Answer all five questions (in English, Finnish, or Swedish). Using a calculator is allowed, but all memory must be cleared!

1. Describe the field-oriented control system of permanent-magnet synchronous motors. Draw also the block diagram of the control system, label the signals in the diagram, and describe the tasks of the blocks.

   **Solution:**

   See lectures and readings.

2. Answer briefly to the following questions:
   (a) What is the expression for the current space vector as a function of the phase currents $i_\mathrm{a}$, $i_\mathrm{b}$, and $i_\mathrm{c}$? Also explain how the vector behaves in the steady state.
   (b) How the physical size of the motor approximately depends on the rated values of the motor?
   (c) What is the operating principle of a synchronous reluctance motor?

   **Solution:**

   See lectures, exercises, and readings.

3. (a) Express the dynamic model of the DC motor.
   (b) Sketch the steady-state characteristics of the flux factor $k_\mathrm{f}$, current $I_\mathrm{a}$, torque $T_\mathrm{M}$, and power $P_\mathrm{M}$ as a function of the speed $\omega_\mathrm{M}$. Include the field-weakening region. Assume that the machine is lossless and equipped with the field winding.

   **Solution:**

   See lectures, exercises, and readings.

4. The datasheet values for a three-phase permanent-magnet synchronous motor are:

| | |
|---|---|
| maximum continuous torque | 15 Nm @ 2 400 r/min |
| voltage constant | 0.159 V/(r/min) |
| number of pole pairs | $p = 4$ |
| stator inductance | $L_\mathrm{s} = 4.86$ mH |
| stator resistance | $R_\mathrm{s} = 0.46\ \Omega$ |

   (a) The motor rotates at the speed of 1 800 r/min. Calculate the mechanical angular speed, electrical angular speed, and supply frequency.
   (b) Calculate the peak-valued phase-to-neutral back-emf induced by the permanent magnets, when the motor rotates at 1 800 r/min. Calculate also the permanent-magnet flux constant $\psi_\mathrm{f}$.
   (c) The torque is $-30$ Nm. Calculate the output power of the motor at the speed of 1 800 r/min and at zero speed.

(d) The control principle $i_\mathrm{d} = 0$ is used. Calculate the stator current $\underline{i}_\mathrm{s}$ and the stator voltage $\underline{u}_\mathrm{s}$ in the following operating points: 1) torque is −30 Nm at 1 800 r/min; 2) torque is −30 Nm at zero speed; and 3) no load at 1 800 r/min.

**Solution:**

(a) The mechanical angular speed of the rotor is

$$\omega_\mathrm{M} = 2\pi n = 2\pi \cdot \frac{1\,800\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 2\pi \cdot 30\ \mathrm{rad/s}$$

The electrical angular speed is

$$\omega_\mathrm{m} = p\omega_\mathrm{M} = 4 \cdot 2\pi \cdot 30\ \mathrm{rad/s} = 2\pi \cdot 120\ \mathrm{rad/s}$$

The supply frequency is

$$f = pn = 4 \cdot \frac{1\,800\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 120\ \mathrm{Hz}$$

(b) The steady-state voltage equation can be represented as

$$\begin{aligned}\underline{u}_\mathrm{s} &= R_\mathrm{s}\underline{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}\underline{\psi}_\mathrm{s} = R_\mathrm{s}\underline{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}(L_\mathrm{s}\underline{i}_\mathrm{s} + \psi_\mathrm{f}) \\ &= (R_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}L_\mathrm{s})\underline{i}_\mathrm{s} + \underline{e}\end{aligned} \tag{1}$$

where the last term $\underline{e} = \mathrm{j}\omega_\mathrm{m}\psi_\mathrm{f}$ is the back-emf induced by the permanent magnets. The back-emf depends only on the speed (but not on the current).

Unless otherwise noted, the voltage values given in the datasheets and nameplates refer to rms-valued line-to-line voltages. Taking this into account, the peak-valued phase-to-neutral back-emf at the given speed is

$$|\underline{e}| = \sqrt{\frac{2}{3}} \cdot 0.159\ \frac{\mathrm{V}}{\mathrm{r/min}} \cdot 1\,800\ \mathrm{r/min} = 233.7\ \mathrm{V}$$

The flux constant (or the permanent-magnet flux linkage) is

$$\psi_\mathrm{f} = \frac{|\underline{e}|}{\omega_\mathrm{m}} = \frac{233.7\ \mathrm{V}}{2\pi \cdot 120\ \mathrm{rad/s}} = 0.31\ \mathrm{Wb}$$

It is worth noticing that the flux constant does not depend on the speed (i.e., the same value for $\psi_\mathrm{f}$ would be obtained at any other speeds).

**Remark:** In order to be able to use the standard equations and equivalent circuits, it is a recommended practice to transform all parameter values to SI units and line-to-line voltages to line-to-neutral voltages (as we did here). The results of the calculations can then be transformed back to the required form.

(c) At the speed of 1 800 r/min, the output power (mechanical power) is

$$P_\mathrm{M} = T_\mathrm{M}\omega_\mathrm{M} = -30\ \mathrm{Nm} \cdot 2\pi \cdot 30\ \mathrm{rad/s} = -5.65\ \mathrm{kW}$$

The negative sign means that the motor is braking (i.e., operates in the regenerating mode). At zero speed, the mechanical power is zero (but the power fed to the stator is positive due to the losses).

(d) The q-axis current $i_\mathrm{q}$ can be solved from the torque expression:

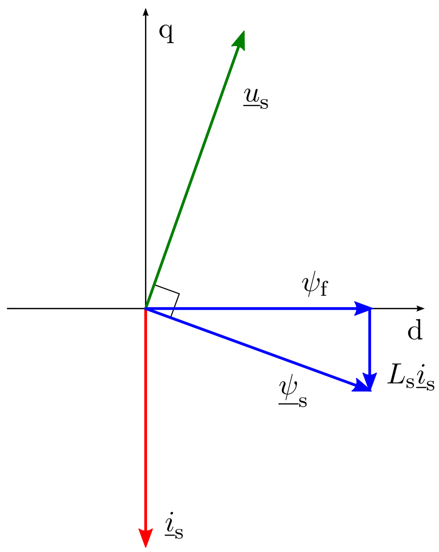
$$T_\mathrm{M} = \frac{3p}{2}\psi_\mathrm{f} i_\mathrm{q} \qquad \Rightarrow \qquad i_\mathrm{q} = \frac{2}{3p\psi_\mathrm{f}} T_\mathrm{M}$$

Due to the control principle $i_\mathrm{d} = 0$, the stator current vector is $\underline{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = \mathrm{j}i_\mathrm{q}$. The stator current is proportional to the torque and independent of the speed.

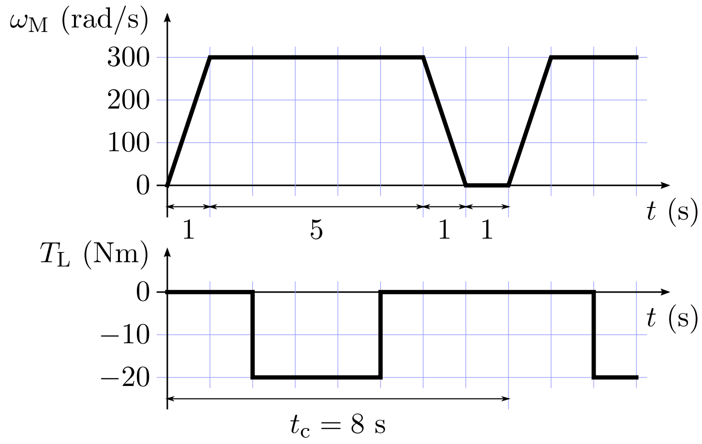
The stator voltage $\underline{u}_\mathrm{s}$ can be calculated from (1) using the previously calculated values for $\omega_\mathrm{m}$ and $\underline{i}_\mathrm{s}$. The results are collected in the table below.

| | $\omega_\mathrm{m} = 0$ | $\omega_\mathrm{m} = 2\pi \cdot 120$ rad/s |
|---|---|---|
| $T_\mathrm{M} = 0$ | $\underline{i}_\mathrm{s} = 0 + \mathrm{j}0$<br>$\underline{u}_\mathrm{s} = 0 + \mathrm{j}0$ | $\underline{i}_\mathrm{s} = 0 + \mathrm{j}0$<br>$\underline{u}_\mathrm{s} = 0 + \mathrm{j}233.7$ V |
| $T_\mathrm{M} = -30$ Nm | $\underline{i}_\mathrm{s} = 0 - \mathrm{j}16.1$ A<br>$\underline{u}_\mathrm{s} = 0 - \mathrm{j}7.4$ V | $\underline{i}_\mathrm{s} = 0 - \mathrm{j}16.1$ A<br>$\underline{u}_\mathrm{s} = 59.0 + \mathrm{j}226.3$ V |

**Remark:** The vector diagram for a negative torque and a positive speed is illustrated in the figure below. The stator resistance $R_\mathrm{s} = 0$ is assumed for simplicity.



5. In periodic duty, the mechanical angular speed $\omega_M$ and load torque $T_L$ vary as shown in the figure. The total equivalent inertia is 0.02 kgm$^2$. The cycle duration is $t_c = 8$ s.
   (a) Draw the conceptual waveforms of the electromagnetic torque $T_M$ and mechanical power $p_M$ for one cycle.
   (b) Calculate the rms value of the electromagnetic torque.
   (c) A permanent-magnet DC motor is applied in this periodic duty. The rated torque and rated armature current of the motor are $T_N = 14.3$ Nm and $I_N = 33$ A, respectively. What is the maximum magnitude of the armature current during the period?




**Solution:**

(a) The required electromagnetic torque is

$$T_M = J\frac{d\omega_M}{dt} + T_L$$

where $J = 0.02$ kgm$^2$ is the total equivalent inertia. The mechanical power is

$$p_M = T_M\omega_M$$

During acceleration at $t = 0 \ldots 1$ s, the electromagnetic torque is

$$T_M = J\frac{\Delta\omega_M}{\Delta t} = 0.02 \text{ kgm}^2 \cdot \frac{300 \text{ rad/s}}{1 \text{ s}} = 6 \text{ Nm}$$

During the loading phase at $t = 2 \ldots 5$ s, the electromagnetic torque has to be $T_M = T_L = -20$ Nm since the speed is constant. The braking torque at $t = 6 \ldots 7$ s is

$$T_M = J\frac{\Delta\omega_M}{\Delta t} = 0.02 \text{ kgm}^2 \cdot \frac{-300 \text{ rad/s}}{1 \text{ s}} = -6 \text{ Nm}$$

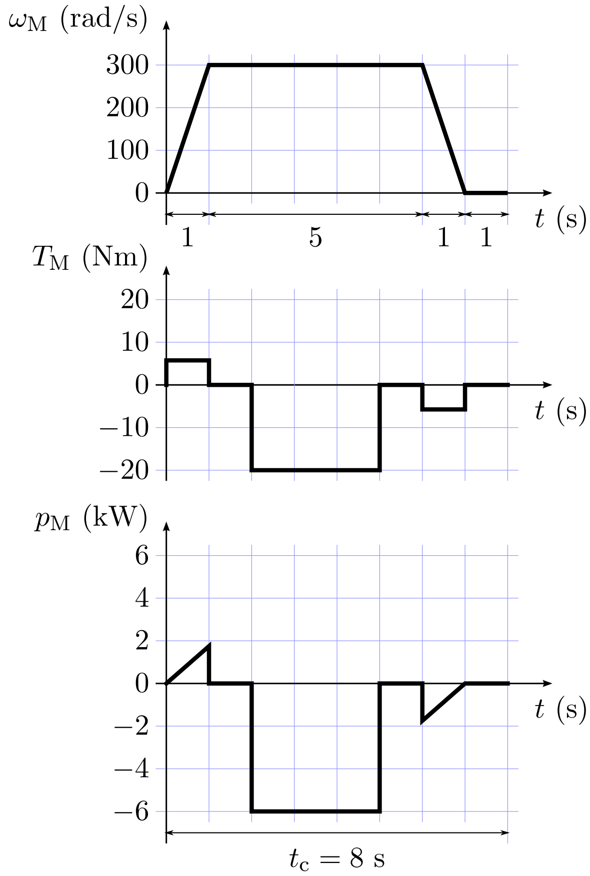
The value of the mechanical power at $t = 1$ s is

$$p_M = 6 \text{ Nm} \cdot 300 \text{ rad/s} = 1.8 \text{ kW}$$

and at $t = 2 \ldots 5$ s it is

$$p_\mathrm{M} = -20\ \mathrm{Nm} \cdot 300\ \mathrm{rad/s} = -6.0\ \mathrm{kW}$$

Based on the above calculations, we can plot the following waveforms.




(b) The rms value of the electromagnetic torque over the period $t_\mathrm{c}$ is

$$T_\mathrm{M,rms} = \sqrt{\frac{1}{t_\mathrm{c}} \int_0^{t_\mathrm{c}} T_\mathrm{M}^2\, \mathrm{d}t}$$
$$= \sqrt{\frac{(6\ \mathrm{Nm})^2 \cdot 1\ \mathrm{s} + (20\ \mathrm{Nm})^2 \cdot 3\ \mathrm{s} + (-6\ \mathrm{Nm})^2 \cdot 1\ \mathrm{s}}{8\ \mathrm{s}}} = 12.6\ \mathrm{Nm}$$

**Remark:** The period $t_\mathrm{c} = 8$ s is much shorter than thermal time constants (several minutes or tens of minutes) of motors in this power range. Hence, the motor can be selected based on the average temperature rise, which leads to the selection criterion $T_\mathrm{N} > T_\mathrm{M,rms}$. Furthermore, the motor should be able to produce the required maximum torque $T_\mathrm{M,max} = 20$ Nm.

(c) The flux factor of the motor is $k_\mathrm{f} = T_\mathrm{N}/I_\mathrm{N} = 14.3\ \mathrm{Nm}/33\ \mathrm{A} = 0.43$ Nm/A. The maximum magnitude of the armature current is $I_\mathrm{a} = T_\mathrm{M}/k_\mathrm{f} = 20\ \mathrm{Nm}/0.43\ \mathrm{Nm/A} = 46.2$ A.