Required math skills: examples you are expected to know how to solve

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I. PURPOSE

I collect here some example math problems that we expect you know how to solve in the course "Kvanttimekaniikka". In case you notice you have problem with these, please look the math up. If you do not know how to solve these, you will likely run into technical difficulties during the course. This list might appear kind of long, but do not be intimidated by it. The math is not hard and some of it you have probably already encountered. I add some links to online material, but by googling you will find more. The material can also be found on about any basic university math text book.

II. COMPLEX NUMBERS

- What is the meaning of $z = x + iy$?
- Write $x+iy$ in terms of phase and amplitude $z = \rho \exp i\phi$. What are x and y expressed in terms of ρ and ϕ?
- If say $z = \sqrt{2} \exp i\pi/3$, what are the real and imaginary part of z?
- If $z = x + iy$, what is z^2 and what is $|z|^2$. When are they equal (if ever)?
- What are the real and imaginary parts of the product of two complex numbers. Express the results both in terms of amplitude and phase as well as in terms of "x:s" and "y:s".
- What does complex conjugation z^* mean?
- If you have a function $f(z, z^*) = z^2 + 2zz^*$, what would you get if you compute the partial derivative $\partial f/\partial z$? What about $\partial f/\partial z^*$?

Some sources online..some links might have been provided ironically...click on the text to reach the page behind the hyperlink:

- Complex numbers in Khan academy
- Complex numbers according to MIT open course ware.
  MIT open courseware complex numbers
- Nicholas Hoell crash course (University of Toronto)

III. DERIVATIVES

- Compute the derivative $df(x)/dx$, when $f(x) = x^n$, $f(x) = \sin(kx)$, $f(x) = \exp i\alpha x$, and $f(x) = \exp(-x^2/2\sigma^2)$
- Compute the derivative $df(x)/dx$, when $f(x) = \sin^2(kx)$ (Chain rule. In Finnish: ketjusääntö) Notice btw. that in case of trigonometric functions one often puts the exponent before the argument the signal that we take the power AFTER computing the sine.
- Compute the partial derivative $\partial\psi(x,t)/\partial x$ of a function

$$\psi(x,t) = \sin(kx)\exp(i\omega t) \tag{1}$$

- Compute the partial derivative $\partial\psi(x,t)/\partial t$ of a function

$$\psi(x,t) = \sin(kx)\exp(i\omega t) \tag{2}$$

Some sources online...click on the text to reach the page behind the hyperlink:

- Khan academy on taking derivatives

IV. INTEGRALS

- Calculate the definite integral: $\int_a^b dx x^n$. Note sometimes notation $\int_a^b x^n dx$ is used for the same thing. The first version is a bit clearer when the function to be integrated is long. You immediately see which variable is integrated over.

- Integration by parts, in Finnish: osittaisintegrointi
- Calculate the classic definite integral $I = \int_{-\infty}^{\infty} dx \exp(-x^2)$. There are many ways to do this. One way is to compute I^2, by noting that the expression is simple in polar coordinates $x = r\cos\phi$, $y = r\sin\phi$. How do you change the integration variables?
- Calculate the definite integrals $\int_{-\infty}^{\infty} dx x \exp(-x^2/\sigma^2)$ and $\int_{-\infty}^{\infty} dx x^2 \exp(-x^2/\sigma^2)$. Integration by parts helps especially in the 2nd one.
- Compute the integrals $\int_0^L dx \cos^2(\pi x/L)$ and $\int_0^L dx \cos(\pi x/L)\sin(\pi x/L)$

Some sources online...click on the text to reach the page behind the hyperlink:

- Khan academy on integration by parts
- Khan academy on integration and trigonometric identities

V. DIFFERENTIAL EQUATIONS

- Confirm that $y(x) = \sin(kx) + b\cos(kx)$ can be a solution for a differential equation

$$\frac{d^2y}{dx^2} + cy(x) = 0 \tag{3}$$

- If boundary conditions are $y(0) = 0$ and $y(L) = 0$, how k and b in the earlier solution should be chosen?

Some sources online...click on the text to reach the page behind the hyperlink:

- Differential equations in Khan academy. Notice that you do not need to know all of this. Differential equations you will encounter during the course are quite simple and often solution is very clear or can be remembered.

VI. LINEAR ALGEBRA

- Solve the linear algebra (eigenvalue) problem:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = E \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{4}$$

  Note: here E is the eigenvalue.

- Skills related to previous: What is the meaning of the product of a matrix and a vector?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{5}$$

  Open up this expression!

- What is the determinant of a matrix

$$detA = det \begin{pmatrix} a & b \\ c & d \end{pmatrix}? \tag{6}$$

- If you have to vectors, what is their sum

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{7}$$

- What is a transpose of a matrix?

- What is a Hermitian conjugate $A^\dagger$ or conjugate transpose of a matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{8}$$

  where the elements of the matrix are complex numbers?

Some sources online...click on the text to reach the page behind the hyperlink:

- Khan academy videos on linear algebra

VII. OTHER POTENTIALLY USEFUL THINGS

- **Euler's formula**: $e^{i\theta} = \cos\theta + i\sin\theta$. Notice that if you do not remember the trigonometric identitities below, you can always derive them using the Euler's formula.

- Trigonometric identity: $\sin 2x = 2\cos x \sin x$

- Trigonometric identity: $\cos 2x = \cos^2 x - \sin^2 x$

- Trigonometric identity: $\sin^2 + \cos^2 = 1$

- $e^{i\pi} = -1$