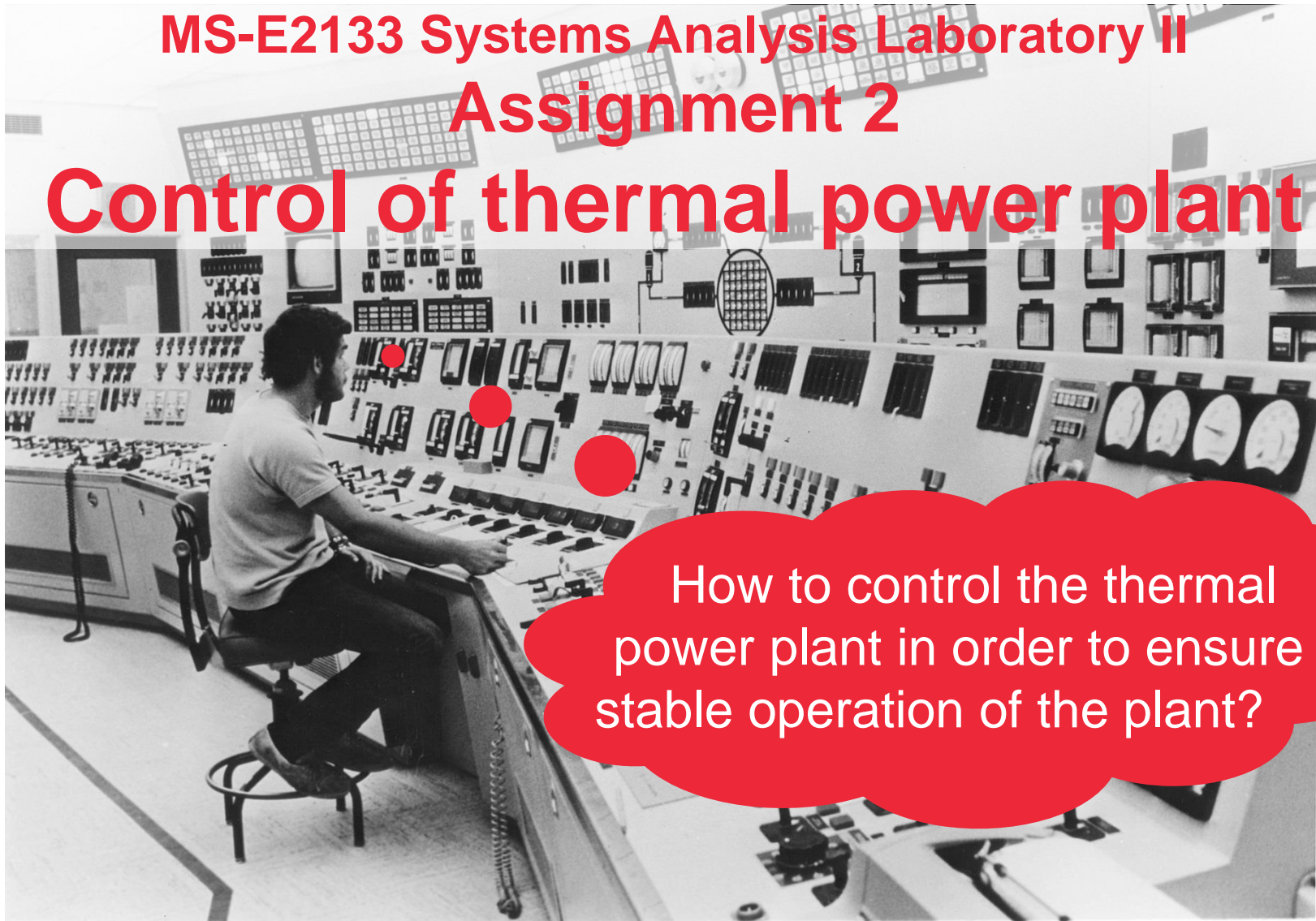


# MS-E2133 Systems Analysis Laboratory II

## Assignment 2

# Control of thermal power plant



How to control the thermal power plant in order to ensure the stable operation of the plant?

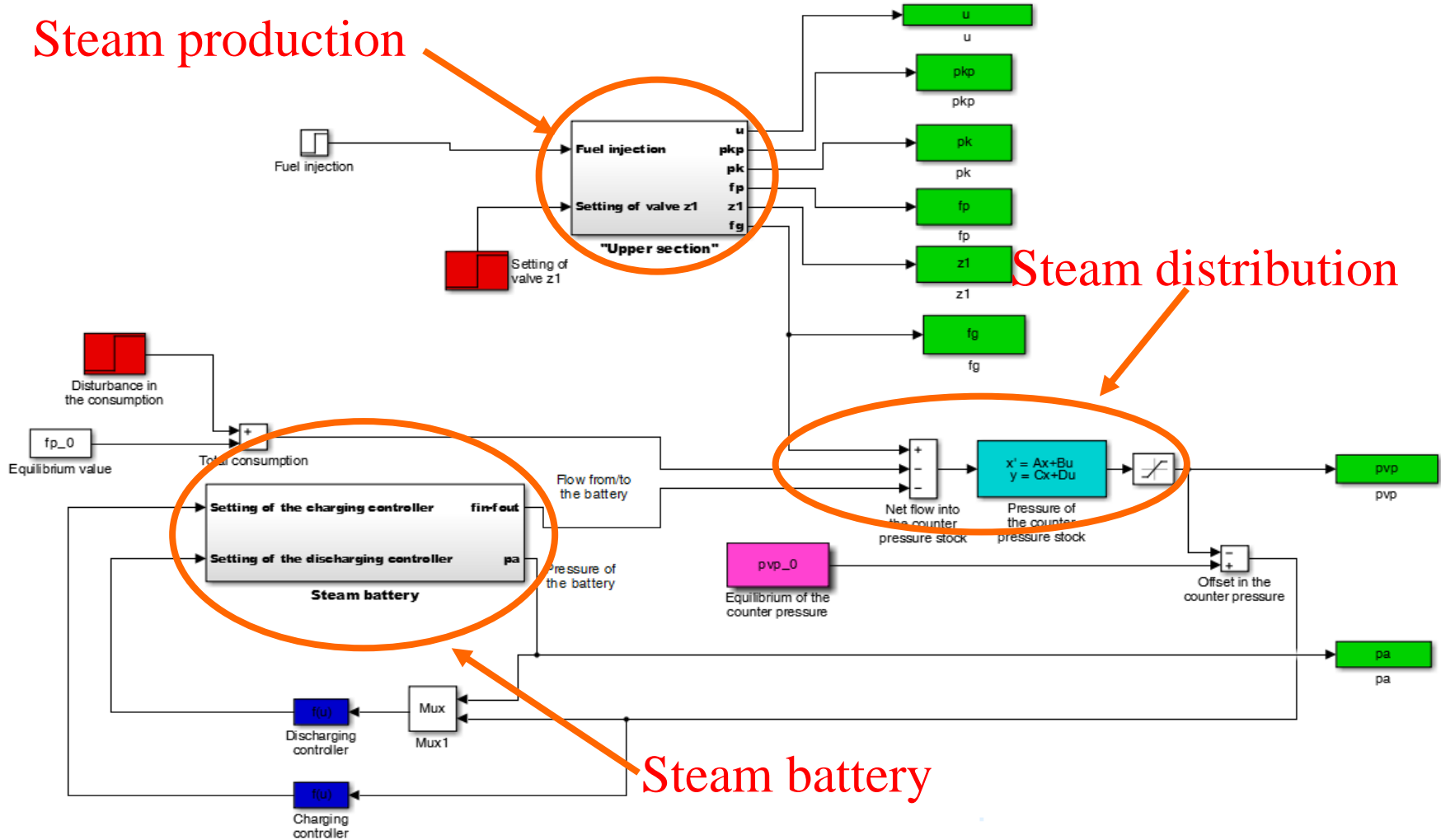
# In the assignment...

- Production of steam in a thermal power plant is analyzed
    - Burning process of fuel => Steam generation in a boiler => Production of high pressure steam => (Turbine) => Distribution of counter pressure steam => Steam battery => Consumption of steam
    - Steam used by, e.g., a paper mill
  - Steam production affected by "disturbance": Steam consumption (also steam flow through the turbine)
  - Steam production stabilized by
    - Controlling fuel injection
    - Controlling steam flow through the turbine
    - Charging and discharging the steam battery
  - PID controller and state feedback controller are used
  - Three sections of the power plant
    1. Steam production; upper section of the plant
    2. Steam distribution; turbine flow & counter pressure stock
    3. Steam battery
  - Analysis and control of a large scale system (in principle)
-

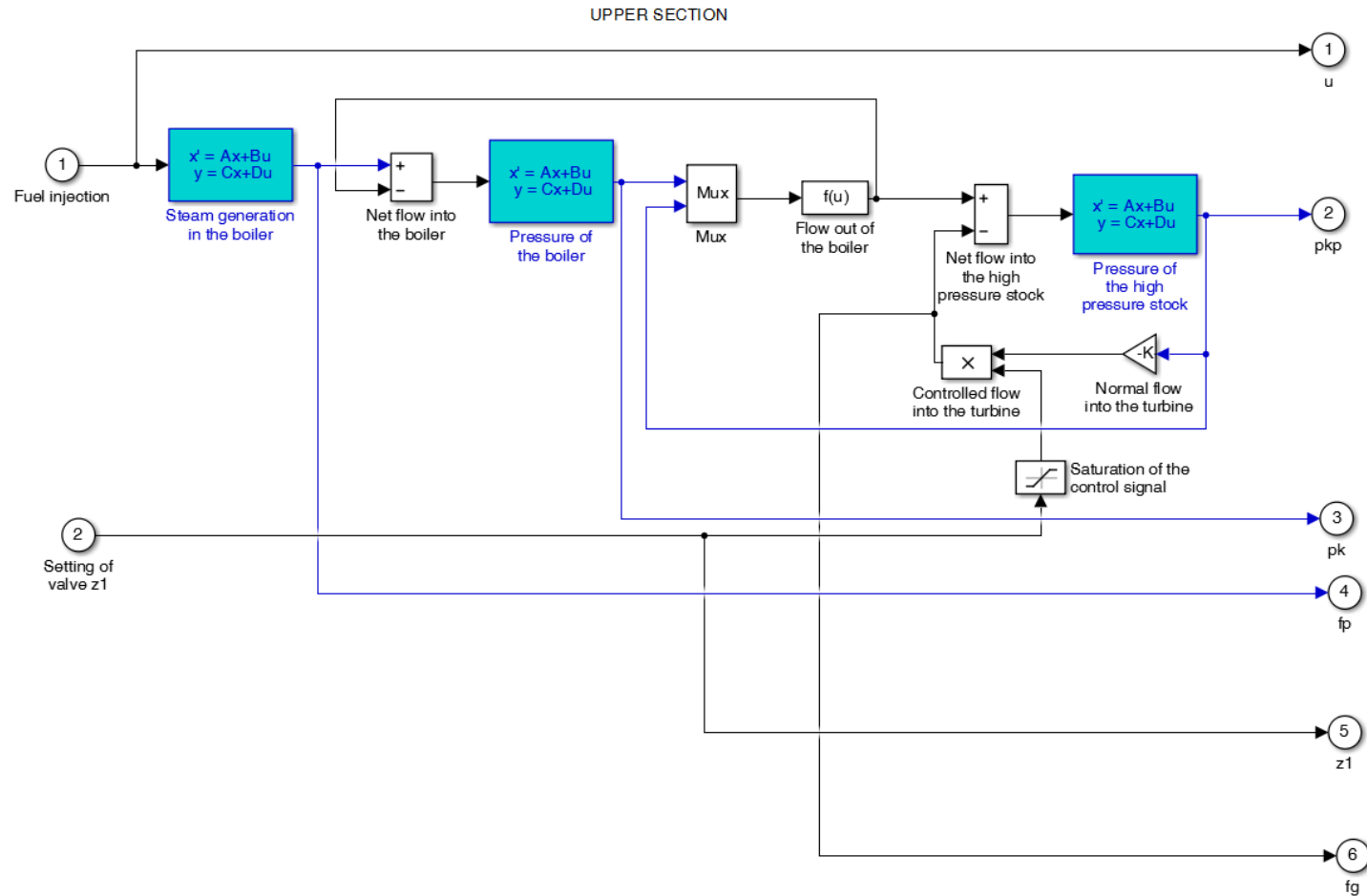
# Simulink model of the power plant

POWER PLANT

Steam production

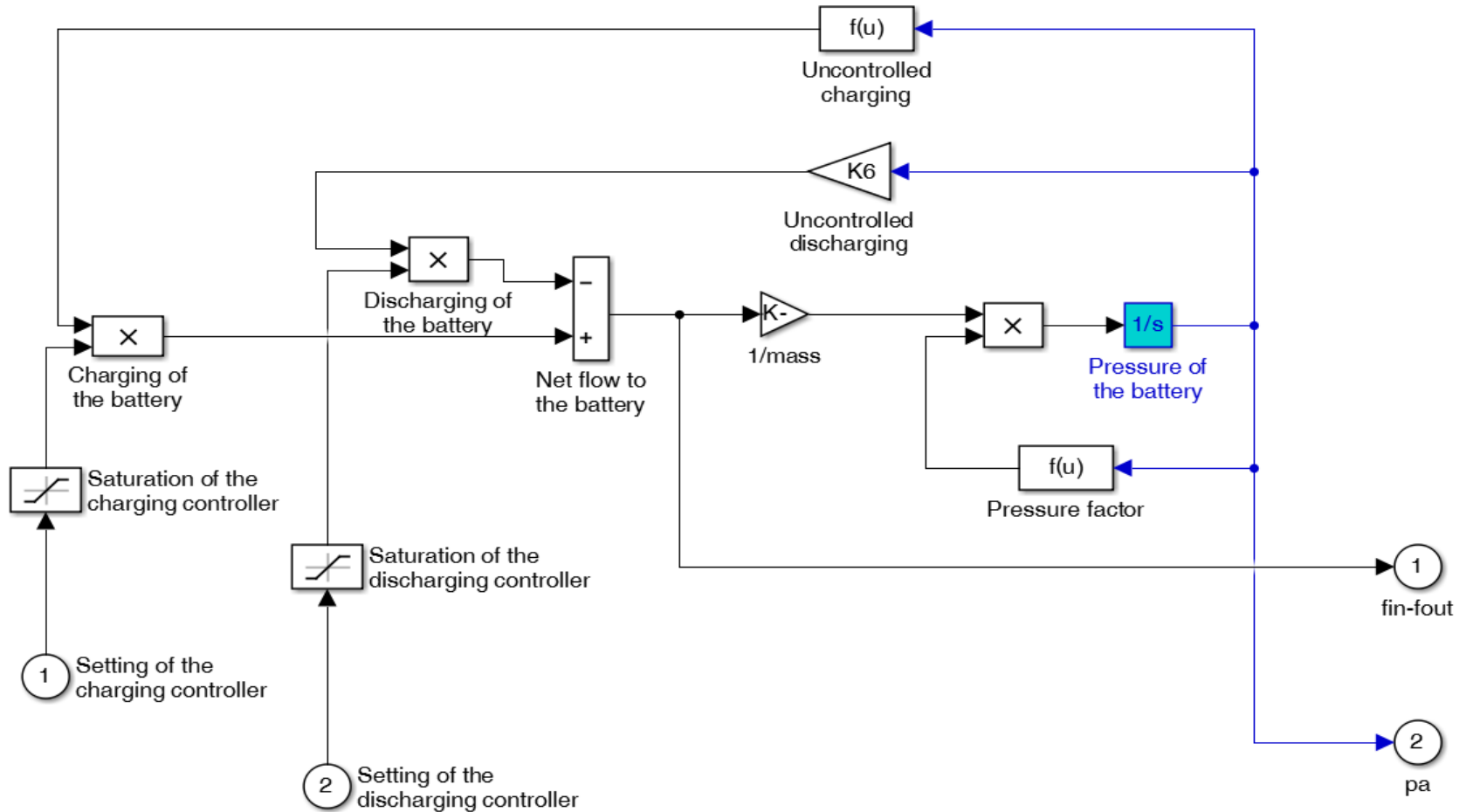


# Steam production ("upper section")



# Steam battery

STEAM BATTERY

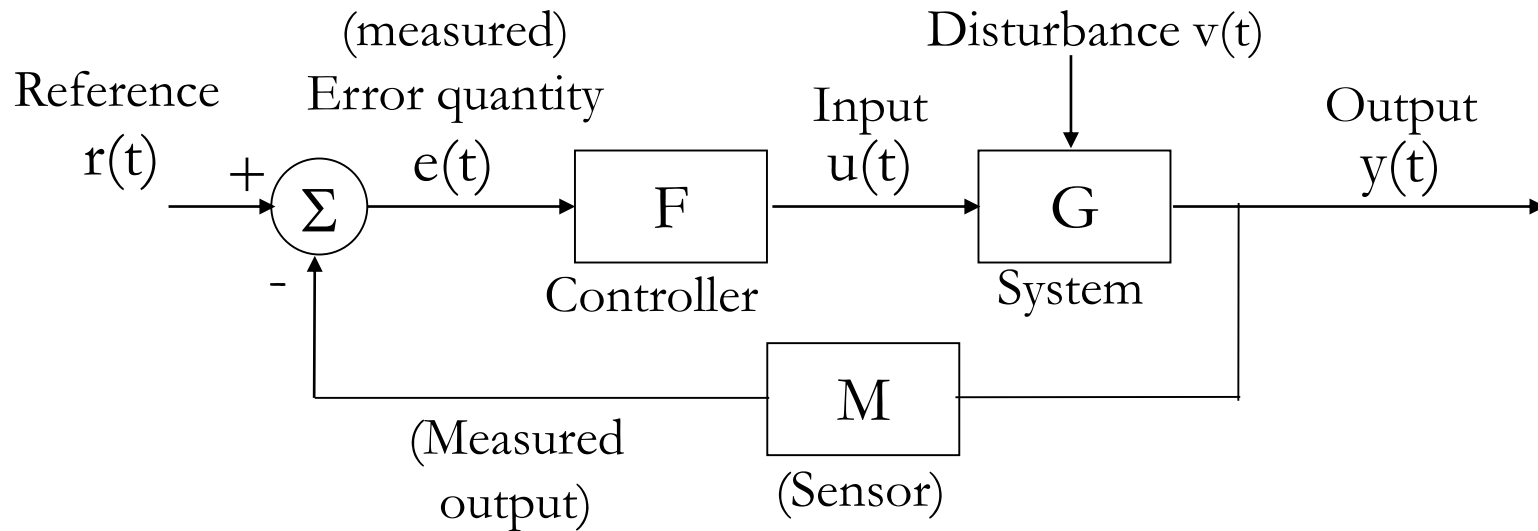


# On control theory

(e.g., Åström & Murray, Chapter 1)

- Feedback:
  - The control (= the input) of a system depends on the output / the state of the system
- Basic problems:
  - Tracking: The output should follow the external reference signal as accurate as possible (focus is to compensate the dynamics of the system)
  - Stabilization: The output should be constant (focus is to compensate disturbances)
- The assignment deals with stabilization

# Idea of feedback



- $r(t)$  is the external reference signal
- Feedback from the output using the error quantity  $e(t)=r(t)-y(t)$ 
  - $e(t)=0 \Rightarrow$  OK! Otherwise: Adjust  $u(t)$  until  $e(t)=0$
- Control problem: Construct the controller
  - Structure
  - Gains, i.e., parameters

# PID-controller

(e.g., Åström & Murray, Chapter 10)

- P = Proportional, I = Integral, D = Derivative
  - P-term: The input depends on the current value of the error quantity
    - Insufficient for steady/constant disturbance
  - I-term: The input depends on the time integral of the error quantity
    - Destabilizes the system; relying on old information
  - D-term: The input depends on the time derivative of the error quantity
    - Stabilizes the system; issues on amplification of high frequency measurement or process noise
- Each term has a gain - tuning parameters  $K_P$ ,  $K_I$  and  $K_D$  of the controller
  - Select the parameters in an appropriate way => Stability of the system

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t)$$



# State feedback controller

(e.g., Åström & Murray, Chapter 6)

- Linear dynamic system  $dx/dt=Ax+Bu$ 
  - External reference signal = 0
  - Control the system such that  $x = 0$
- Controls = Linear combinations of states:  $u=-Kx$ 
  - Closed loop system:  $dx/dt=(A-BK)x$
- System matrix of the closed loop system:  $A-BK$
- The open loop system is controllable (see, e.g., Åström & Murray, Chapter 6) => Arbitrary dynamics for the closed loop system
- The problem: Select gain  $K$
- The state feedback controller does not have necessarily integrating feature
  - Integration should be augmented if needed (step disturbances)

# Optimal state feedback controller

(e.g., Kirk, Chapter 5.2)

- Select  $u$  such that the functional

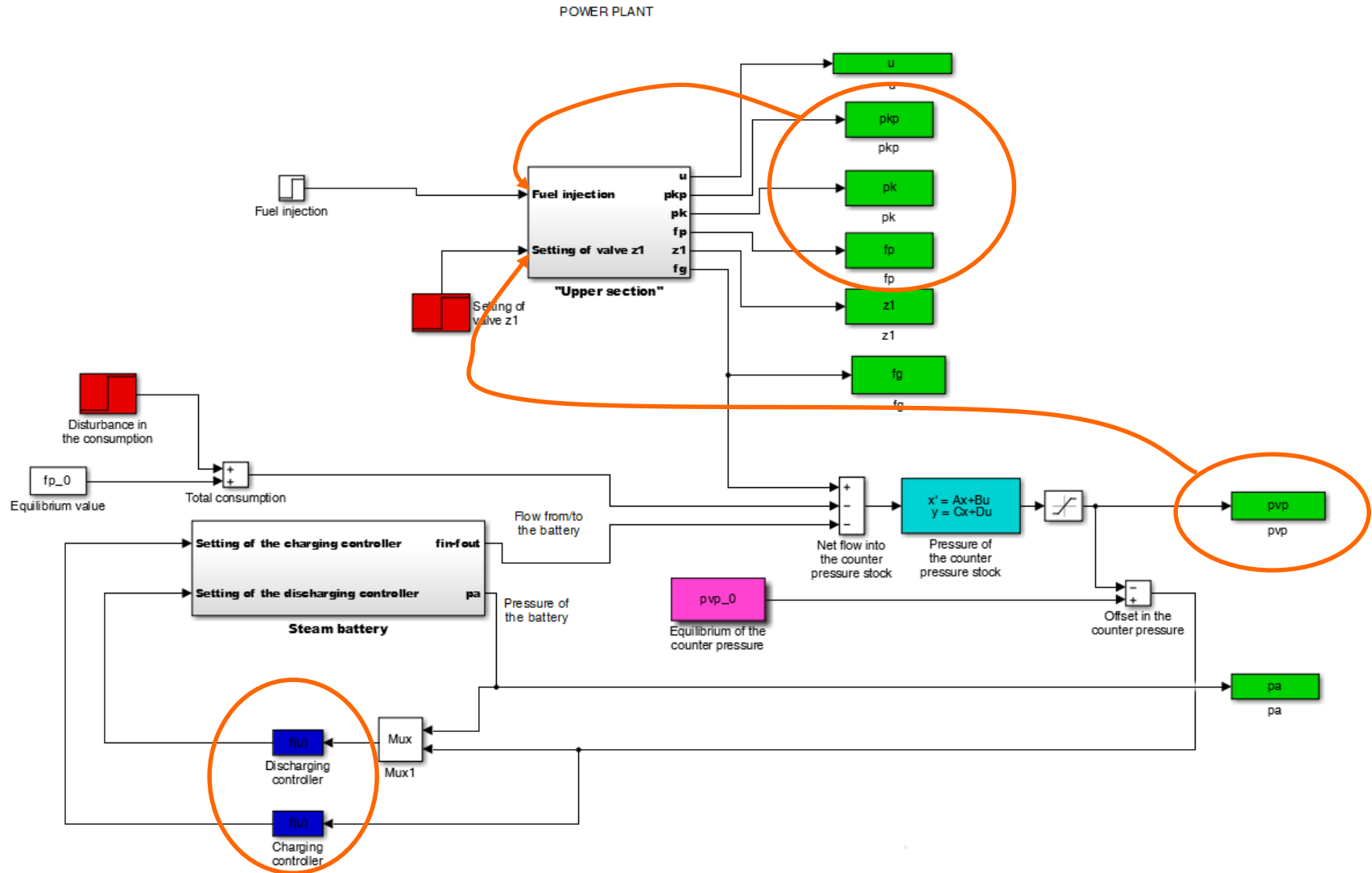
$$J[u] = \frac{1}{2} \int_0^T x(t)^T R x(t) + u(t)^T Q u(t) dt + \frac{1}{2} x(T)^T P x(T)$$

is minimized (linear-quadratic (LQ) problem)

- Weights  $R \Rightarrow$  penalty related to large states, weights  $Q \Rightarrow$  penalty related to large controls, weights  $P \Rightarrow$  penalty related to large terminal states
- Feedback solution obtained by deriving and solving the necessary conditions for the optimal control
  - State equation, co-state equation, optimal control (see the material of the MS-E2148 course)

- .....
- Assume that the co-state is of form  $S(t)x(t)$   
=> Riccati equation for  $S$ 
    - The optimal control is the time variant linear combination of the states:  $u^*=-K^*(t)x(t)$
- ⇒ Solution: Integrate Riccati equation backward =>  $S(t)$  => the optimal feedback gain  $K^*(t)$  => Employ the control  $u(t)=-K^*(t)x(t)$
- $S$  typically stabilized quickly => Time invariant (but suboptimal) gain  $K^*$  obtained by solving algebraic Riccati equation (derivatives of  $S$  are set to be zeros)
    - (Matlab: `lqr/lqr2`)

# Feedbacks in the assignment



# Linearization (e.g., Åström & Murray, Chapter 5.4)

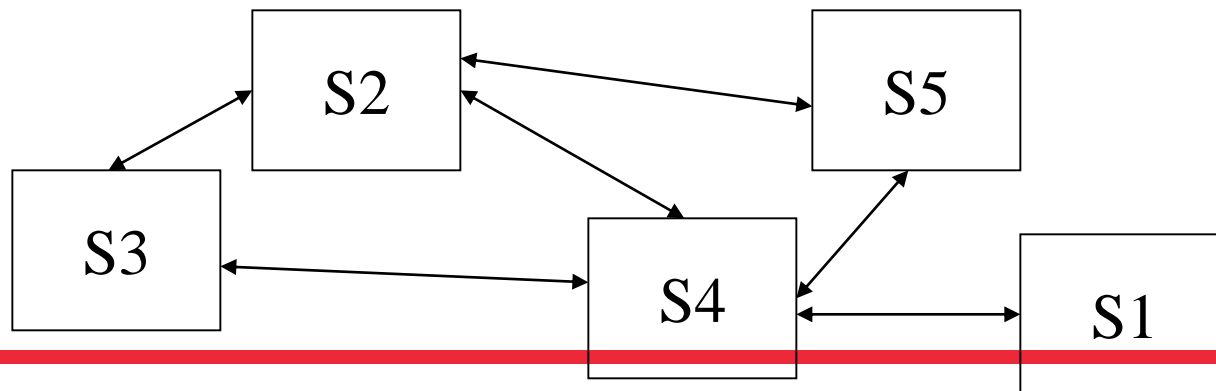
- Controllers can be used (cautiously) with nonlinear systems
    - PID-controller can be tuned by using a real-life system – a system model is not needed but can be used
  - Tuning of a State feedback controller – also PID if a model is used - requires a linear system model => Nonlinear systems must be linearized
  - Nonlinear system  $dx/dt=f(x(t),u(t))$ ,  $y(t)=g(x(t),u(t))$ 
    - Analyze the stationary/equilibrium point  $(x_0,u_0)$  (and the corresponding  $y_0$ ) and small differences  $\Delta x=x-x_0$ ,  $\Delta u=u-u_0$ ,  $\Delta y=y-y_0$
    - It holds
$$d\Delta x(t)/dt= \partial f/\partial x\Delta x(t)+\partial f/\partial u\Delta u(t)$$
$$\Delta y(t)=\partial g/\partial x\Delta x(t)+\partial g/\partial u\Delta u(t)$$
    - Jacobians evaluated at  $(x_0,u_0,y_0)$
  - Note: Valid domain of linearization?
-

# On large scale systems

- Large-scale system = System consists of several subsystems connected each other loosely
  - thousands of variables
  - analysis and synthesis using direct methods challenging or impossible
- "Theory" of large-scale systems: Approaches, methods and techniques for tackling such systems
  - "divide and conquer"
- Typical applications areas: optimization and simulation
- More esoteric themes dealing with large-scale systems:
  - decentralized control, coordination, autonomous agents, agent simulation, self-organization, artificial life,...



- Basic idea: subsystems treated separately by taking into account interactions and dependences between the subsystems
- Interactions and dependences treated in an iterative way
  - Subsystems treated with wrong (but hopefully converging) assumptions on interactions and dependences
- Typically two level algorithms
  - Upper level: Updating interactions and dependences with fixed subsystems
  - Lower level: Updating subsystems with fixed interactions and dependences



# Structural versus mathematical large-scale system

- Often subsystems interacting identifiable wholes
  - For example, multi-part mechanical systems: Parts and subsystems interact through different articulations; interactions due to supporting forces
- "Large-scale system" can also be originated from mathematical analysis
  - For example, discretization of a continuous time dynamic optimization problem: each discretization point depends only on proximate points – points far away from each other loosely coupled
- Regardless of origin of a large-scale system, mathematical description of the system has utilizable structure
  - For example, the Jacobian of the constraints of a discretized dynamic optimization almost block diagonal



# Important solution paradigms

- Analysis of large-scale systems using decentralizing methods enables parallel and distributed computation
  - A single processor for each single subsystem (lower level)
  - One processor coordinates computation (upper level)
- Algorithms and data structures for sparse matrices
  - Decentralization on algorithm level

# How do large-scale systems relate to the assignment?

- The thermal power plant is a large-scale system
  - Three subsystems connected each other loosely: Steam production, Steam distribution, Steam battery
- First: Each subsystem is tuned separately
- Second: Interactions between the subsystems are taken into account and the tuning is updated
- Iterative process

# Comments / hints, Exercises 1-2

- Exercise 1:
  - Read the work instructions!
  - Familiarize yourself with the Simulink model of a power plant
  - The system is initially in a steady/equilibrium state – the steady state values of the variables are given in the work instructions
- Exercise 2:
  - Control variables of the upper section: fuel injection  $u$ , setting of the valve of the turbine flow  $z_1$
  - Turbine flow increases (=”disturbance”) => How much the setting of the valve change? => What happens to the pressure of the high pressure stock?

# Comments / hints, Exercise 3

## Exercise 3:

- Operation of the upper section of the plant is only analyzed!
- Offset of the pressure of the high pressure stock from the steady state pressure must be below 2%
- Rapid changes in the fuel injection is not preferable (such "control" fuels are expensive)  
=> Multiple objective optimization problem (stable pressure of the high pressure stock is more important!)
- ***Control of the fuel injection ( $u$ ) using feedback from the pressure of the high pressure stock ( $p_k$ )***
- Modify the Simulink model
- Tune P-controller with the step response experiment ( $u=+1\text{kg/s}$ ) => How the controller works when turbine flow is changed? => No good! => Tune PI => How it works? => Tune PID => How it works?
- Tuning of PID (e.g., Åström & Murray, Chapter 10)
- Maximum value of the derivative of the response using, e.g., difference approximation

# Comments / hints, Exercise 4

- Exercise 4:
  - State feedback controller for the upper section
  - **Control, i.e., the fuel injection ( $u$ ) is the linear combination of the steam generation in the boiler ( $fp$ ), the pressure of the boiler ( $pk$ ) and the pressure of the high pressure stock ( $pkp$ )**
  - The model of the upper section must be linearized! A linear open loop system is controllable  $\Rightarrow$  arbitrary dynamics for the closed loop system  $\Rightarrow$  stabilization possible
  - Modify the simulink model
  - Tune the gains of the controller such that the eigenvalues of the system matrix of the closed loop system are on the left-half complex plane
  - Linear quadratic dynamic optimization problem  $\Rightarrow$  Riccati differential equation  $\Rightarrow$  algebraic Riccati equation (Matlab's functions `lqr`, `lqr2`)
  - Select the weight matrices of the criterion appropriately; compare different matrices; study eigenvalues of  $A-BK$  (should be on the left-half complex plane)
  - Other means for defining appropriate eigenvalues laborious!!!

# Comments / hints, Exercises 5-6

- Exercise 5:
  - Issue on the state feedback controller – fixed offset in the output from the steady state output, cf. P-controller
  - Extend the system by taking into account a new state variable
  - Time derivative of the new state variable is the error quantity, i.e., the variable is the integral of the error quantity
  - Tune the controller using the solution of the LQ problem – selection of the weight matrices of the criterion – comparisons
  - How the controller works?
- Exercise 6:
  - Compare and discuss the application of the PID-controller and the state feedback controller – Which one is better?
  - Go with PID

$$\tilde{x} = \int_0^t (p_k p_0 - p_k p) dt$$
$$\dot{\tilde{x}} = p_k p_0 - p_k p$$

# Comments / hints, Exercise 7

- Exercise 7:
  - Reduction of the upper section model into a first order system
  - Linearize the upper section; write the linearized model in the form of a transfer function (Laplace-transformation; frequency space)
  - Construct the Pade approximation for the transfer function
    - See, Norton pp. 225-227
    - Derive the Taylor series of the transfer function
    - Set the series equal to the first order Pade approximation (rational function approximation) => Parameters of Pade
  - State space representation into transfer function: Matlab's function `ss2tf`
  - Taylor series using, e.g., Mathematica's function "series"
  - Create a simple Simulink model containing only the transfer functions of the Pade approximation (1-3 functions)
  - Compare the step responses provided by the original system and the approximation
  - The approximation is not used in the following exercises

# Comments / hints, Exercises 8-9

- Exercise 8:
  - The upper section of the plant works fantastically (after exercise 6)
  - Consumption flow of steam (=”disturbance”) suddenly changes => How does this affect the pressure of the counter pressure stock? (Should be within 10% of the equilibrium value)
- Exercise 9:
  - ***Control of the turbine flow ( $z_1$ ) using feedback from the pressure of the counter pressure stock ( $p_{vp}$ )***
  - Modify the Simulink model
  - P, PI, I, ID, PID controllers – tune such that the counter pressure within acceptable limits, i.e., close to the equilibrium value
  - How different controllers work? Only experiments - e.g., tuning of the controllers based on the step response is not required!
  - Controls of the turbine flow and the fuel injection take care of low frequency and large amplitude disturbances in steam consumption



# Comments / hints, Exercise 10

- Exercise 10:
  - The steam battery is included in the analysis
  - ***Control of the input and output flow of the steam battery (charge and discharge valves z2 and z3) using feedback from the pressure of the counter pressure stock (pvp)***
  - Ready made P-controllers in the Simulink model; saturation limits for Z2 and Z3 have so far been zero => modify them to appropriate values given in the work instructions
  - Find appropriate gains (Kin ja Kout) such that the controllers of the steam battery and the turbine flow provide jointly the good behaviour of the plant – try & test alternative disturbances in steam consumption
  - The steam battery takes care of large frequency disturbances in steam consumption => compensate sudden consumption changes => no variations in the fuel injection

# Comments / hints, Exercise 11-12

- Exercise 11:
  - Adjust the gains of all the controllers such that
    - The counter pressure stays within the given limits
    - Fluctuations in the pressure of the high pressure stock as small as possible
    - Use of expensive "control" fuel in the heating of the boiler minimized
  - Test the operation of the plant as a whole
  - Different step and ramp disturbances in the steam consumption – also frequency responses (high frequencies, low frequencies)
  - Give a general recommendation to the consumer of steam regarding
    - The amplitudes of disturbances allowed at high and low frequencies
    - The size of step disturbances allowed with the given limits of the counter pressure
- Exercise 12:
  - Write the report – see the work instructions

# References

- An Introduction to Identification; Norton J.P., Academic Press, 2009
- Feedback Systems; Åström K.J. & Murray R.M., Princeton University Press, 2008
- Optimal Control Theory; Kirk D.E., Prentice-Hall, 2004