

**Problem 1: Zero sequence and the space vector**

Consider the phase voltages

$$u_a = u'_a + u_0 \quad u_b = u'_b + u_0 \quad u_c = u'_c + u_0$$

where  $u'_a + u'_b + u'_c = 0$  holds and  $u_0$  is the zero-sequence component. Show that the zero sequence disappears in the space-vector transformation.

**Solution**

Let us insert the phase voltages into the space-vector transformation:

$$\begin{aligned} \underline{u}_s^s &= \frac{2}{3} (u_a + u_b e^{j2\pi/3} + u_c e^{j4\pi/3}) \\ &= \frac{2}{3} [(u'_a + u_0) + (u'_b + u_0) e^{j2\pi/3} + (u'_c + u_0) e^{j4\pi/3}] \\ &= \frac{2}{3} (u'_a + u'_b e^{j2\pi/3} + u'_c e^{j4\pi/3}) + \frac{2}{3} \underbrace{(1 + e^{j2\pi/3} + e^{j4\pi/3})}_{=0} u_0 \\ &= \frac{2}{3} (u'_a + u'_b e^{j2\pi/3} + u'_c e^{j4\pi/3}) \end{aligned}$$

It is geometrically clear that  $1 + e^{j2\pi/3} + e^{j4\pi/3} = 0$  holds (draw these three unit vectors to see it). Hence, the zero sequence does not affect the space vector.

**Problem 2: Synchronous machine model in rotor coordinates**

(a) Equations for the stator voltage and stator flux linkage in stator coordinates are

$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt} \quad \underline{\psi}_s^s = L_s \underline{i}_s^s + \psi_f e^{j\vartheta_m}$$

Express these equations in rotor coordinates.

(b) Express the previous equations in rotor coordinates in steady state.

(c) Starting from

$$T_M = \frac{3p}{2} \text{Im} \left\{ \underline{i}_s \underline{\psi}_s^* \right\}$$

derive the torque expression in rotor coordinates as a function of  $i_d$  and  $i_q$ .

**Solution**

(a) Inserting  $\underline{u}_s^s = \underline{u}_s e^{j\vartheta_m}$ ,  $\underline{i}_s^s = \underline{i}_s e^{j\vartheta_m}$ , and  $\underline{\psi}_s^s = \underline{\psi}_s e^{j\vartheta_m}$  into the stator voltage equation leads to

$$\begin{aligned} \underline{u}_s e^{j\vartheta_m} &= R_s \underline{i}_s e^{j\vartheta_m} + \frac{d}{dt} (\underline{\psi}_s e^{j\vartheta_m}) \\ &= R_s \underline{i}_s e^{j\vartheta_m} + \frac{d\underline{\psi}_s}{dt} e^{j\vartheta_m} + \underline{\psi}_s \frac{d e^{j\vartheta_m}}{dt} \\ &= R_s \underline{i}_s e^{j\vartheta_m} + \frac{d\underline{\psi}_s}{dt} e^{j\vartheta_m} + \underline{\psi}_s e^{j\vartheta_m} \frac{d(j\vartheta_m)}{dt} \end{aligned}$$

Dividing the both sides of the equation by  $e^{j\vartheta_m}$  and noting that  $d\vartheta_m/dt = \omega_m$ , the voltage equation in rotor coordinates becomes

$$\underline{u}_s = R_s \underline{i}_s + \frac{d\underline{\psi}_s}{dt} + j\omega_m \underline{\psi}_s \quad (1)$$

Inserting  $\underline{\psi}_s^s = \underline{\psi}_s e^{j\vartheta_m}$  and  $\underline{i}_s^s = \underline{i}_s e^{j\vartheta_m}$  into the flux linkage equation gives

$$\underline{\psi}_s e^{j\vartheta_m} = L_s \underline{i}_s e^{j\vartheta_m} + \psi_f e^{j\vartheta_m}$$

Dividing the both sides by  $e^{j\vartheta_m}$  gives the equation in rotor coordinates,

$$\underline{\psi}_s = L_s \underline{i}_s + \psi_f \quad (2)$$

- (b) In rotor coordinates, the vectors are constant in steady state and  $d/dt = 0$  holds. The voltage equation (1) reduces to

$$\underline{u}_s = R_s \underline{i}_s + j\omega_m \underline{\psi}_s$$

The flux equation (2) holds in steady state as it is.

- (c) Inserting the stator flux linkage in (2) into the torque expressions yields

$$\begin{aligned} T_M &= \frac{3p}{2} \operatorname{Im} \left\{ \underline{i}_s \underline{\psi}_s^* \right\} = \frac{3p}{2} \operatorname{Im} \left\{ \underline{i}_s (L_s \underline{i}_s + \psi_f)^* \right\} \\ &= \frac{3p}{2} \operatorname{Im} \left\{ L_s \underline{i}_s \underline{i}_s^* + \psi_f \underline{i}_s \right\} = \frac{3p}{2} \operatorname{Im} \left\{ \psi_f (i_d + j i_q) \right\} \\ &= \frac{3p}{2} \psi_f i_q \end{aligned}$$

It can be seen that the torque is proportional to  $i_q$  while  $i_d$  does not contribute to the torque at all.

### Problem 3: Operating points of a permanent-magnet synchronous motor

The datasheet values for a three-phase permanent-magnet synchronous motor are:

maximum continuous torque	15 Nm @ 2 400 r/min
voltage constant	0.159 V/(r/min)
number of pole pairs	$p = 4$
stator inductance	$L_s = 4.86$ mH
stator resistance	$R_s = 0.46$ $\Omega$

- (a) The motor rotates at the speed of 2 400 r/min. Calculate the mechanical angular speed, electrical angular speed, and supply frequency.
- (b) Calculate the peak-valued phase-to-neutral back-emf induced by the permanent magnets, when the motor rotates at 2 400 r/min. Calculate also the permanent-magnet flux constant  $\psi_f$ .
- (c) The torque is 15 Nm. Calculate the output power of the motor at the speed of 2 400 r/min and at zero speed.

- (d) The control principle  $i_d = 0$  is used. Calculate the stator current  $\underline{i}_s$  and the stator voltage  $\underline{u}_s$  in the following operating points: 1) torque is 15 Nm at 2400 r/min; 2) torque is 15 Nm at zero speed; and 3) no load at 2400 r/min.

### Solution

- (a) The mechanical angular speed of the rotor is

$$\omega_M = 2\pi n = 2\pi \cdot \frac{2400 \text{ r/min}}{60 \text{ s/min}} = 2\pi \cdot 40 \text{ rad/s}$$

The electrical angular speed is

$$\omega_m = p\omega_M = 4 \cdot 2\pi \cdot 40 \text{ rad/s} = 2\pi \cdot 160 \text{ rad/s}$$

The supply frequency is

$$f = pn = 4 \cdot \frac{2400 \text{ r/min}}{60 \text{ s/min}} = 160 \text{ Hz}$$

- (b) The steady-state voltage equation can be represented as

$$\begin{aligned} \underline{u}_s &= R_s \underline{i}_s + j\omega_m \underline{\psi}_s = R_s \underline{i}_s + j\omega_m (L_s \underline{i}_s + \psi_f) \\ &= (R_s + j\omega_m L_s) \underline{i}_s + \underline{e}_s \end{aligned} \quad (3)$$

where the last term  $\underline{e}_s = j\omega_m \psi_f$  is the back-emf induced by the permanent magnets. The back-emf depends only on the speed (but not on the current).

Unless otherwise noted, the voltage values given in the datasheets and nameplates refer to rms-valued line-to-line voltages. Taking this into account, the peak-valued phase-to-neutral back-emf at the given speed is

$$|\underline{e}_s| = \sqrt{\frac{2}{3}} \cdot 0.159 \frac{\text{V}}{\text{r/min}} \cdot 2400 \text{ r/min} = 311.6 \text{ V}$$

The flux constant (or the permanent-magnet flux linkage) is

$$\psi_f = \frac{|\underline{e}_s|}{\omega_m} = \frac{311.6 \text{ V}}{2\pi \cdot 160 \text{ rad/s}} = 0.31 \text{ Vs}$$

It is worth noticing that the flux constant does not depend on the speed (i.e., the same value for  $\psi_f$  would be obtained at any other speeds).

**Remark:** In order to be able to use the standard equations and equivalent circuits, it is a recommended practice to transform all parameter values to SI units and line-to-line voltages to line-to-neutral voltages (as we did here). The results of the calculations can then be transformed back to the required form.

- (c) At the speed of 2400 r/min, the output power (mechanical power) is

$$P_M = T_M \omega_M = 15 \text{ Nm} \cdot 2\pi \cdot 40 \text{ rad/s} = 3.77 \text{ kW}$$

At zero speed, the mechanical power is zero (but the power fed to the stator is positive due to the losses).

(d) The q-axis current  $i_q$  can be solved from the torque expression:

$$T_M = \frac{3p}{2} \psi_f i_q \quad \Rightarrow \quad i_q = \frac{2}{3p\psi_f} T_M$$

Due to the control principle  $i_d = 0$ , the stator current vector is  $\underline{i}_s = i_d + j i_q = j i_q$ . The stator current is proportional to the torque and independent of the speed.

The stator voltage  $\underline{u}_s$  can be calculated from (3) using the previously calculated values for  $\omega_m$  and  $\underline{i}_s$ . The results are collected in the table below.

	$\omega_m = 0$	$\omega_m = 2\pi \cdot 160 \text{ rad/s}$
$T_M = 0$	$\underline{i}_s = 0 + j0$ $\underline{u}_s = 0 + j0$	$\underline{i}_s = 0 + j0$ $\underline{u}_s = 0 + j311.6 \text{ V}$
$T_M = 15 \text{ Nm}$	$\underline{i}_s = 0 + j8.1 \text{ A}$ $\underline{u}_s = 0 + j3.7 \text{ V}$	$\underline{i}_s = 0 + j8.1 \text{ A}$ $\underline{u}_s = -39.4 + j315.4 \text{ V}$

**Remark:** The vector diagram for a positive torque and a positive speed is illustrated in the figure below. The stator resistance  $R_s = 0$  is assumed for simplicity. Could you sketch the vectors in the previous operating points. What would happen, if the speed or the torque were reversed?

