HYDROCYCLONE

1. GENERAL

Hydrocyclones have been discussed for example in McCabe et al. (1993) in pages 1062-3 and in Svarovsky (1984). Hydrocyclones are used in (Svarovsky, 1984, s. 1):

- Clarification of slurry (removing solids) and thickening (removing liquids)
- Sorting of solids
- Washing of solids
- Removing gases from liquid
- Removing immiscible liquid from liquid

Hydrocyclone is a device where solid particles or immiscible liquids are separated from liquid (which is usually water (hydro!)). Separation is based on density gradient between the liquid and the matter to be separated. The separating force is radial acceleration caused by strong rotating motion. Generally the device is a cyclone and the continuous phase is a fluid (gas or liquid). In figure 1 a schema of a cyclone and its characteristic dimensions are shown. The double vortex of a cyclone is illustrated in figure 2.

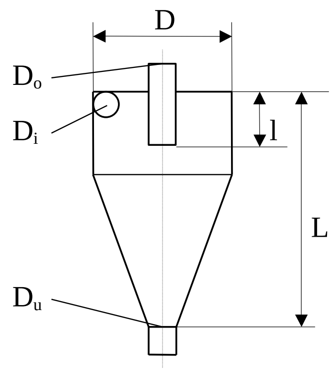


Figure 1. Dimensions of a cyclone.

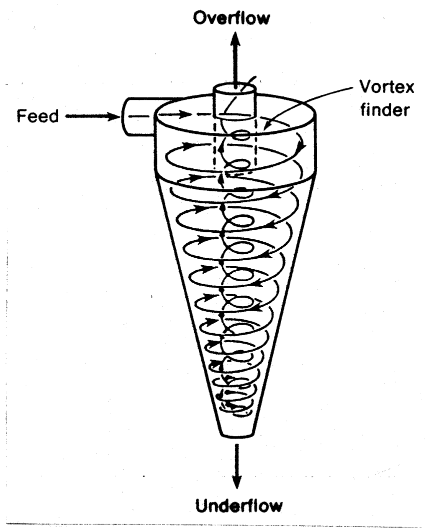


Figure 2. Double vortex

Feed containing solid particles divides into overflow, which has the main part of the fluid, and into underflow, which contains the most of the solids. Separation depends on the size distribution of the solids. This is characterized by the cut size, which is the size of a particle, which has equal probability to end up either in overflow or underflow.

The advantages and disadvantages of hydrocyclone are (Svarovsky, 1984, p. 2):

+ They are very versatile.
+ They are simple and cheap to buy, install and operate.
+ They need only a little space.
+ Great shear forces are advantageous for example if dispersing clustered particles or handling thixotropic liquids.

– They are not flexible.
– The resolution and power of separation is quite poor.
– Great flow rate may cause weariness.
– Great shear forces are disadvantageous because flocculation cannot be used to enhance the separation.

2. CALCULATIONS

Only simple mass balances are considered in this work. Then, separation power can be calculated from the balances. Also the cut size or the dependency of separation power on particle size is studied based on a simple residence time theory. Finally energy balances are considered, that is how the mechanical energy of a fluid changes into heat.

2.1 MASS BALANCE

The flow chart of a cyclone is shown in figure 3.

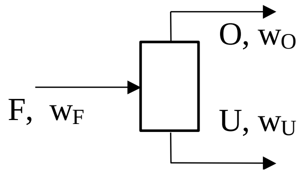


Figure 3. Flow chart of a cyclone.

The concentrations of underflow U and overflow O are determined by weighing samples taken from both of them before and after evaporation. Thus, the concentrations are obtained as mass fractions, which are marked with *w*.

Since concentrations are given in mass fractions, all the flows O, U, and F (feed) are given in mass flows. A volume flow rate $\dot{V}_F$ is measured from the feed flow and it is converted to mass flow by multiplying it with the density of the feed, which is assumed to be (accurate enough)

$$\rho_F = 1000 \text{ kg/m}^3 \tag{1}$$

So, the mass flow rate is

$$F = \rho_F \dot{V}_F \tag{2}$$

Balance calculations are done in mass fraction coordinates where total flows F, O, and U are total mass flows. Then the total mass balance and the solid mass balance for the process shown in figure 3 is:

$$F = O + U \tag{3}$$

$$Fw_F = Ow_O + Uw_U \tag{4}$$

After measurements only O and U remains unknown. These can be easily solved from equations (3) and (4) by multiplying equation (3) with either $-w_O$ or $-w_U$, and adding up the two equations gives

$$F(w_F - w_O) = O(w_O - w_O) + U(w_U - w_O) = U(w_U - w_O) \tag{5a}$$

$$F(w_F - w_U) = O(w_O - w_U) + U(w_U - w_U) = O(w_O - w_U) \tag{5b}$$

From these O and U are obtained:

$$U = \frac{(w_F - w_O)}{(w_U - w_O)} F \tag{6a}$$

$$O = \frac{(w_F - w_U)}{(w_O - w_U)} F \tag{6b}$$

2.2 Separation Efficiencies

Separation efficiencies are usually expressed as a function of particle size. Here no size distribution is measured, so the efficiencies are based only on total and solid balances. Depending on the use of the cyclone, the efficiencies can be defined as follows:

$$E_1 = w_O \qquad E_2 = w_U \tag{7a,b}$$

$$E_3 = \frac{O}{F} \qquad E_4 = \frac{Ow_O}{Fw_F} \tag{8a,b}$$

$$E_5 = \frac{U}{F} \qquad E_6 = \frac{Uw_U}{Fw_F} \tag{9a,b}$$

$$E_7 = \frac{O}{U} \qquad E_8 = \frac{Ow_O}{Uw_U} \tag{10a,b}$$

2.3 Cut Size

Flows inside the cyclone are considered to be as in figure 2 and the cross-section of the cyclone on the height of the feed inlet as in figure 4.

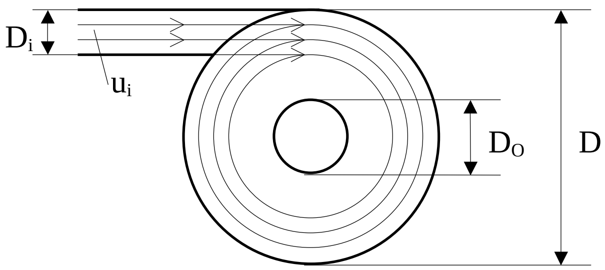


Figure 4. Cross-section of a cyclone.

Feed (here a mixture of water and solid particles) incomes to the cyclone tangentially and is sucked into rotating motion. In first stage it flows against the wall of the cyclone as a vortex to the bottom of the cyclone where the underflow separates from the overflow and the underflow exits the cyclone. After this overflow, which has a diameter about the same size as the outlet pipe, flows as a vortex upwards to the top of the cyclone where overflow exits the cyclone.

The essential thing for the separation in the cyclone is the first stage from the feed point to the bottom of the cyclone. Next, only this stage is considered. According to the residence time –theory (Svarovsky, 1984, 47) a particle ends up to underflow if it has enough time to move to the wall of the cyclone. The time that can be taken to this transfer is called the residence time, which in this case is the time elapsed when feed flows from feed point to the bottom of the cyclone. The driving force of this transfer is the radial acceleration due to the vortex and the velocity of the transfer is the sinking speed due to this acceleration. Since the sinking speed depends on the particle size the probability of separation depends on particle size, too. This probability is handled with concept of cut size.

2.3.1 Cut Size

Cut size is defined as (Svarovsky, 1984, p 23):

- *Cut size d_n is a diameter d for particles, which have a probability of n % to end up in underflow.*

So cut size d_{90} is a particle size, which has a probability of 90 % to end up in underflow.

A rough estimate for cut size is determined here from following condition with which the probability of ending up in underflow can be calculated:

> *The probability of ending up in the underflow is n %,*
> *if the particle sinks n % of the radial depth within the residence time.*

Let's assume that the vortex of the top of the cyclone is the annulus constrained by the diameters D and D_o, so the radial depth of the top is half of the difference of the diameters D and D_o.

Now d_{100} is the size of particles, which sink 100 % of the distance from the inner diameter of the vortex Do to the outer diameter of the vortex D (1.0*0.5(D-D_o)) during the residence time. Respectively d_{50} is half of that distance or 0.5*0.5(D-D_o).

Svarovsky (1984, p 47-50) shows another way to apply the residence time –theory to calculate the separation efficiency. Next, an example how to calculate a rough estimate for cut size d_n can be calculated, is shown.

2.3.2 Normal Acceleration

Separation in hydrocyclone is based on the normal acceleration of the vortex. Since the flows inside the cyclone are known only qualitatively, it is reasonable to calculate the normal acceleration only in the upper section of the cyclone when based on the inner diameter of the cyclone and on the velocity of the incoming fluid.

Since the flow has a depth in the direction of the radius of the cyclone, the maximum acceleration is greater than calculated here (if rotation speed remains the same, is the acceleration greater near the inner wall than near the exterior wall where the radius is greater.) Also the cyclone narrows downward, so the acceleration is greater in bottom.

Normal acceleration based on velocity u_i of incoming liquid and the inner diameter D of the top is obtained from equation (Alonso and Finn, 1980, p 98):

$$a = \omega^2 R = \frac{v^2}{R} = \frac{u_i^2}{D/2} \tag{11}$$

2.3.3 Sinking Speed of a Particle in Liquid

The density gradient and normal acceleration in the direction of the radius of the cyclone produce a force, which makes the particles to sink and end up into underflow. Although the acceleration is great, are the particles so small that the sinking speed can be calculated from *Stokes* equation (McCabe et al. (1993) page 160) in which case the *Reynolds* number needs to be smaller than 1.

$$u_p = \frac{(\rho_s - \rho_l) d_p^2}{18\eta} a \qquad \mathrm{Re}_p = \frac{\rho_l u_p d_p}{\eta} < 1.0 \tag{12}$$

2.3.4 Length of Sinking

The radial depth of the flow in the top of the cyclone is *S*:

$$S = 0.5(D - D_o) \tag{13}$$

Length of sinking is a distance that a particle has to sink in order to end up in the exterior wall of the cyclone. Here the length of sinking is defined as

$$S_n = n * 0.01 * 0.5(D - D_o) \tag{14}$$

where S is the length of sinking and *n* is %.

This definition enables the definition of the cut size as it was defined earlier, in other words the probabilities *n* of length of sinking S_n and the cut size d_n are equal.

This means that cut size d_{100} is size of a particle which has a 100 % probability to end up into underflow or it has enough time to sink trough the whole depth of the flow 0.5*(D-D_o). Correspondingly cut size d_{50} has time to sink half of the whole depth of the flow (0.5*0.5*(D-D_o)).

2.3.5 Residence Time

Available time for sinking is the residence time, which in this case is the time in which the feed flows from the inlet section to the bottom of the cyclone.

Let's assume that:

- The axial cross-sectional area of flow in the cyclone is the annulus constrained by diameters D and D_o.
- The length of the flow is the length L of the cyclone.

Now the residence time τ in the cyclone can be calculated:

$$\tau = \frac{L}{u_A} = \frac{L}{\frac{\dot{V}_F}{A}} = \frac{L}{\frac{\dot{V}_F}{\frac{\pi(D^2 - D_O^2)}{4}}}, \tag{15}$$

where L is height of the cyclone from the inlet point to the bottom, u_A is axial flow velocity, and A is axial cross-sectional area of the flow in the top of the cyclone. Here axial is in the direction of vertical axis of the cyclone.

2.3.6 Sinking Time of Particle

When the particle diameter is known can the sinking time through the whole radial depth be calculated:

$$t_p = \frac{S}{u_p} = \frac{0.5(D - D_o)}{\frac{(\rho_s - \rho_l)d_p^2}{18\eta}a} \tag{16}$$

If $t_p < \tau$ then particle has time to sink all the way to the exterior wall and ends up in the underflow. Analogically with equation (16) the sinking time through a certain depth can be calculated:

$$t_{p,n} = \frac{S_n}{u_{p,n}} = \frac{n*0.01*0.5(D - D_o)}{\frac{(\rho_s - \rho_l)d_p^2}{18\eta}a} \tag{17}$$

If $t_{p,n} < \tau$ then within sinking time a particle sinks n % of the radial depth of the top of the cyclone, so the particle has *n %* probability to end up into the underflow. If stating that $t_{p,n} = \tau$, a condition for cut size d_n is obtained:

$$t_{p,n} = \tau = \frac{n*0.01*0.5(D - D_o)}{\frac{(\rho_s - \rho_l)d_p^2}{18\eta}a}, \tag{18}$$

so

$$d_p = \sqrt{\frac{n*0.01*0.5(D - D_o)18\eta}{(\rho_s - \rho_l)\tau a}} = \sqrt{\frac{\frac{S_n}{\tau}18\eta}{(\rho_s - \rho_l)a}}. \tag{19}$$

2.4 Losses of Energy

2.4.1 Losses of Mechanical Energy in the Cyclone

The balance of mechanical energy between points a and b is

$$\rho\eta_p W_p = (p_b - p_a) + \rho g(z_b - z_a) + \rho\left(\alpha_b \frac{\bar{u}_b^{\,2}}{2} - \alpha_a \frac{\bar{u}_a^{\,2}}{2}\right) + \rho(h_{PIPING} + h_{DEVICE}), \tag{20}$$

where h is the loss of mechanical energy per mass unit of the fluid due to friction. Applying this equation to one process unit, which is a cyclone and has no pump, gives:

$$0 = (p_b - p_a) + \rho g(z_b - z_a) + \rho\left(\alpha_b \frac{\bar{u}_b^{\,2}}{2} - \alpha_a \frac{\bar{u}_a^{\,2}}{2}\right) + \rho h_{CYCLONE} \tag{21}$$

The losses of mechanical energy per mass unit of the fluid $h_{CYCLONE}$ caused by the cyclone can be calculated from this equation. When this is known, the power losses $P_{h,CYCLONE}$ caused by this process unit can be calculated from equation:

$$P_{h,CYCLONE} = \dot{m}h_{CYCLONE} = \dot{V}\rho h_{CYCLONE} \tag{22}$$

In figure 5 is shown the balance area of the hydrocyclone. Point *a* is the inlet point and *b* is both the outlets.

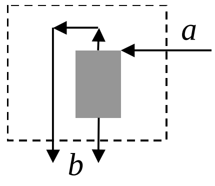


Figure 5. Balance area of the cyclone.

Now the following assumptions are made in equation (21):

Level difference is negligible (when compared to other terms), so $(z_b - z_a) = 0$
The kinetic energies of the exiting flows are negligible, so $\bar{u}_b = 0$

Now equation (21) reduces to:

$$\rho h_{CYCLONE} = (p_a - p_b) + \rho\alpha_a \frac{\bar{u}_a^{\ 2}}{2} \tag{23}$$

and equation (22) to:

$$P_{h,CYCLONE} = \dot{V}\rho h_{CYCLONE} = \dot{V}\left[(p_a - p_b) + \rho\alpha_a \frac{\bar{u}_a^{\ 2}}{2}\right] \tag{24}$$

This is the rate that mechanical energy is lost from the system when fluid flows through the hydrocyclone. This energy must be covered with a pump to maintain the flow.

2.4.2 Losses in System

The losses of mechanical energy to heat energy in the system can be calculated from the total mechanical energy balance.

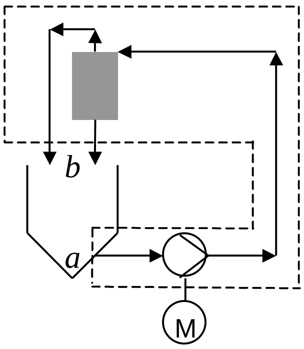


Figure 6. Pumping system

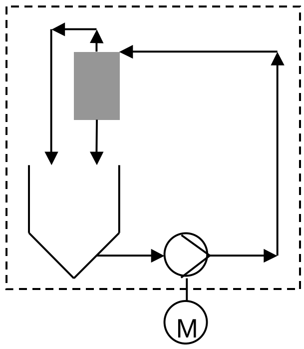


Figure 7. Pumping system

Mechanical energy balance for flow in figure 6 is:

$$\rho\eta_p W_p = (p_b - p_a) + \rho g(z_b - z_a) + \rho\left(\alpha_b \frac{\bar{u}_b^{\,2}}{2} - \alpha_a \frac{\bar{u}_a^{\,2}}{2}\right) + \rho(h_{PIPING} + h_{CYCLONE}) \tag{25}$$

Now it is assumed that:

- Level difference is negligible, so $(z_b - z_a) = 0$
- Difference in kinetic energy in the balance boundaries is negligible, so $\bar{u}_a = \bar{u}_b = 0$
- Pressure in the feed tank on the suction height (in point a) is air pressure (in other words suction point is not in to deep), so $(p_b - p_a) = 0$ because pressure in point b is air pressure.

Now equation (25) reduces to:

$$\rho\eta_p W_p = \rho h_{PIPING} + \rho h_{CYCLONE} \tag{26}$$

Besides these there are energy losses in the pump. These losses are taken into account with the mechanical efficiency of the pump. This loss of brake power in the pump is:

$$P_{h,PUMP} = \dot{V}\rho(1-\eta_p)W_p \tag{27}$$

So, there are three different sources for mechanical energy losses in the system:

Losses of the cyclone: $P_{h,CYCLONE} = \dot{V}\rho h_{CYCLONE} = \dot{V}(p_a - p_b) + \rho\alpha_a \frac{\bar{u}_a^{\,2}}{2}$

Losses of the piping system: $P_{h,PIPING} = \dot{V}\rho h_{PIPING} = \dot{V}\rho\left(h_\xi + h_\zeta\right) = \dot{V}\rho\left(\xi \frac{L}{D} + \sum \zeta_i\right)\frac{\bar{u}^2}{2}$

Losses of the pump: $P_{h,PUMP} = \dot{V}\rho(1-\eta_p)W_p$

From figure 7 can be seen that these three losses are equal to the brake power of the pump, since there are no other source of mechanical energy in the system and the whole brake power is consumed in the system:

$$P_B = P_{h,PIPING} + P_{h,CYCLONE} + P_{h,PUMP} \tag{28}$$

Brake power P_B in the pump depends on the electrical power P_E and the electrical efficiency η_E of the pump:

$$P_E = \frac{P_B}{\eta_E} \tag{29}$$

The electrical power of the motor is marked on the rating plate.

2.4.3 Rise of Temperature in the System

The losses of mechanical energy in the system are calculated above. Actually, the mechanical energy does not disappear but it only translates into heat. So, the power loss of mechanical energy is equal to generation power of heat energy. Since the total losses of mechanical energy are equal to the break power of the pump, it is obtained that:

$$P_{H,GEN} = P_h = P_B = \eta_E P_E \tag{30}$$

If the fluid volume of the system is V, is the speed of temperature rise:

$$\frac{dT}{dt} = \frac{P_{H,GEN}}{V\rho c_p} \tag{31}$$

3. EQUIPMENT

3.1 HYDROCYCLONES

There are two hydrocyclone, delivered by Dorr-Oliver N.V in 1965, in the laboratory: A hydrocyclone “Dorrclone Type T 104 II” and a multicyclone “Multicyclone Type TM1-A10/15-80” which has four hydrocyclones in parallel. For the present only the hydrocyclone is used to this laboratory work. The cut-away drawing of the cyclone is shown in appendix 1.

According to the drawing H52910/2 of Dorr-Oliver the diameter of the top of the cyclone is D = 25 mm, the length between the feed point and underflow exit point is L = 120 mm, and the diameter of the overflow outlet pipe is D_o = 9 mm. The diameter of the feed pipe is assumed to be D_i = 10 mm. The characteristics of the pump used in the equipment are shown in appendix 3. The diameter of the impeller is 169 mm.

3.2 ARRANGEMENTS

- Feed slurry is a mixture of water and calcium carbonate.
- Properties of calcium carbonate are shown in appendix 2.
- Solids content is about 2 w-% or about 20 g/l.

3.3 MEASURING EQUIPMENT

There are already attached in the laboratory equipment:

- Rotameter
- Pressure gauge

In the measuring range 35%-100% the rotameter gives the volume flow rate in percent of the maximum flow rate, which is 0,0015 m^3/s.

Also

- 3+8*2 = 19 evaporating dishes

- thermometer
- measuring tape

are needed for the work.

4. OPERATING THE CYCLONE

4.1 Starting the Work

- Close the bottom valve from the feed tank.
- Fill up the feed tank to the level shown by the assistant.
- Measure and write down the amount of water.
- Open the pox water valve.
- Close the control valve (gray).
- Switch the pump on.
- Open the recycle valve (red) a little in order to generate a flow.
- Pour calcium carbonate an amount given by the assistant into the feed tank.
- Recycle and mix a while.
- Take three (3) samples from the mixture to an evaporating dish.
- Measure the temperature of the mixture.
- Write down time and initial temperature.

4.2 Measurements

- Adjust the flow rate to given value and close the recycling valve.
- Let it stabilize for 1 minute.
- Take a sample from both over- and underflow to an evaporating dish.
- Write down the flow rate and pressure drop.
- Measure the temperature of the mixture.
- Write down time and temperature.
- If temperature is greater than 50 ºC, open the cooling water valve.
- Repeat 8 times in total.

4.3 Ending the Work

- Stop the pump.
- Inform the assistant.
- After permission empty and flush the feed tank.
- Fill up the tank about to half way.
- Switch the pump on and circulate the water for a while.
- Stop the pump and empty the feed tank.
- Close the pox water and cooling water valves.
- Calculate the length of the piping system and its inner diameter.
- Write down all friction losses due to valves, fittings etc.
- Write down the electrical power of the motor of the pump.
- **Clean up** the environment of the laboratory equipment and return all the things you have used to where they belong.

5. SPECIFICATION

5.1 Calculations

- Calculations are done with MS-EXCEL workbook 13-sykl.xls.

5.2 Short Work

- Fill up the form.
- Each member of the group shows calculations for different cases.
- Show O/U, w_O, and w_U graphically as a function of the feed in an appendix.
- Show the change of the most essential efficiency graphically as a function of the feed in an appendix.
- Show the cut size d_{50} graphically as a function of feed in an appendix. Compare your results to the data of appendix 2.
- Only a qualitative incorrect estimate is required.

5.3 Extensive Work

1. Make the calculations, fill up the form, and show the graphs as in brief work. The form is added to the specification.

2. Euler number is defined (Svarovsky, 1984, page 8)

$$Eu = \frac{\Delta p}{0.5\rho u_i^2}$$

Check, how the following correlation holds true for measurement data (Svarovsky, 1984, page 8)

$$Eu = 24.38\,\mathrm{Re}^{0.3748}$$

Assess the accuracy of four significant figures.

6. APPENDICES

1. A cut-away drawing of a hydrocyclone (Dorr-Oliver N.V. part list H.52752/4 31.10.1963)
2. Properties of calcium carbonate.
3. Characteristic of the pump.

7. REFERENCES

McCabe, W.L., 1993, Smith J.C. and Harriot, P., Unit Operations of Chemical Engineering, 5 edition, McGraw-Hill
Svarovsky, L., 1984, Hydrocyclones, Technomic Publ. Co. Inc., Lancaster, PA, USA

8. NOMENCLATURE

a	acceleration, m^2/s
c_p	heat capacity in constant pressure, J/kgK
d_p	particle diameter, m
d_n	diameter of a particle, which has a probability of n % to end up in underflow, m

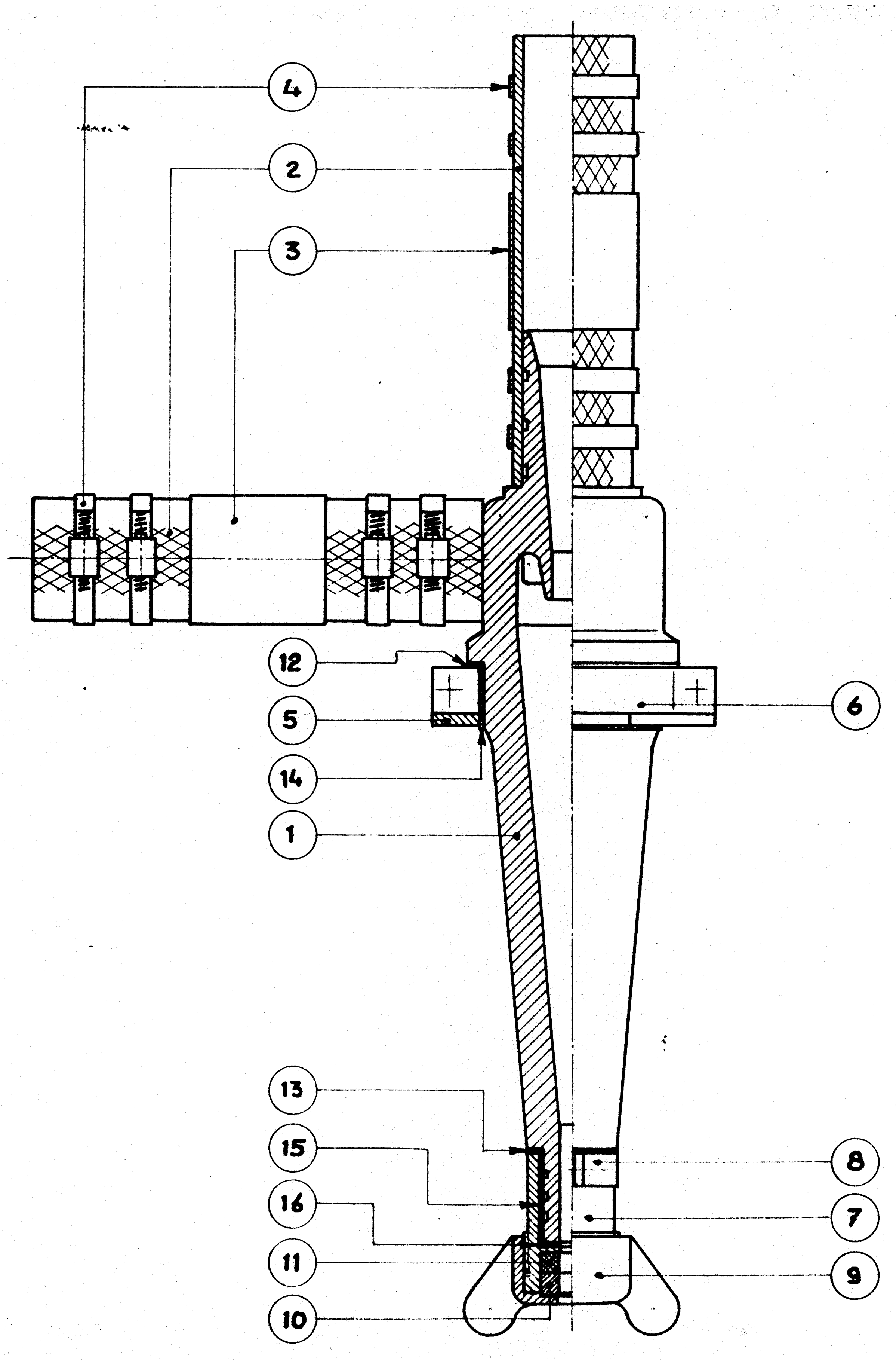
D	inner diameter of the cyclone, m
D_i	inner diameter of the feed pipe, m
D_O	inner diameter of the overflow outlet pipe, m
D_U	inner diameter of the underflow outlet pipe, m
E	efficiency, dimensionless
F	feed flow rate, kg/s
h	loss of mechanical energy per mass unit of the fluid, J/kg
$\dot{m}$	mass flow, m^3/s
p	pressure, Pa
Δp	pressure difference OUT-IN, Pa
l	"vortex finder" length, m
L	length of the cyclone, m
O	overflow rate, kg/s
P_h	power loss, W
$P_{H,GEN}$	generation power of heat energy, heat power, W
Re	Reynolds number, dimensionless
S	distance, m
t	time, s
u	velocity, m/s
u_p	particle velocity, m/s
$u_{p,n}$	velocity of a particle, which diameter is d_n, m/s
U	underflow rate, kg/s
z	height, m
V	volume, m^3
$\dot{V}$	volume flow rate, m^3/s
w	mass fraction, dimensionless

Subindexes

A	axial
F	feed
i	inlet pipe
l	liquid
n	probability n%
p	particle
s	solid

Greek letters

α	factor of kinetic energy, dimensionless
ρ	density, kg/m^3
η	viscosity, Pas
τ	residence time, s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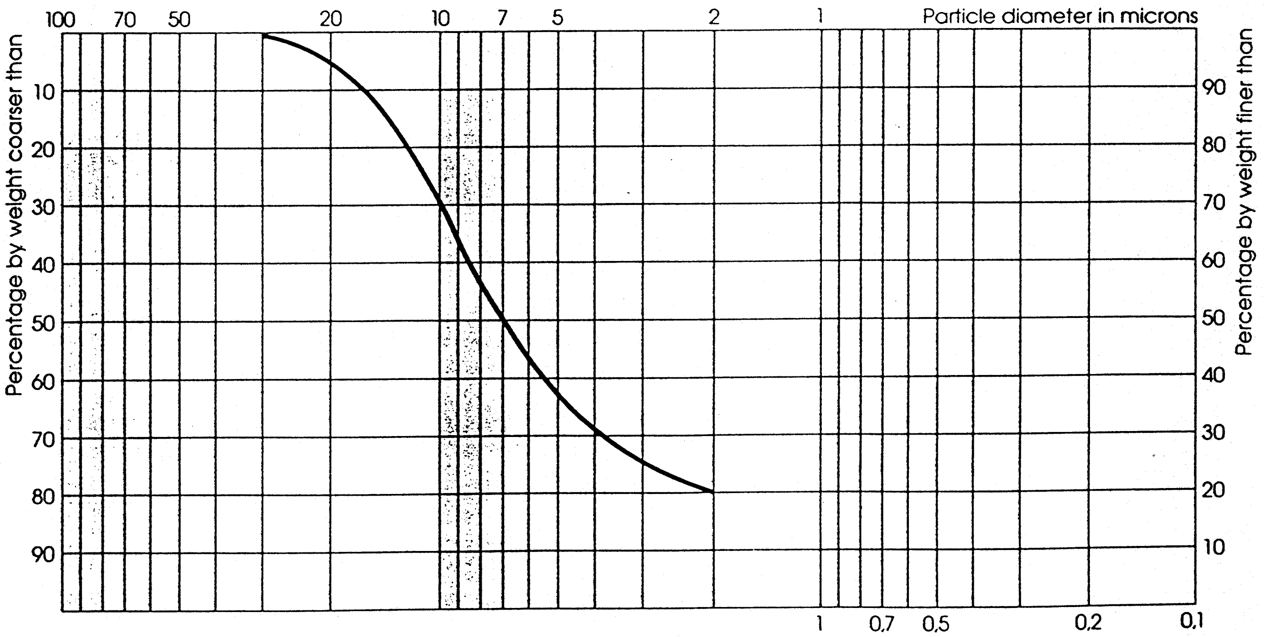
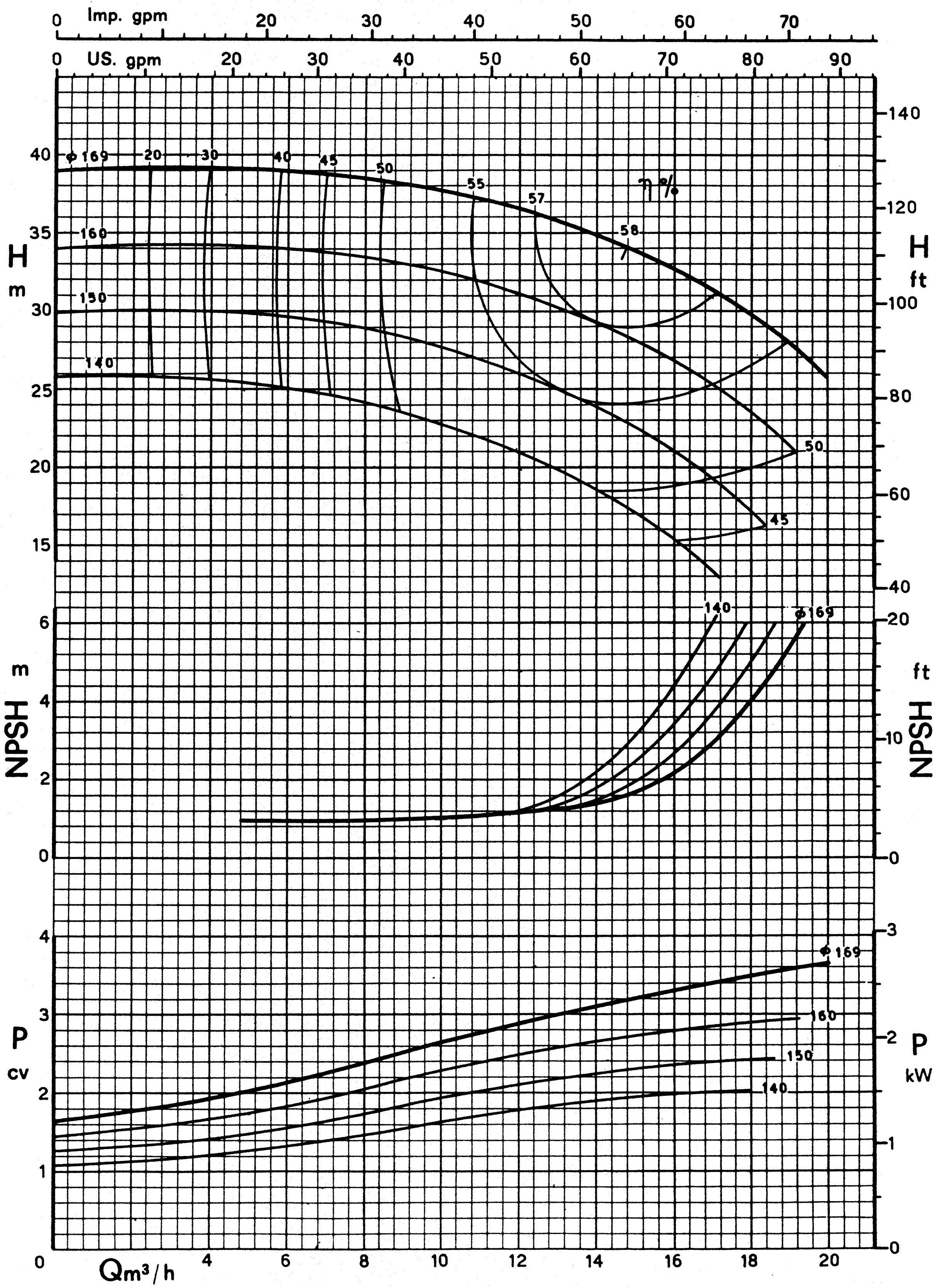
OMYACARB 10-GU

PRODUCTION PLANT:	Gummern / Austria		
SHORT DESCRIPTION OF THE PRODUCT:	Fine, high purity, white marble powder.		
CHEMICAL ANALYSIS OF THE RAW MATERIAL:	Carbonate content	≥ 98	%
	Fe_2O_3	≤ 0.15	%
	HCl insoluble content	≤ 2	%
PHYSICAL PROPERTIES OF THE RAW MATERIAL:	Density (ISO 787/10)	2.7 g/cm^3	
	Refractive index	1.59	
	Hardness (Mohs)	3	

OMYACARB 10-GU
Particle size distribution (Cilas 920)

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