1. Mix design of light weight concrete

2. Estimation of the strength development of concrete

Exercise 9
13.11.2015

Mix design of light weight concrete
**Lightweight concrete mix proportioning**

**Step 1.** Choice of concrete slump  
**Step 2.** Choice of max. aggregate size  
**Step 3.** Estimation of mixing water & Air content  
**Step 4.** Water cement ratio  
**Step 5.** Calculation of the amount of cement  
**Step 6.** Estimation of coarse aggregate  
**Step 7.** Calculation of fine aggregate

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**LWC exercise**

- 100 mm LWC slab  
- Required compressive strength at 28 days is 24 MPa  
- **From light weight aggregate manufacturer:**  
  - Coarse aggregate factor (CAF) is 516 kg/m³ at a 15% moisture content (“as-is” condition)  
  - Specific gravity factor = 1.48 at 15% moisture content  
- **From sand supplier:**  
  - Specific gravity = 2.6, fineness modulus = 2.8
Step 1. choice of concrete slump

- Min. slump = 25 mm
- Max. Slump = 75 mm

➤ **Slump: 75 mm; ordinary placement**

<table>
<thead>
<tr>
<th>Types of construction</th>
<th>Slump, (mm)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Minimum</td>
<td></td>
</tr>
<tr>
<td>Beams and reinforced walls</td>
<td>100</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Building columns</td>
<td>100</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Floor slabs</td>
<td>75</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Step 2. max. aggregate size

- Max. aggregate size $< \frac{slab \ depth}{3}$
- Max. aggregate size $< \frac{100}{3}$
- Max. aggregate size = 19 mm
Step 3. **Water & Air content**

- Concrete slab is normally non air-entrained concrete
- Max. aggregate size = 19 mm & Slump = 75 mm
  - **Water** = 202 kg/m³
  - **Air content** = 2%

<table>
<thead>
<tr>
<th>Slump, (mm)</th>
<th>Mixing Water (kg/m³) of concrete for indicated nominal maximum sizes of aggregates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9.5 mm</td>
</tr>
<tr>
<td>Air-entrained concrete</td>
<td>181</td>
</tr>
<tr>
<td>25 to 50</td>
<td>202</td>
</tr>
<tr>
<td>75 to 100</td>
<td>211</td>
</tr>
<tr>
<td>125 to 150</td>
<td></td>
</tr>
</tbody>
</table>

**Recommended average total air content, percent, for level of exposure**

- Mild exposure = 4.5
- Moderate exposure = 6.0
- Extreme exposure = 7.5

<table>
<thead>
<tr>
<th>Slump, (mm)</th>
<th>Non Air-Entrained Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>208</td>
</tr>
<tr>
<td>25 to 50</td>
<td>229</td>
</tr>
<tr>
<td>75 to 100</td>
<td>237</td>
</tr>
<tr>
<td>125 to 150</td>
<td></td>
</tr>
</tbody>
</table>

Approximate amount of entrapped air in non-air-entrained concrete, percent

<table>
<thead>
<tr>
<th>Approximate amount of entrapped air in non-air-entrained concrete, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Step 4. **Water cement ratio**

- Design strength (28d) = 24 Mpa
- 8 MPa (required over design – standard deviation strength)
- The target mean strength = 24 MPa + 8 MPa = 32 MPa
  - **W/C** = \( \frac{0.57-0.48}{34.5-27.6} \times (34.5 - 32) \) + 0.48 = 0.51

<table>
<thead>
<tr>
<th>Compressive strength at 28 days, (MPa)</th>
<th>Approximate water-cement ratio, by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nonair-entrained concrete</td>
</tr>
<tr>
<td>41.4</td>
<td>0.41</td>
</tr>
<tr>
<td>34.5</td>
<td>0.48</td>
</tr>
<tr>
<td>27.6</td>
<td>0.57</td>
</tr>
<tr>
<td>20.7</td>
<td>0.68</td>
</tr>
<tr>
<td>13.8</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Step 5. cement amount

- Water = 202 kg/m³
- W/C = 0.51

\[
\text{Cement} = \frac{202}{0.51} = 396 \text{ kg/m}^3
\]

Step 6. coarse aggregate

- From light weight aggregate manufacturer:
  - Coarse aggregate factor (CAF) is 516 kg/m³ at a 15% moisture content
- From sand supplier:
  - specific gravity = 2.6, fineness modulus = 2.8
- The table indicates that 0.70 m³ of coarse aggregate, on a dry-loose basis, may be used for each cubic meter of concrete

\[
\text{Coarse aggregate volume} = 0.7 \times 1 \text{ m} = 0.7 \text{ m}^3
\]

\[
\text{Coarse aggregate dry weight} = 0.7 \text{ m} \times 516 \text{ kg/m} = 361.2 \text{ kg}
\]
Step 7. fine aggregate

- Cement volume = \( \frac{396}{3.1 \times 1000} \) = 0.128 m³
- Water = 0.202 m³
- Air = 0.020 m³
- Coarse aggregate
  - Specific gravity factor = 1.48 15% moisture content
  - Volume = \( \frac{\text{weight}}{SGF \times \rho_w} = \frac{361.2}{1.48 \times 1000} \) = 0.244 m³

- Fine aggregate (sand) volume = 1 – 0.128 – 0.202 – 0.02 – 0.244 = 0.406 m³
- Fine aggregate (sand) weight = 0.406 m \( \times \) 2670 kg/m
  = 1048 kg

final LWC mic

Weights: 1 m³

- Cement 396 kg
- LWA (as is) 361 kg
- Sand (dry) 1048 kg
- Water 202 kg

Total 2007 kg/m³
Estimation of the strength development of concrete

Strength development

Nominal strength
Moulds removing
Freezing strength

From: by201, p. 347
Factors affecting concrete strength

- Concrete porosity
  - the more porous the concrete, the weaker it will be

- Water/cement ratio
  - In mixes where the w/c is greater than approximately 0.4, all the cement can, in theory, react with water to form cement hydration products.
  - At higher w/c ratios it follows that the space occupied by the additional water above w/c=0.4 will remain as pore space filled with water, or with air if the concrete dries out.

- Soundness of aggregate
  - if the aggregate in concrete is weak, the concrete will also be weak

Factors affecting concrete strength

- Aggregate-paste bond
  - the integrity of the bond between the paste and the aggregate is critical.
  - If there is no bond, the aggregate effectively represents a void; which are a source of weakness in concrete.

- Cement composition related parameters
  - alite (Ca$_3$SiO$_5$ = C$_3$S) content: more alite should give better early (7d) strengths
  - alite and belite (Ca$_2$SiO$_4$ = C$_2$S) reactivity
  - cement sulfate content: both the clinker sulfate and added gypsum, retards the hydration of the aluminate phase
  - cement surface area and particle size distribution
Factors affecting concrete strength

• Curing temperature
  – Higher curing temperatures promote an early strength gain in concrete but may decrease its 28-day strength.

![Graph showing curing temperature vs. compressive strength](image)

T > 40 °C
→ rapid strength development
→ strength depletion (lujuuskato) of about 10...20 %

What happens when concrete freezes?

• Temperature around -1 °C
  – Pore water in concrete starts to freeze
  – As some water freezes the ion concentration in the unfrozen water goes up, further depressing the freezing point.

• Temperature around -3 to -4°C
  – enough of the pore water will freeze → hydration will completely stop, and thus the development of the compressive strength
  – Water expands up to 9% of its volume when it freezes, causing cracks in the concrete matrix
  – Up to 50% strength reduction can occur if concrete freezes before reaching 5 MPa
What happens when concrete freezes?

Equivalent age (ekvivalentti-ikä)

- the actual age of the concrete is converted to its equivalent age, in terms of strength gain, at the reference temperature.
- In European practice, the reference temperature is usually taken to be 20 °C,
- whereas in North American practice it is usually taken to be 23 °C.
For example:

If the rate of strength development doubles at +50 °C compared to the strength development at +20 °C

the equivalent age of the concrete at the age of 1 d at 50 °C is 2d at 20°C.

Concrete maturity age

Maturity function for computing a maturity age from the recorded temperature history of the concrete. The function was based on the Arrhenius equation that is used to describe the effect of temperature on the rate of a chemical reaction

\[ f = e^{-\frac{E}{R} \left(\frac{1}{T} - \frac{1}{T_r}\right)} \]

where

- \( f \) is the chemical reaction rate
- \( E \) is the activation energy, 33.5 kJ/mol (from a Danish study)
- \( T \) is the temperature [°K] = (°C+273)
- \( R \) is the universal gas constant 8,314 J/mol·K
- \( T_r \) is the absolute reference (20°C) temperature [K]

The equation is also suitable for the estimation of strength development rate of concrete.
Concrete maturity age

• Sadgrove Method

\[ t_{20} = \left( \frac{T + 16 \, ^\circ C}{36 \, ^\circ C} \right)^2 \times t \]

Where
- \( t_{20} \) is the concrete maturity age [d]
- \( T \) is the concrete temperature at a time \( t \) [°C]
- \( t \) is the time for the hardening [d]

\[ T_{20} = k \times (T + 10 \, ^\circ C) \times t \]

Where
- \( T_{20} \) is the concrete maturity age [d]
- \( T \) is the concrete temperature at a time \( t \) [°C]
- \( t \) is the time for the hardening [d]
- \( k \) is coefficient
  - \( k = 1 \) when \( +50 \, ^\circ C \geq T \geq 0 \, ^\circ C \)
  - \( k = 0,4 \) when \( 0 \, ^\circ C > T \geq -10 \, ^\circ C \)
  - \( k = 0 \) when \( T < -10 \, ^\circ C \)
Strength development of concretes made of Normal (CEM I 42.5 N) and SR (CEM I 42.5 N - SR) cements as a function of maturity age
Strength development of concretes made of Rapid (CEM II/A-LL 42,5 R) and Mega (CEM I 52,5 N) cements as a function of maturity age

CEM I 52,5 R
Portland cement with high initial strength

Kuva 8.9  Erittäin nopeasti kovettuvaa sementitä käytettäessä betonin suhdeellinen lujuuden kohitys kypsyyssä funktiona. Betoni on valmistettu Pikasementtiä käyttäen.
Formulas for the estimation of strength development

Sadgrove

\[ f = \left( \frac{T+16}{36} \right)^2 \]

Arrhenius

\[ f = e^{-\left( \frac{E(293-T)}{293RT} \right)} \]

EXERCISE 01
1. The temperature of a K30 (CEM I 42,5 N) concrete right after the casting was measured at +15 °C. The temperatures were measured also after the casting and the results of measurements were:

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h</td>
<td>+15 °C</td>
</tr>
<tr>
<td>6h</td>
<td>+20 °C</td>
</tr>
<tr>
<td>24h</td>
<td>+25 °C</td>
</tr>
<tr>
<td>2d</td>
<td>+20 °C</td>
</tr>
<tr>
<td>3d</td>
<td>+10 °C</td>
</tr>
<tr>
<td>4d...28d</td>
<td>+5 °C</td>
</tr>
</tbody>
</table>

Calculate using the Sadgrove equation

a) At what time did the concrete reach its freezing strength?

b) At what time did the concrete reach its disassembly strength of the moulds? The construction load was 1,5 MN/m² and the design load 2,5 MN/m²

c) What was the strength of the concrete at the age of 28d?

\[ t_{20} = \left( \frac{T + 16 \degree C}{36 \degree C} \right)^2 \times t \]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Hardening-</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>0-0,25</td>
</tr>
<tr>
<td>0,25</td>
<td>20</td>
<td>0,25-1</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>1-2</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2-3</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>3-4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4-5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5-28</td>
</tr>
</tbody>
</table>
a) At what time did the concrete reach its freezing strength?

Freezing strength = 5 Mpa
Concrete K30 strength = 30MPa
\( \frac{f_{c,\text{freezing}}}{f_{c,28d}} = \frac{5}{30} = 0.17 = 17\% \) of the strength
Cement in normal cement

\[ T_{20}(17\%) = 0.8\, d \]

<table>
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<tr>
<th>Measurement</th>
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<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t ) [d]</td>
<td>( T ) [°C]</td>
<td>period [d]</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>( 0-0.25 )</td>
</tr>
<tr>
<td>0.25</td>
<td>20</td>
<td>( 0.25-1 )</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>( 1-2 )</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>( 2-3 )</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>( 3-4 )</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>( 4-5 )</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>( 5-28 )</td>
</tr>
</tbody>
</table>

0.8d is realized during the second period
\( \frac{(0.8-0.21)}{(1.07-0.21)} = 0.68605 \)
The duration of the second period is 0.75 d
Thus the time is completed at 0.75d * 0.68605 = 0.5145 d
Starting from time 0 : 0.25d + 0.5145d = 0.7645 d
In hours 24h/d * 0.7645d = 18.3 h
b) At what time did the concrete reach its disassembly strength of the moulds? 
The construction load was 1,5 MN/m² and the design load 2,5 MN/m²?

\[ K_m = K \times \frac{F}{F_k} = 30 \times \frac{1,5}{2,5} = 18 \text{ MPa} \]
\[ \frac{18}{30} = 60\% \]
\[ t_{20} \text{ (60\%)} = 5,6 \text{ d} \]

is realized after 5d

\[
\Delta t_{20} = (\frac{(T+16)}{36})^2 \times t
\]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Hardening-</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>t [d]</td>
<td>T [°C]</td>
<td>period [d]</td>
</tr>
<tr>
<td>0</td>
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<td>1-2</td>
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<tr>
<td>2</td>
<td>20</td>
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<td>3</td>
<td>10</td>
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<tr>
<td>5</td>
<td>5</td>
<td>5-28</td>
</tr>
</tbody>
</table>

5.6 d is realized during the 7th period

\[
\frac{(5,6-3,73)}{(7,83-0,34)} = 0,24
\]

The duration of the 7th period is 28-5=23 d

Thus the time is completed at 23d * 0,24 = 5,52 d

Starting from time 0 : 5d + 5,52d = 10,52 d
c) What was the strength of the concrete at the age of 28d?

<table>
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<tr>
<th>Measurement</th>
<th>Hardening-</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>t [d]</td>
<td>T °C</td>
<td>period [d]</td>
</tr>
<tr>
<td>0</td>
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<td>10</td>
<td>3-4</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>5</td>
<td>5-28</td>
</tr>
</tbody>
</table>

Strength at t<sub>20</sub> = 11,55

Strength from the curve about 77%

0.77 * 30 = 23.1 MPa

EXERCISE 02
2. A concrete with a mix design of 1:4,5:0,42 was hardening for 1 day at a temperature of 40 °C.
   • What was the strength of the concrete after this?
   • How long would it have taken to reach this strength at a temperature of 20 °C?
   • Calculate using both the Sadgrove and the Arrhenius equations.

1:4,5:0,42
→ \( \frac{c}{3.1} + \frac{4.5}{2.68}c + 0.42c + 20 = 1000 \) l

c = 405 kg

Water-air/cement -ratio
(0,42*405+20)/405 = 0,47
K_s = 47,5 MPa

Let’s assume it equals to nominal strength of K45
Using the Sadgrove formula

\[ t_{20} = \left(\frac{40+16}{36}\right)^2 \times 1 = 2.42 \]

From the figure
strength of about 50 %

\[ \Rightarrow 22 \text{ MPa} \]

Using the Arrhenius equation:

Reaction speed \( e^{-\frac{E}{RT}} \)

\( E = 33.5 \text{ kJ/mol} \)
\( R = 8.314 \text{ J/mol·K} \)
\( T = ? \text{ °K} \)

1 d +40 °C: let’s calculate the equivalent age at +20 °C

\[
1d \times \frac{e^{-\frac{33.5\times10^3}{8.314\times313}}}{e^{-\frac{33.5\times10^3}{8.314\times293}}} = 1d \times \frac{2.566 \times 10^{-6}}{1.066 \times 10^{-6}}
\]

= 2.41 d
So 1 d at +40 °C is equal to 2.4 d at +20 °C

The strength development at +20 °C is known from the maturity figures:

2.4d at 20 °C is about 50 % of the strength = 22 MPa

EXERCISE 03
3. A concrete with a mix design of 1:6,5:0,56 was hardening for 10 hours at a temperature of 60 °C (the temperature increased by 5 °C / h).

- What was the strength of the concrete after this period (18h)?
- What was the strength of the concrete at the age of 7d when it was cured in +20 °C water?

\[
\text{c} = \frac{3,1}{2,68} + \frac{6,5}{0,56} \cdot \text{c} + 20 = 1000\text{l}
\]
\[
\text{c} = 296 \text{ kg}
\]

Water-air-cement -ratio
\[
(0,56 \times 296 + 20) / 296 = 0,62
\]
\[K_s = 38 \text{ MPa}
\]

Let’s assume it equals to nominal strength of K35
• What was the strength of the concrete after (18h)?

Using the Arrhenius equation:

18 h calculated in two sections:
8h (+40 °C ) + 10h (+60 °C)

\[ t_{20} = 8 \times e^{\frac{313-293}{293+R+313}} + 10 \times e^{\frac{333-293}{293+R+333}} \]

\[ t_{20} = 8 \times 2.41 + 10 \times 5.2 = 71h \approx 3d \]

• What was the strength of the concrete after (18h)?

≈ 50 % of strength (from the maturity figures)

\[ f_c = 0.5 \times 35 = 17.5 \text{ MPa} \]

The heat treatment causes strength depletion (lujuuskato) of about 10...20 % - assume 15% depletion

\[ f_c = (100\% - 15\%) \times 17.5 = 15 \text{ MPa} \]
What was the strength of the concrete at the age of 7d when it was cured in +20 °C water?

Equivalent age

– (18h in 40°C and 60°C is equal to 3d in (+20°C)
– The rest of the 7 days = 7d – 18h/24 = 6.25d in (+20°C)

The equivalent age = 3d + 6,25d = 9,25d in (+20 °C)

\[ 0.78 \times 35 = 27.3 \text{ MPa} \]