j^2 + \sum i$).
8. Calculate the moment of inertia of the total section about the NA ($I_{NA} = I_{BL} - Ah_{NA}^2$).
9. Calculate the section modulus for deck and keel.

Example

Consider the ship section shown in the figure. Calculate:

- The sectional modulus at the deck and the bottom.
- The maximum bending stress ($M = 105.7\ MN \cdot m$).
- The factor of safety (FOS) ($\sigma_y = 248\ MPa$).



Solution:

- We can follow the same procedure discussed above but in a tabular format to avoid any mistakes:

Item	Scantling $(b \times d)$ (mm)	Area a_j (mm^2)	Height h_j (mm)	1st moment $a_j h_j$ (mm^3)	2nd moment @BL $a_j h_j^2$ (mm^4)	2nd moment @ own centroid i (mm^4)
Deck	1829x25	45725	6082.5	2.78E+08	1.69E+12	2.38E+6
Side Shell	25x6045	151125	3047.5	4.61E+08	1.40E+12	4.6E+11
Bottom	3810x25	95250	12.5	1.19E+06	1.49E+07	4.96E+6
Total		2.92E+05		7.40E+08	3.10E+12	4.6E+11

- The height of the NA above the baseline: $NA = \frac{\sum a_j h_j}{a_j} = \frac{7.4E+8}{2.92E+5} = 2534.2\ mm$
- The total moment of inertia about the NA:

$$I_{NA} = I_{BL} - A \cdot h_{NA}^2 = \sum i + \sum a_j h_j^2 - \sum a_j \cdot h_{NA}^2$$
$$= (4.6E+11) + (3.1E+12) - (2.92E+05 \times (2534.2)^2) = 1.685E+12\ mm^4 = 1.685\ m^4$$

- Noted that this is only the half-section, so $I = 2 \times 1.685 = 3.37\ m^4$

- The distance from the neutral axis to the deck and the bottom:

$$z_{deck} = 3.561\ m,\quad z_{bottom} = 2.534\ m$$

- The sectional modulus at the deck and the bottom respectively:

$$Z_{deck} = \frac{I}{z_{deck}} = \frac{3.37}{3.561} = 0.946\ m^3$$

$$Z_{bottom} = \frac{I}{z_{bottom}} = \frac{3.37}{2.534} = 1.33\ m^3$$

- Since $z_{deck} = z_{\max}$; the maximum bending stress is at the deck:

$$\sigma_{deck} = \frac{M}{Z_{deck}} = \frac{105.7}{0.946} = 111.7\ MPa$$

- Finally, the factor of safety is given by:

$$FOS = \frac{\sigma_y}{\sigma_{deck}} = \frac{248}{111.7} = 2.22$$