1 Helpful physics formulas

Debye length	$\lambda_D^2 = \frac{\epsilon_0 T}{e^2 n_0}$	(1)
Plasma parameter	$\Lambda = \frac{4}{3} n_0 \pi \lambda_D^3$	(2)
Plasma frequency	$\omega_p^2 = \frac{e^2 n_0}{m_e \epsilon_0}$	(3)
Larmor frequency	$\Omega = \frac{qB}{m}$	(4)
Larmor radius	$r_L = \frac{m v_\perp}{qB}$	(5)
Magnetic moment	$\mu = \frac{m v_\perp^2}{2B}$	(6)
$\boldsymbol{E} \times \boldsymbol{B}$ drift	$\boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}} = \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}$	(7)
Gradient drift	$\boldsymbol{v}_{\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\boldsymbol{B} \times \nabla B}{B^2}$	(8)
Diamagnetic drift	$\boldsymbol{v}_D = -\frac{\nabla p \times \boldsymbol{B}}{q n_0 B^2}$	(9)
Maxwell-Boltzmann	$f(\boldsymbol{v}) = \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{m v^2}{2T}\right)$	(10)
	$h(E) = \frac{2}{T^{3/2}} \sqrt{\frac{E}{\pi}} e^{-E/T}$	(11)
Convective derivative	$\frac{\mathrm{d}n(\boldsymbol{r},t)}{\mathrm{d}t} = \frac{\partial n(\boldsymbol{r},t)}{\partial t} + \boldsymbol{v} \cdot \nabla n(\boldsymbol{r},t)$	(12)
Collision frequency	$\nu = \frac{e^4 \ln \Lambda}{4\pi \epsilon_0^2 \sqrt{m}} \frac{n}{T^{3/2}}$	(13)
Gauss's law	$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$	(14)
Gauss's law for magnetism	$\nabla \cdot \boldsymbol{B} = 0$	(15)
Maxwell-Faraday equation	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	(16)
Ampère's circuital law	$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t}$	(17)
Sound speed in ideal gases	$v_s = \sqrt{\frac{E_{\text{therm}}}{m}}$	(18)
Sound speed in plasma	$v_s = \sqrt{\frac{E_{\text{therm, e}}}{M} + \frac{\gamma_i E_{\text{therm, i}}}{M}}$	(19)

2 Vector identities

1. $\boldsymbol{A} \cdot \boldsymbol{B} \times \boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} \cdot \boldsymbol{C} = \boldsymbol{B} \cdot \boldsymbol{C} \times \boldsymbol{A}$
2. $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{C} \times \boldsymbol{B}) \times \boldsymbol{A} = \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$
3. $(\boldsymbol{A} \times \boldsymbol{B}) \cdot (\boldsymbol{C} \times \boldsymbol{D}) = (\boldsymbol{A} \cdot \boldsymbol{C})(\boldsymbol{B} \cdot \boldsymbol{D}) - (\boldsymbol{A} \cdot \boldsymbol{D})(\boldsymbol{B} \cdot \boldsymbol{C})$
4. $(\boldsymbol{A} \times \boldsymbol{B}) \times (\boldsymbol{C} \times \boldsymbol{D}) = \boldsymbol{C}(\boldsymbol{A} \times \boldsymbol{B} \cdot \boldsymbol{D}) - \boldsymbol{D}(\boldsymbol{A} \times \boldsymbol{B} \cdot \boldsymbol{C})$

5. $\nabla \cdot (f\boldsymbol{A}) = f(\nabla \cdot \boldsymbol{A}) + \boldsymbol{A} \cdot (\nabla f)$
6. $\nabla \times (f\boldsymbol{A}) = f(\nabla \times \boldsymbol{A}) - \boldsymbol{A} \times (\nabla f)$
7. $\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B})$
8. $\nabla \times (\boldsymbol{A} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + \boldsymbol{A}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{A})$
9. $\nabla(\boldsymbol{A} \cdot \boldsymbol{B}) = \boldsymbol{A} \times (\nabla \times \boldsymbol{B}) + \boldsymbol{B} \times (\nabla \times \boldsymbol{A}) + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A}$
10. $\nabla \cdot (\boldsymbol{A}\boldsymbol{B}) = (\nabla \cdot \boldsymbol{A})\boldsymbol{B} + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B}$
11. $\nabla \cdot (f\mathbf{T}) = \nabla f \cdot \mathbf{T} + f\nabla \cdot \mathbf{T}$
12. $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$
13. $\nabla^2 \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla \times \nabla \times \boldsymbol{A}$
14. $\nabla(fg) = \nabla(\phi g) = f\nabla g + g\nabla f$
15. $\nabla \cdot (\nabla f \times \nabla g) = 0$
16. $\nabla \cdot \nabla f = \nabla^2 f$
17. $\nabla \times \nabla f = 0$

18. $\int_V \nabla f \, \mathrm{d}V = \int_S f \, \mathrm{d}\boldsymbol{S}$
19. $\int_V \nabla \times \boldsymbol{A} \, \mathrm{d}V = \oint_S \mathrm{d}\boldsymbol{S} \times \boldsymbol{A}$
20. $\int_S \mathrm{d}\boldsymbol{S} \cdot \nabla \times \boldsymbol{A} = \oint_C \mathrm{d}\boldsymbol{l} \cdot \boldsymbol{A}$
21. $\oint_C \mathrm{d}\boldsymbol{l} \times \boldsymbol{A} = \int_S (\mathrm{d}\boldsymbol{S} \times \nabla) \times \boldsymbol{A}$

3 Curvilinear coordinates

Cylindrical Coordinates (r, θ, z)

1. $\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$

2. $\nabla \cdot \boldsymbol{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$

3. $\nabla \times \boldsymbol{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{\boldsymbol{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial f}{\partial z}\right)\hat{\boldsymbol{\theta}} + \left(\frac{1}{r}\frac{\partial f}{\partial z}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{z}}$

4. $\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates (r, θ, ϕ)

5. $\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$

6. $\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$

7. $\nabla \times \boldsymbol{A} = \left(\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol{r}} + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}} + \left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$

8. $\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial^2 f}{\partial d\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$

4 Gaussian Integrals

Definite integral relations of Gaussian integrals

1. $\int_0^\infty e^{-ax^2}\mathrm{d}x = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2}$
2. $\int_{-\infty}^\infty e^{-ax^2}\mathrm{d}x = \left(\frac{\pi}{a}\right)^{1/2}$
3. $\int_{-\infty}^\infty e^{-ax^2}e^{-2bx}\mathrm{d}x = \left(\frac{\pi}{a}\right)^{1/2}e^{\frac{b^2}{a}} \qquad \text{for } a > 0$
4. $\int_{-\infty}^\infty xe^{-a(x-b)x^2}\mathrm{d}x = b\left(\frac{\pi}{a}\right)^{1/2}$
5. $\int_{-\infty}^\infty x^2e^{-ax^2}\mathrm{d}x = \frac{1}{2}\left(\frac{\pi}{a^3}\right)^{1/2}$
6. $\int_0^\infty x^n e^{-ax^2}\mathrm{d}x = \begin{cases} \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)/a^{(n+1)/2} & a > 0 \\ \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} & n = 2k, a > 0 \\ \frac{k!}{2a^{k+1}} & n = 2k+1, a > 0 \end{cases}$

5 Useful Fourier transforms

1. $\mathcal{F}\left[\frac{\partial f}{\partial t}\right] = -i\omega\tilde{f}$
2. $\mathcal{F}[\nabla f] = -i\boldsymbol{k}\tilde{f}$