AGITATION/MIXING

1. INTRODUCTION

Agitation and mixing is discussed in McCabe et al. (1993) chapter 9 pages 235–287. Viscosity and non-Newtonian fluid are discussed in McCabe et al. (1993) chapter 9 pages 44–48.

Agitation:	Induced motion of a material in a specific way, usually in a circulatory pattern inside some sort of container.
Mixing:	Mixing is random distribution, into and through one another, of two or more initially separate phases.
Blending:	Agitation of two miscible liquid phase
Suspension:	Suspending solid particles into liquid.
Dispersion:	Dispersing a gas through the liquid in the form of small bubbles.

In this work we are talking about agitation.

Purpose of this work is to compare different impellers and compare measurements to values from literature.

2. VISCOSITY

Concept of viscosity and its definitions are not straightforward. In this case, only basic ideas will be described. Viscosity depicts ability of fluid to resist shear-stress (Figure 1.).

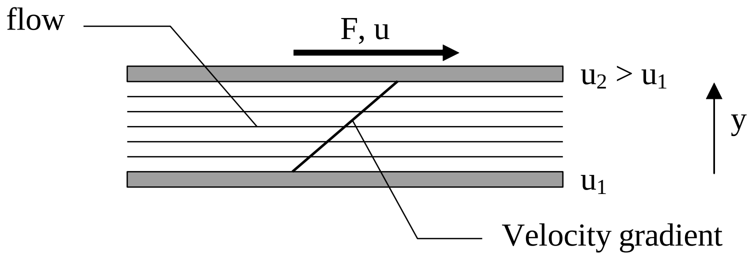


Figure 1. Newtonian fluid between two plates.

In Figure 1, top plate has a velocity u_2. Because of friction, a force is needed to keep the top plate in motion. This force causes a velocity gradient (= shear stress) which is equal to force *F* divided by area of the plate *A*.

$$\tau = \frac{F}{A} \tag{1}$$

Thus fluid can not resist fully it begins to flow. Change of velocity function of distance of plates (*y*) is velocity gradient:

$$\text{velocity gradient} = \frac{du}{dy} \tag{2}$$

2.1.1 Newtonian fluid

Assume that the velocity gradient is proportional to the shear rate, so

$$\tau = \frac{F}{A} = \eta \frac{du}{dy} \tag{3}$$

This is the definition of a Newtonian fluid. The proportionally constant η is called the viscosity.

Note this does not imply that the viscosity is constant, because the viscosity of real fluid is a function temperature (and small extent of pressure).

The velocity gradient of Newtonian fluid between two plates is constant:

$$\frac{du}{dy} = \frac{\tau}{\eta} = \frac{1}{\eta}\frac{F}{A} \tag{4}$$

2.1.2 Non-Newtonian fluid

The fluid is non-Newtonian if the viscosity of the fluid does depend on the velocity gradient.

$$\eta = f\left(\frac{du}{dy}\right) \tag{5}$$

This is a definition of a non-Newtonian fluid. It is a consequence of this definition that the velocity distribution is non-linear in laminar flow between two plates.

This work is focused only fluids, which are assumed to follow the Ostwald-de Waele –equation. Assume that dependence of velocity gradient and shear stress is following:

$$\tau = \frac{F}{A} = K'\left(\frac{du}{dy}\right)^{n'} \tag{5}$$

where *K’* and *n’* are experimentally determined parameters. Fluids that follow this power law are called power-law fluid.

In the case of non-Newtonian fluid, apparent viscosity η' is used instead of viscosity. The apparent viscosity is analogical to the viscosity of Newtonian fluids. From eq (3):

$$\tau = \frac{F}{A} = K'\left(\frac{du}{dy}\right)^{n'-1}\left(\frac{du}{dy}\right) \tag{6}$$

where the apparent viscosity can be derived:

$$\eta' = K'\left(\frac{du}{dy}\right)^{n'-1} \tag{7}$$

In fig. 2 the relation of shear stress to velocity gradient for Newtonian and non-Newtonian fluids when n’ > 1 and n’ < 1 is presented.

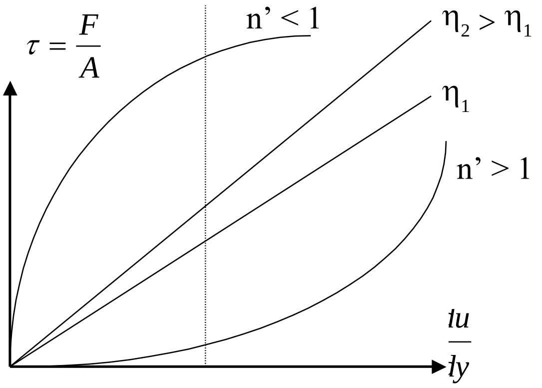


Figure 2.a:
Shear stress as function of velocity gradient

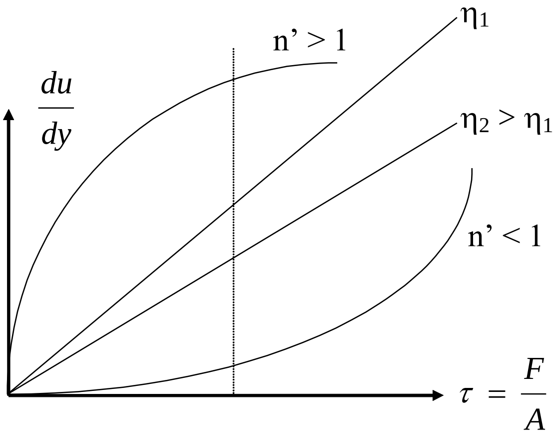


Figure 2.b:
Velocity gradient as function of shear stress

If fluid is following eq. (6) and *n'* > 1, it is called dilatant fluid and shear rate thickening. If fluid is following eq. (6) and *n'* < 1, it is called pseudoplastic fluid and shear rate thinning.

2.1.3 Non-fluids

McCabe et al. (1993) give definition: "A fluid is a substance that does not permanently resist distortion". Streeter (1961) gives another definition: "A fluid is a substance that, no matter how viscous, will yield in time to the slightest stress."

For example in Figure 3.2, McCabe et al. (1993), fluids in slope B "Bingham plastics" are non-fluids. For these fluids dependence between velocity gradient and shear stress is following

$$\tau = \frac{F}{A} = \tau_0 + K\frac{du}{dy} \tag{8}$$

where τ_0 and K are constants. The velocity gradient is following:

$$\frac{du}{dy} = \frac{\tau - \tau_0}{K} \tag{9}$$

It can be seen that a Bingham plastics can resist a distortion or stress smaller than τ_0, so it is non-fluid.

2.2 DEFINING THE VISCOSITY

In a Newtonian fluid, the viscosity is constant in constant state. The viscosity of a Newtonian fluid although depends on temperature and pressure.

For Newtonian fluid

$$\frac{F}{A} = \eta = \frac{F/A}{\Delta u/\Delta y} \tag{10}$$

is constant in specific condition. So, only one (F/A) / ($\Delta u/\Delta y$) determination is needed to define viscosity of a Newtonian fluid.

Defining apparent viscosity of non-Newtonian fluids is more complicated, because ratio (F/A) / ($\Delta u/\Delta y$) is changing. This ratio depends on two parameters, K' and n'. At least two determinations are needed to define those parameters. In practice, more determinations are made.

In this work, the viscosity is defined with rotating viscometer (rotating sensor). By changing rotational speed and diameter of the sensor, the velocity gradient can be changed. With Newtonian fluid these changes do not have effect on values of measured viscosity. With non-Newtonian fluids measured viscosity is changing and parameters K' and η' are calculated based on measurements.

In addition, the velocity gradient caused by rotating sensor is needed to define parameters K' and n' from Eq. (8). If sensor is rotating in a vessel, which is very large comparing to sensor, the velocity gradient can be calculated from following equation (Streeter, 1961, pages 7-12):

$$\frac{du}{dy} = \frac{4\pi n}{n'} \tag{11}$$

From eq. (8):

$$\eta' = K'\left(\frac{4\pi n}{n'}\right)^{n'-1} \tag{12}$$

where η' is apparent viscosity. When measurements are done with different rotational speed, parameters K' and n' can be defined from eq. (13).

3. POWER CONSUMPTION

3.1 DEFINING POWER CONSUMPTION

Purpose of this work is to study how rotational speed, impeller type and viscosity of fluid affect to power consumption. In this work, torque caused by agitation and rotational speed of impeller are measured. The power of agitation a.k.a. power input, P_B, can be obtained following:

$$P_B = \omega M = \omega F l = \omega m g l = 2\pi n m g l \tag{13}$$

where ω is angular velocity, M is torque, F is power, l is torque arm and g is gravitional acceleration.

The power input is the power transformed to the fluid; the power which increases mechanical energy of the fluid. The power of electric motor, P_E, is greater than power input and can be obtained following:

$$P_E = \frac{P_B}{\eta_E \eta_{MEK}} \tag{14}$$

3.2 DIMENSIONLESS NUMBERS

Power number Po, agitator Reynolds number Re and Froude number Fr are needed when researching power consumption.

Those dimensionless number are defined to impeller, which takes the power P and which diameter is D_a and which rotational speed is n in fluid, which density and viscosity are ρ and η, by following equations:

$$\mathrm{Re} = \frac{nD_a^2\rho}{\eta} \tag{15}$$

$$Po = \frac{P}{n^3 D_a^5 \rho} \tag{16}$$

$$Fr = \frac{n^2 D_a}{g} \tag{17}$$

Agitator Reynolds number Re describes the type of flow near the impeller.

Power number *Po* describes ability of impeller (D_a) transport mechanical power (P) with specific rotational speed (n) to fluid (η). The greater the Power number is more power ca be transport to the fluid and agitation is more efficient.

Froude number *Fr* is significant, when surface of the fluid is not even that will say when waves occur (ships) or when there is a vortex in the agitator.

3.2.1 Reynolds number, Newtonian

According to McCabe et al. (1993, s. 253) the flow is laminar when agitator Reynolds number is smaller than 10 and turbulent when agitator Reynolds number is greater than 10 000.

If diameter of impeller used is around 0.1 m and if agitated fluid is water (ρ = 1000 kg/m^3, η = 1 mPas), rotational speed is:

$$n = Re\frac{\eta}{D_a^2\rho} = 10\frac{1\cdot 10^{-3}}{0.1^2\cdot 1000}\frac{\mathrm{kg/ms}}{\mathrm{m}^2\cdot \mathrm{kg/m}^3} = 0.001\frac{1}{\mathrm{s}} \tag{18}$$

in laminar case and

$$n = Re\frac{\eta}{D_a^2\rho} = 10\,000\frac{1\cdot 10^{-3}}{0.1^2\cdot 1000}\frac{\mathrm{kg/ms}}{\mathrm{m}^2\cdot \mathrm{kg/m}^3} = 1\frac{1}{\mathrm{s}} \tag{19}$$

in turbulent case.

Fiqures in McCabe et al. (1993, s. 250-1) show that Power number is not depended to agitator Reynolds number when Reynolds number is greater than 1000 (or even 100). This can be assumed as a limit for turbulent flow.

3.2.2 Reynolds number, non-Newtonian

Apparent viscosity (η') of non-Newtonian fluid can be calculated from Eq. (8):

$$\eta' = K'\left(\frac{du}{dy}\right)^{n'-1} \tag{8}$$

When using apparent viscosity, agitator Reynolds number can be obtained:

$$Re = \frac{nD_a^2\rho}{\eta'} = \frac{nD_a^2\rho}{K'\left(\frac{du}{dy}\right)^{n'-1}} \tag{20}$$

Because the apparent viscosity is changing when velocity gradient is changing, average value for velocity gradient has to be used in Eq. (21). Velocity gradient in agitated vessels has discussed in McCabe et al. (1993) pages 246–7. The value of maximum gradient with standard turbine is 19*n*.

After McCabe et al. (1993, s. 256) calculating Reynolds number the value of average gradient is 11*n*. Following equation can be obtained:

$$Re = \frac{nD_a^2\rho}{K'(11n)^{n'-1}} = \frac{n^{2-n'}D_a^2\rho}{K'11^{n'-1}} \tag{21}$$

3.2.3 Corresponding rotational speed

If one wants compare agitation of Newtonian and non-Newtonian fluid, the same agitator Reynolds number should be used. If rotational speed for agitating Newtonian fluid is n_N, the rotational speed n for non-Newtonian fluid with same agitator Reynolds number is following:

$$\frac{n_N D_a^2\rho}{\eta_N} = \frac{n^{2-n'}D_a^2\rho}{K'11^{n'-1}} \tag{22}$$

From which can be obtained to the same impeller:

$$\frac{n_N}{\eta_N} = \frac{n^{2-n'}}{K'11^{n'-1}} \tag{23}$$

and finally:

$$n = \sqrt[2-n]{\frac{K'11^{n'-1}}{\eta_N}n_N} \tag{24}$$

where the rotational speed can be calculated, when parameters *K’* and *n’*, and rotational speed n_N are known.

3.2.4 Power Number

With larger Reynolds number (Re > 10 000 or Re > 1000 or even Re > 100) the power number is independent to Reynolds number, and it is specific parameter K_T to each impeller type, McCabe et al. (1993, s. 253):

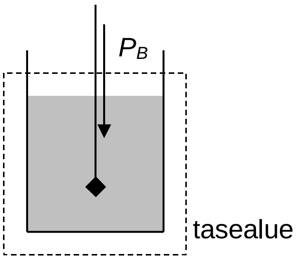
$$Po = K_T \tag{25}$$

from Eq. (17):

$$P = K_T n^3 D_a^5 \rho \tag{26}$$

3.3 RISE OF TEMPERATURE

In steady state, when flow velocities in the vessel are constant, all mechanical energy delivered changes to heat energy.



Kuva 3.

The power loss of mechanical energy is equal to the generation power of heat energy. Because, the total power loss of mechanical energy is equal to the input power of agitator, it can be obtained that:

$$P_{H,GEN} = P_B \tag{27}$$

When the fluid volume of the system is V, the rate of temperature rise is following:

$$\frac{dT}{dt} = \frac{P_{H,GEN}}{V\rho c_p} \tag{28}$$

4. EQUIPMENT USED IN THE LABORATORY

4.1 MIXING EQUIPMENT

- Mixing equipment consist of mixing vessels, shaft, different impellers, and bearing-mounted and torque shaft equipped electric motor.
- Water and wallpaper paste is used as liquids.

4.2 MEASURING APPARATUS

- Torque measuring apparatus is closely installed to the mixing equipment.
- Thermometer in needed to measure temperatures of liquids.
- Viscometer is needed to measure viscosity of viscous fluid.
- Rotational speed can be read from display of the frequency transformer.

5. OPERATING THE MIXER

5.1 STARTING THE WORK

- Fill (or drain) the mixing vessels so that height of the fluid is equal to diameter of the vessel.
- Measure the temperature of the water and check viscosity and density from Keskinen (1989).
- Measure density of viscous fluid (mass/volume). Repeat twice.
- Measure apparent viscosity with viscometer. Use as many rotational speeds as possible.

5.2 MEASUREMENTS WITH WATER

- Four (4) impellers are used in measurements.
- For each impeller six (6) different rotational speeds are used.
- Install first impeller to the mixer.
- Adjust the rotational speed.
- Set weights to the cage so that measuring scales are parallel.
- Write down length of shaft and weight (cage + weights).
- Repeat with every impeller and every rotational speed.

5.3 MEASUREMENTS WITH VISCOSE FLUID

- Measurements are made only with one impeller.
- Six (6) different rotational speeds are used.
- Perform the same measurements as with water.

6. SPECIFICATION

6.1 SHORT REPORT

1. Fill up the form.
2. Only qualitative incorrect estimate is needed.

6.2 EXTENSIVE WORK

1. Fill up the form; it will be attached to the specification as an appendix. Only qualitative incorrect estimate is needed.
2. Calculate the mixing time, which is needed to complete agitation (99 %)" in all measurements to water. Look McCabe et al. (1993) pages 258-9.
3. Calculate what would be power consumption in gassed liquid, when superficial velocity of air is 20 mm/s.

- What would be holdup of air in the tank?
- What would be mean residence time in the tank?
- What would be mean diameter of air bubble?
- What would be surface area per volume unit of bubbles? Look McCabe et al. (1993) page 269-72.

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- Keskinen, K.I., Kemian laitetekniikan taulukoita ja piirroksia, Otakustantamo, 1989
- McCabe, W.L., Smith, J.C. and Harriot, P., Unit Operations of Chemical Engineering, 5th ed., McGraw-Hill, 1993.
- Streeter, V.L., ed., Handbook of Fluid Dynamics, McGraw-Hill, 1961

. APPENDICES

1. Measuring viscosity with Brookfield - viscometer

9. NOMENCLATURE

A	area, m^2
D_a	diameter of impeller, m
D_t	tank diameter, m
E	height of impeller above vessel floor, m
F	force, N
Fr	Froude number, dimensionless
g	gravitational acceleration, m/s^2
H	depth of liquid in vessel, m
J	width of baffles, m
K'	viscosity parameter, Pas
K_T	parameter, dimensionless
L	length of impeller blades, m
l	shaft, m
m	mass, kg
M	torque = Fl, Nm
n	rotational speed, rounds per s, 1/s
n'	viscosity parameter, dimensionless
P_B	input power, W
Po	power number, dimensionless
P_E	power, power of electric motor, W
Re	agitator Reynolds number, dimensionless
rpm	rotational speed, rounds per minute, 1/min
$S1$ - $S6$	shape factors, dimensionless
u	velocity, m/s
W	impeller width, m
y	distance, m

Greek letters

ω *(w)*	radian velocity, rad/s
τ *(tau)*	shear stress = F/A, N
τ_0	boundary shear stress, Pas
η *(eta)*	viscosity, Pas
η'	apparent viscosity, Pas
ρ *(roo)*	density, kg/m^3
η_E	efficiency of electric motor, dimensionless
η_{MEK}	mechanical efficiency, dimensionless

Viskositeetin mittaaminen Brookfield-LVT -viskosimetrillä

Aluksi viskosimetriin liitetään mittauselin, joka on kiinnitetty pyöreään telineeseen. Mittauselimet lähtevät irti telineestä painamalla niitä alaspäin. Mittauselimiä tulee käsitellä varoen, niitä ei missään tapauksessa saa taivuttaa. Huomaa että mittauselimessä kierteet ovat vastapäivään (eli vastoin yleistä käytäntöä).

Mittauksia suoritettaessa on varottava, ettei viskosimetri kastu mittauselimen kiinnityskohdasta tai sen yläpuolelta. Mittaria ei siis saa kääntää ylösalaisin, koska tällöin nestettä pääsee valumaan laitteen sisäosiin.

Haluttu pyörimisnopeus valitaan mittarin oikealla puolella olevasta vivusta. Kun pyörimisnopeutta vaihdetaan, tulee moottorin olla *käynnissä*.

Mittauselin upotetaan tutkittavaan nesteeseen mittauselimessä olevan merkin syvyydelle. Laitteen vasemmassa reunassa olevan vesivaa'an avulla asetetaan mittarin akseli pystysuoraan.

Mittauksen aikana mittarin lukema voidaan lukita painamalla kahvan yläpuolella olevaa vipua. Mittarin lukema pysyy lukittuna niin kauan kun vipua painetaan. Luotettavien tulosten saamiseksi tulee laitteen antaa stabiloitua noin puoli minuuttia.

Mittarin näyttämästä lukemasta saadaan mitattu viskositeetti senttipoiseina (cP) kertomalla lukema taulukon 1 kertoimella.

Taulukko 1. Brookfield-LVT viskosimetrin kertoimet

Kierrosnopeus kierrosta/min	Mittauselin			
	1	2	3	4
0.3	200	1000	4000	20000
0.6	100	500	2000	10000
1.5	40	200	800	4000
3	20	100	400	2000
6	10	50	200	1000
12	5	25	100	500
30	2	10	40	200
60	1	5	20	100

Esimerkki. Kun käytetään mittauselintä nro 2 ja pyörimisnopeutta 3 rpm ja saadaan mittarin lukemaksi 33,5. Tällöin on viskositeetti 100 * 33,5 = 3350 cP.

Mittausvirhe

Valmistajan mukaan mittauksen **absoluuttinen virhe** ei riipu mittarin lukeman suuruudesta ja on 1% koko mitta-asteikosta. Eli, jos mittarin lukema on 100, on **suhteellinen virhe** 1%, mutta jos mittarin lukema on 1, on **suhteellinen virhe** 100% !

Tästä seuraa, että mittaukset on suoritettava siten, että mittarin lukema on mahdollisimman suuri eli kerroin mahdollisimman pieni.