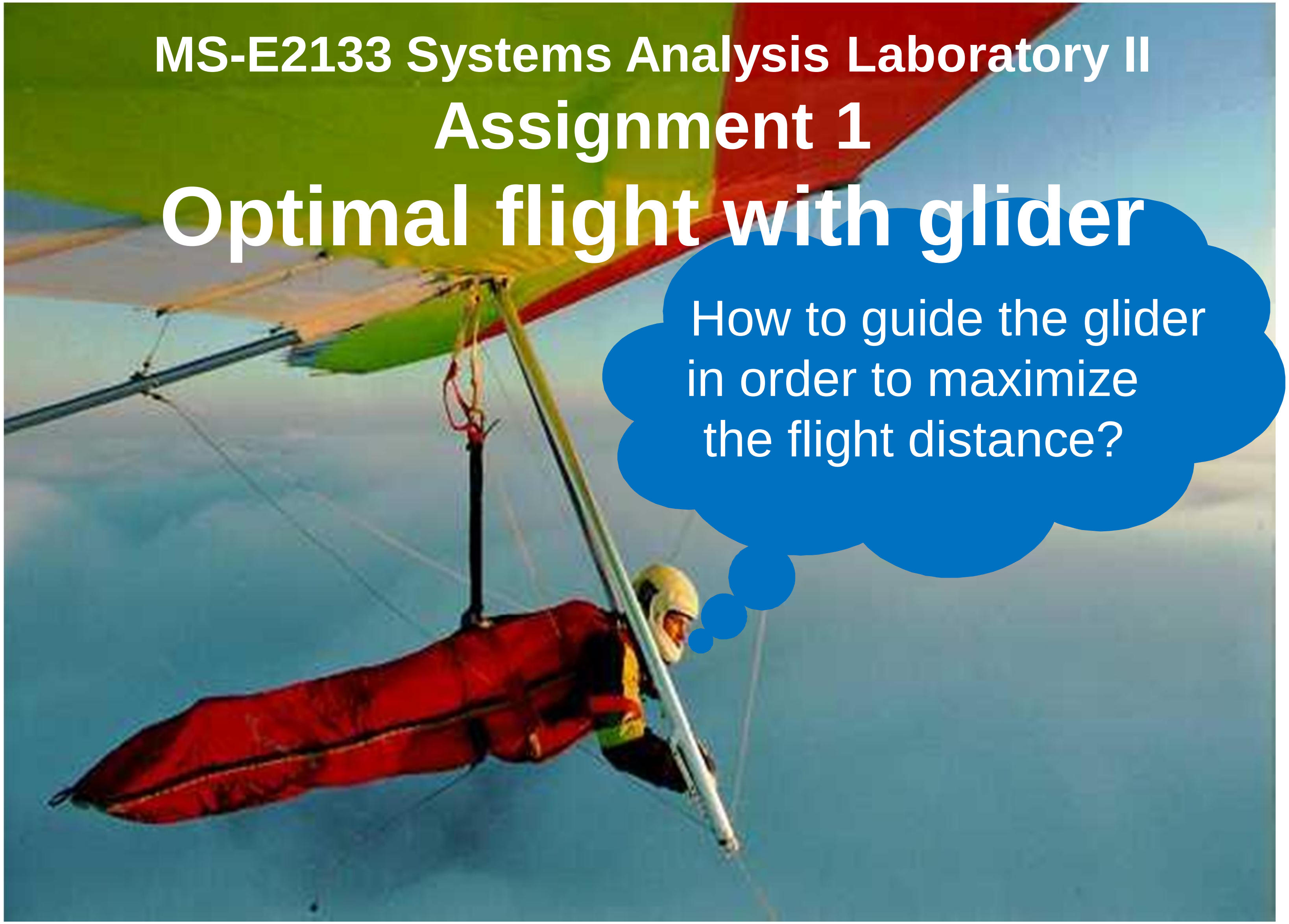
MS-E2133 Systems Analysis Laboratory II

Assignment 1

Optimal flight with glider



Aalto University
School of Science

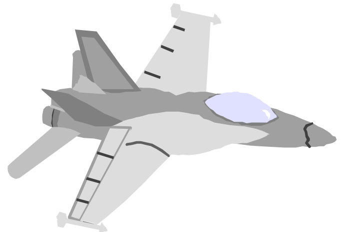
Systems Analysis Laboratory

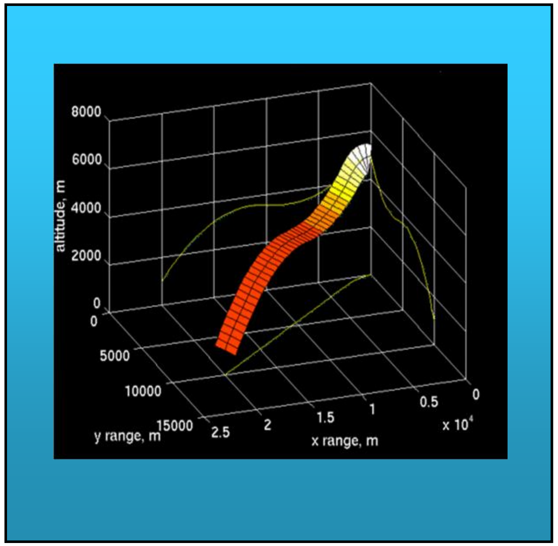
In the assignment…

- Model of the glider constructed
  - State space representation of a dynamic system – state equations
- Flight of the glider simulated and optimized using the model
  - In windless condition
  - In thermal (upward airflow)
- We learn…
  - Grey box modelling
  - Formulation, analysis and numerical solution of dynamic optimization problems (optimal control problems)
  - Solution of nonlinear optimization problems using an existing optimization routine (MATLAB, Optimization Tool Box)

Dynamic optimization problem – solution process

Find the best possible way to control a dynamic system





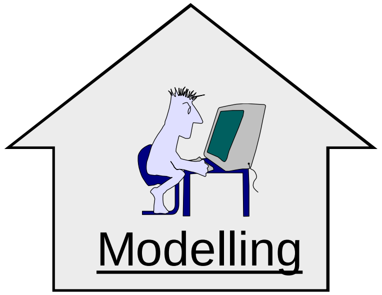
System model

- System of first order diff eqs
- Fixed initial and terminal states
- Feasible controls
- Feasible states

Numerical solution method

- Indirect
- Direct

Visualization of solution



Criterion to be optimized

On theory of flight - forces

- Flight vehicle is affected by four forces:

Lift L=L(angle of attack α, velocity **v**, altitude **h**)

- Lift coefficient $C_L=C_L(\alpha)$
- Reference area S
- Kinetic pressure q=q(**h,v**)

Thrust

How about the glider?

Drag

D=D(angle of attack α, velocity **v**, altitude **h**)

- Total drag coefficient $C_D=C_D(\alpha)$
- Reference area S
- Kinetic pressure q=q(**h,v**)

Weight/Gravity

G = mass **m***gravitational acceleration **g**

Level flight & Constant velocity
<=>
Lift = Gravity & Thrust = Drag

On theory of flight – angle of attack

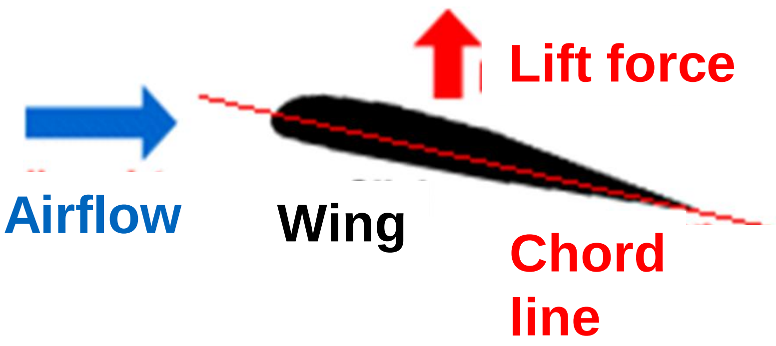
- Chord line and velocity vector of a flight vehicle not parallel

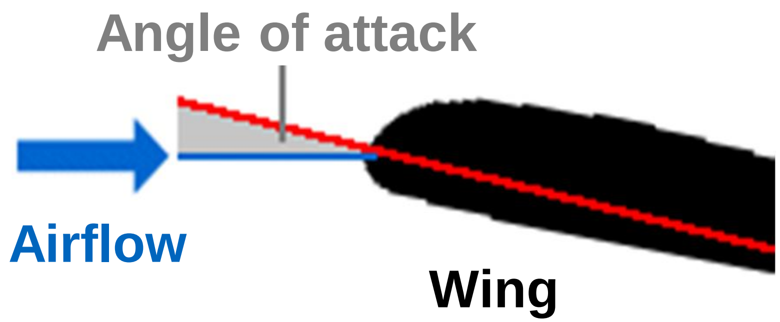
  => Lift force

- Angle of attack α = Angle between chord line and velocity vector
- Lift coefficient $C_L = C_L(\alpha)$

  => Lift force is controlled by angle of attack

- C_L = the control variable in the model of the glider!

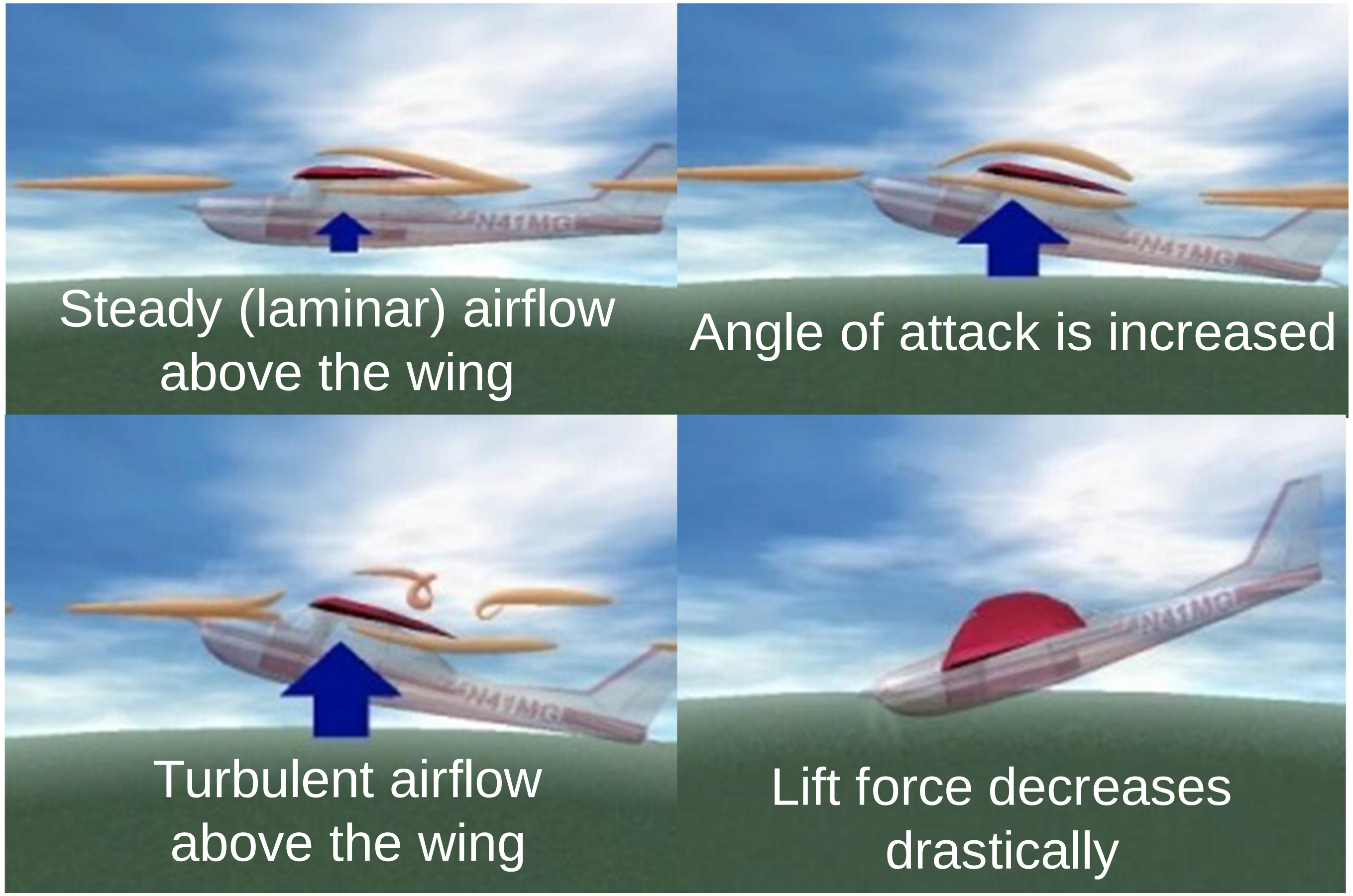




Model of glider

- Movement dynamics (no rotation dynamics)
- Flight in vertical plane
- State variables: x-coordinate, altitude, velocity, flight path angle
- Control variable: lift coefficient
- *Free body diagram*
- *F=ma, v=dx/dt etc. => state equations*
- *Validation of the model using simulation; Effects of parameters*
- *Stall?*

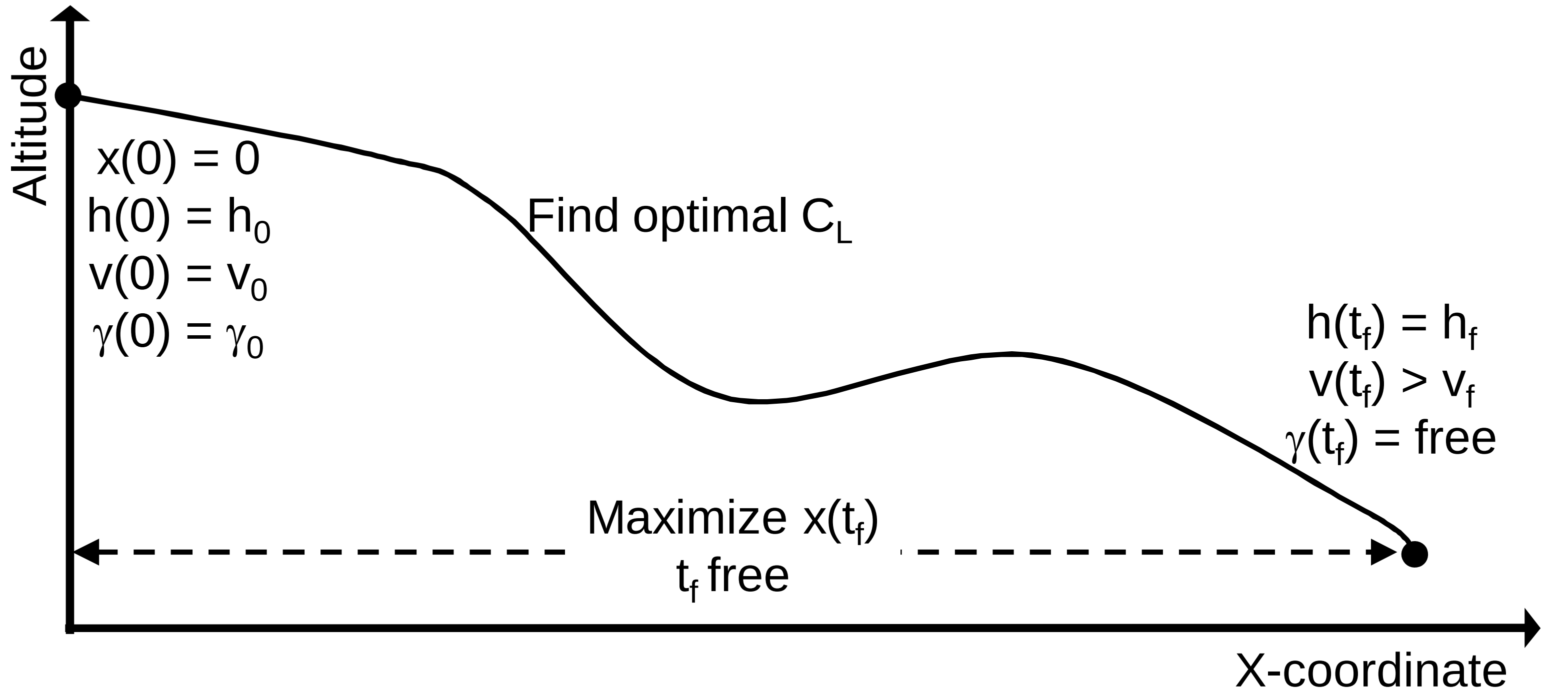
Stall (http://vfinn.fsnordic.net/)



Stall velocity in level flight ⇔
Maximal lift force is equal to gravity force

Optimization of flight

Find the control such that one glides as far along the x-coordinate as possible for each unit of lost altitude



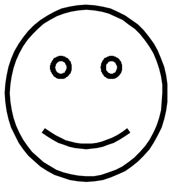
Optimization problems

- "Static" optimization problem:
  - max Δx**/**$-\Delta h$, multiply Δt**/**Δt and $\Delta t \to 0$, therefore max **??**
  - Simplify state equations
  - Maximize the objective function with respect to C_L
  - Verify:

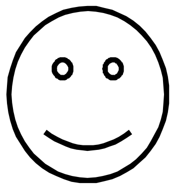
    Maximal distance per one altitude unit is glided when total drag coefficient C_D **/** lift coefficient C_L is as low as possible
- Dynamic optimization problem with the free final time
  - max $x(t_f)$ ….. can be expressed in other forms
- Comparison of solutions

On solution of dynamic optimization problems

- Open-loop solution / open-loop optimal control
- Indirect solution methods:
  - ***Derive*** *(see the material of the MS-E2148 course)* and solve the necessary conditions for the optimal control
  - Multiple-point boundary value problem
  - e.g., **multiple-point shooting method** *(see additional material)*



- Direct solution methods
  - Discretization + nonlinear programming



- *Comparison of the solution methods (see additional material)*

Discretization methods

- Controls are discretized
  - State equations are integrated explicitly
  - “control parameterization”, “direct shooting”
- Controls and states are discretized
  - Implicit integration, number of decision variables increases
  - Euler, Runge-Kutta, **direct collocation** *(see additional material)*
  - “direct transcription”

YEAH!!

- States are discretized
  - Controls are eliminated
  - Discrete state is achievable from the previous state
  - “difference inclusion”

Pros & cons of discretization

- Derivation of necessary optimality conditions not required
  - Initial guesses of Lagrange multipliers/co-state variables not needed
  - Switching structure not needed
- Existing routines for solving nonlinear optimization problems
  - ***Rough initial guess is adequate*** (see ready-made Matlab files)
- Automated solution
- Approximate solution – accuracy depends on the order of discretization and ΔT
  - Higher order => more constraints
  - Smaller ΔT => more decision variables, more constraints
- Increasing accuracy of solution
  - Adaptive non-uniform discretization points (=> estimation of error)
  - ***Continuation with respect to the number of points*** *(see ready-made Matlab files)*
- Constraints satisfied only at discretization points

Solution of discrete time dynamic optimization problems (DTDOPs)

- Discretization => Nonlinear constrained optimization
- ***SQP (sequential quadratic programming)***

  (see the material of the MS-E2139 course)
  - Most used method for solving DTDOPs
  - NPSOL, NAG, FSQP, LANCELOT
  - ***MATLAB - fmincon-routine*** *(see ready-made Matlab files)*

    - Scaling of decision variables!
  - Numerical gradients calculated automatically in several implementations
- Matrices are sparse in DTDOPs
  - Calculation eased in large scale problems

Analysis and comparison of the optimal solutions

- “Static” versus dynamic problem in windless condition
- Direct collocation versus multiple-point shooting
  - Reference solution
  - Co-states approximated by Lagrange multipliers
- Dynamic problem in thermal
  - Rising velocity of airflow as a function of the x-coordinate
  - State equations are modified => new state variables v_x ja v_h
  - Solution with direct collocation and SQP
  - Ready-made Matlab files

Report

- Written in a scientific, academic and professional manner
  - Sections
  - Legends for figures/tables; every figure/table must be referred to from the text
  - Substance important, not fancy layout
  - Understandable, easily readable; rational structure
- Introduction
  - Background and goal of the problem solving setting at hand
- Answers to all exercises and questions in the work instructions
  - Appropriate amount of figures dealing with simulation and optimization results
- Conclusion and discussion
  - Comments on models and methods
  - Comments on the assignment; suggestions for future improvements