Estimation of the strength development of concrete

Exercise 10
Estimation of the strength development

- Normal and winter conditions:
  \( T = +5 \ldots +40 \, ^\circ C \)
  → the rate of the strength development is strongly affected but the final strength \( \approx \) same

- Heat treatment
  \( T > 40 \, ^\circ C \)
  → rapid strength development → strength depletion (lujuuskato) of about 10...20 %
From: by201, p. 347

- **Nominal strength**
- **Disassembly strength of the moulds**
- **Freezing strength**

Graph showing:
- Strength (Lujus) vs. Time (Aika)
- Key points:
  - 5 MN/m²
  - ≥ 0.6 K
  - Nimellislujus
  - Muottien purkamislujus
  - Jäätmislujus, kantamattomat muotinosat voidaan purkaa
  - Älä poista kantavia muotteja tai tukirakenteita
  - Älä anna betonin jäätyä

Graph source: by201, p. 347
What Happens When Concrete Freezes?

• Pore water in concrete starts to freeze around -1°C
• As some water freezes the ion concentration in the unfrozen water goes up, further depressing the freezing point.
• At around -3 to -4°C, enough of the pore water will freeze so that hydration will completely stop, and depending on the extent of hydration, and thus the strength of the concrete, the forces generated by the expansion of ice (ice occupies ~9% more volume than water) may be detrimental to the long term integrity of the concrete.
What Happens When Concrete Freezes?

Kuva 8.2 Jäätymisen vaikutus betonin K20 ljuudenkehitykseen erään tutkimuksen mukaan.
Käyrä 1: Säilytyslämpötila + 20 °C
Käyrä 2: Säilytyslämpötila − 8 °C
Käyrä 3: Säilytyslämpötila 7d − 8 °C, sitten + 20 °C (alusa on veden jäätymisen antamaa varelujuutta, joka häviää kun jää sulaa)
Estimation of strength development in normal and winter conditions

Arrhenius equation:

The rate of many chemical reactions can be estimated with the Arrhenius equation:

$$e^{-\frac{E}{RT}}$$

In which  
- $E$ = activation energy
- $T$ = temperature [°K]
- $R$ = universal gas constant 8,314 J/mol·K

The equation is also suitable for the estimation of strength development rate of concrete
Equivalent age (ekvivalentti-ikä)

Since the strength development in normal conditions is usually known, the calculated strength development at other temperatures is commonly compared to this known strength development.

Thus an equivalent age is calculated, which is the age at which the concrete reaches the strength (maturity) equivalent to the strength (maturity) achieved by a curing period at a temperature of 20 °C (23 °C).
For example:

If the rate of strength development doubles at +50 °C compared to the strength development at +20 °C, the equivalent age of the concrete at the age of 1 d at 50 °C is 2d.

For normal hardening cement the activation energy is about 33,5 kJ/mol (from a Danish study)
Estimation of strength development in normal and winter conditions

- In Finland a popular way for the estimation is the Sadgrove equation

\[ t_{20} = \left( \frac{(T + 16 \, ^\circ \text{C})}{36 \, ^\circ \text{C}} \right)^2 \times t \]

In which:
- \( T \) is the concrete temperature at a time \( t \) [^\circ \text{C}]
- \( t \) is the time for the hardening [d]
- \( t_{20} \) is the concrete maturity [d]
N = k \( (T + 10 \, ^{\circ}C) \times t \)

In which

- \( T \) is the concrete temperature at a time \( t \) [\(^{\circ}C\)]
- \( t \) is the time for the hardening [d]
- \( k = 1 \) when \( +50 \, ^{\circ}C \geq T \leq 0 \, ^{\circ}C \)
- \( k = 0.4 \) when \( 0 \, ^{\circ}C > T \geq -10 \, ^{\circ}C \)
- \( k = 0 \) when \( T < -10 \, ^{\circ}C \)
Kuva 8.7 Normaalisti kovettuvaa sementtiä käytettäessä betonin suhteellinen lujuuden kehitys kypsyysiän funktiona. Betoni on valmistettu Yleis- tai SR-sementtiä käyttäen.
Kuva 8.8  Nopeasti kovettuvaa sementtiä käytettäessä betonin suhteellinen lujuuden kehitys kypsyysiaan funktiona. Betoni on valmistettu Rapid- tai Mega-sementtiä käyttäen.
Kuva 8.9 Erittäin nopeasti kovettuvaa sementtiä käytettäessä betonin suhteellinen lujuuden kehitys kypsyysin funktiona. Betoni on valmistettu Pikasementtiä käyttäen.
Formulas for the estimation of strength development

Saul
\[ f = \frac{T+10}{30} \]

Rastup
\[ f = 2^{\frac{T-20}{10}} \]

Sadgrove
\[ f = \left( \frac{T+16}{36} \right)^2 \]

Arrhenius
\[ f = e^{\frac{E(T-293)}{293*R*T}} \]
Betonin kypsyysastefunktioiden vertailu (lujuudenkehitys/lämpötila- riippuvuus)

\[
t_{20} = e^{\frac{E \cdot (T-293)}{293 \cdot R \cdot T}}
\]

\[
t_{20} = \left( \frac{T + 16}{36} \right)^2 \cdot t
\]

\[
t_{20} = k(T + 10) \cdot t
\]
Betonin kypsyysastefunktioiden vertailu
(lujuudenkehitys/lämpötila-riippuvuus)

\[ t_{20} = e^{\left( \frac{E \cdot (T-293)}{293 \cdot R \cdot T} \right)} \]

\[ t_{20} = \left( \frac{T+16}{36} \right)^2 \cdot t \]

\[ t_{20} = k(T+10) \cdot t \]
1. The temperature of a K30 (CEM II A) concrete right after the casting was measured at +15 °C. The temperatures were measured also after the casting and the results of measurements were:

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h</td>
<td>+15 °C</td>
</tr>
<tr>
<td>6h</td>
<td>+20 °C</td>
</tr>
<tr>
<td>24h</td>
<td>+25 °C</td>
</tr>
<tr>
<td>2d</td>
<td>+20 °C</td>
</tr>
<tr>
<td>3d</td>
<td>+10 °C</td>
</tr>
<tr>
<td>4...28d</td>
<td>+5 °C</td>
</tr>
</tbody>
</table>

Calculate using the Sadgrove equation

a) At what time did the concrete reach its freezing strength?

b) At what time did the concrete reach its disassembly strength of the moulds? The construction load was 1.5 MN/m² and the design load 2.5 MN/m²

c) What was the strength of the concrete at the age of 28d?
\[
t_{20} = \frac{(T+16)}{36}^2 \cdot t
\]

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Hardening-</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>T</td>
<td>period</td>
</tr>
<tr>
<td>[d]</td>
<td>°C</td>
<td>[d]</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>0,25</td>
<td>20</td>
<td>0-0,25</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>0,25-1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>1-2</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>2-3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3-4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4-5</td>
</tr>
<tr>
<td>28</td>
<td>5</td>
<td>5-28</td>
</tr>
</tbody>
</table>
a) At what time did the concrete reach its freezing strength?

\[
5\text{MPa}/30\text{MPa} = 0.17 = 17\% \text{ of the strength}
\]

\[
17\% = 0.8d, t_{20}
\]
0,8d is realized during the second period
(0,8-0,21)/(1,07-0,21) = 0,68605

The duration of the second period is 0,75 d
Thus the time is completed at 0,75d * 0,68605 = 0,5145 d
Starting from time 0 : 0,25d + 0,5145d = 0,7645 d

In hours 24h/d*0,7645d = 18,3 h

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<tr>
<td>0</td>
<td>15</td>
<td>0-0,25</td>
</tr>
<tr>
<td>0,25</td>
<td>20</td>
<td>0,25-1</td>
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<tr>
<td>1</td>
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<td>20</td>
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<td>3</td>
<td>10</td>
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<td>4</td>
<td>5</td>
<td>4-5</td>
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<tr>
<td>5</td>
<td>5</td>
<td>5-28</td>
</tr>
</tbody>
</table>

Measurement Hardening- Maturity
\[ t_{20} = \frac{(T+16)}{36} \]
b) At what time did the concrete reach its disassembly strength of the moulds? The construction load was 1,5 MN/m² and the design load 2,5 MN/m²?

\[
K_m = K \times \frac{F}{F_k} = 30 \times \frac{1,5}{2,5} = 18 \text{ MPa}
\]

\[
18/30 = 60 \%
\]

60 % = 5,6 d, \( t_{20} \)

is realized after 5d
The duration of the period is $28-5 = 23$ d
Thus the time is completed at $23 \times 0.2401 = 5.76$ d
Starting from time 0: $5d + 5.76d = 10.76d$

$\left(\frac{5.6-3.72}{7.83}\right) = 0.2401$
c) What was the strength of the concrete at the age of 28d?

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<tr>
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<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>t [d]</td>
<td>T °C</td>
<td>period [d]</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
<td>0,25</td>
</tr>
<tr>
<td>0,25</td>
<td>20</td>
<td>0,25-1</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>1-2</td>
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<tr>
<td>2</td>
<td>20</td>
<td>2</td>
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Strength at t<sub>20</sub> = 11,55 MPa
Strength from the curve about 77 %
0,77 * 30 = 23,1 MPa
2. A concrete with a mix design of 1:4.5:0.42 was hardening for 1 day at a temperature of 40 °C. What was the strength of the concrete after this? How long would it have taken to reach this strength at a temperature of 20 °C? Calculate using both the Sadgrove and the Arrhenius equations.
1:4,5:0,42

\[ \rightarrow \frac{c}{3,1} + \frac{4,5}{2,68} c + 0,42c + 20 = 1000l \]

c = 405 kg

Water-air/cement -ratio

\[ (0,42 \times 405 + 20)/405 = 0,47 \]

\[ K_s = 47,5 \text{ MPa} \]

Let´s assume it equals to nominal strength of K45
Using the Sadgrove formula

\[ t_{20} = \left( \frac{40+16}{36} \right)^2 \times 1 = 2.42 \]

From the figure:
- Strength of about 50%
- \( \rightarrow 22 \) MPa
Using the Arrhenius equation:

\[ \text{Reaction speed } e^{-\frac{E}{RT}} \]

\[ E = 33.5 \text{ kJ/mol} \]

\[ R = 8,314 \text{ J/mol} \cdot \text{K} \]

\[ T = ? \text{ °K} \]

1 d +40 °C: let’s calculate the equivalent age at +20 °C

\[ 1 \text{ d} \times e^{-\left(\frac{33,5 \times 10^3}{8,314 \times 313}\right)} = 1 \text{ d} \times \frac{2,566 \times 10^{-6}}{1,066 \times 10^{-6}} \]

\[ = 2.41 \text{ d} \]
So 1 d at +40 °C is equal to 2,4 d at +20 °C

The strength development at +20 °C is known from the maturity figures:

2,4d at 20 °C is about 50 % of the strength = 22 MPa
3. A concrete with a mix design of 1:6,5:0,56 was hardening for 10 hours at a temperature of 60 °C (the temperature increased by 5 °C / h). What was the strength of the concrete after this period (18h)? What was the strength of the concrete at the age of 7d when it was cured in +20 °C water?
$c = 296 \text{ kg}$

Water-air-cement ratio

$(0,56 \times 296 + 20)/296 = 0,62$

$K_s = 38 \text{ MPa}$

Let’s assume it equals to nominal strength of $K_{35}$
Using the Arrhenius equation:

18 h calculated in two sections:
8 h (+40 °C) + 10 h (+60 °C)

\[
8h \times e^{\frac{E(313-293)}{293 R \times 313}} + 10h \times e^{\frac{E(333-293)}{293 R \times 333}}
\]

= 8*2.41 + 10h*5.2 = 71h ≈ 3d

≈ 50% of strength (from the maturity figures)

0.5*35 = 17.5 MPa

The heat treatment causes strength depletion (lujuuskato) of about 10...20%

0.5*0.85*35 ≈ 15 MPa
Equivalent age at 7 d
3d + 6.25d = 9.25d (+20 °C)

0.78 * 35 = 27.3 MPa