Limit of a sequence

Definition A sequence (a_n) converges to a limit $a \in \mathbb{R}$, if

$$|a_n - a| \xrightarrow[n \to \infty]{} 0 .$$

Formal: (ε, δ) - version

For every $\varepsilon > 0$ there exists an index $n = n(\varepsilon)$, such that

$$|a_n - a| < \varepsilon \quad \text{for every } n \geq n(\varepsilon).$$

We write: $\lim_{n \to \infty} a_n = a.$

Example $\lim_{n \to \infty} \frac{1}{n^2} = 0$

$$\left| \frac{1}{n^2} - 0 \right| = \frac{1}{n^2} < \varepsilon, \quad \text{if } n > 1/\sqrt{\varepsilon}$$

We can choose the next integer! $n(\varepsilon) = \lceil 1/\sqrt{\varepsilon} \rceil$

We can take $\varepsilon > 0$ and arbitrarily small!

Functions

Definition Function f is a mapping from a domain to its range.

$$f : A \to B$$

Domain: A
Range : B

Often : $f_A = \{ f(a) \mid a \in A \} \subset B$

is the image.

Definition Continuity

The function f is continuous at some point $a \in A$, if :

Always when $a_n \in A$ and $\lim_{n \to \infty} a_n = a$, then

$$\lim_{n \to \infty} f(a_n) = f(a).$$

Let $f : A \to \mathbb{R}$ and $x_0 \in \mathbb{R}$ such that a sequence (a_n) exist where $a_n \in A \setminus \{x_0\}$ for all n and further $\lim_{n \to \infty} a_n = x_0$.

Definition Limit

The limit exists, if

$$\lim_{n \to \infty} f(a_n) = L \quad \text{always when } a_n \in A \setminus \{x_0\}$$

$$\text{and } \lim_{n \to \infty} a_n = x_0.$$

Notation: $\lim_{x \to x_0} f(x) = L$, where L is the limit.