Taylor polynomial and limits

Example

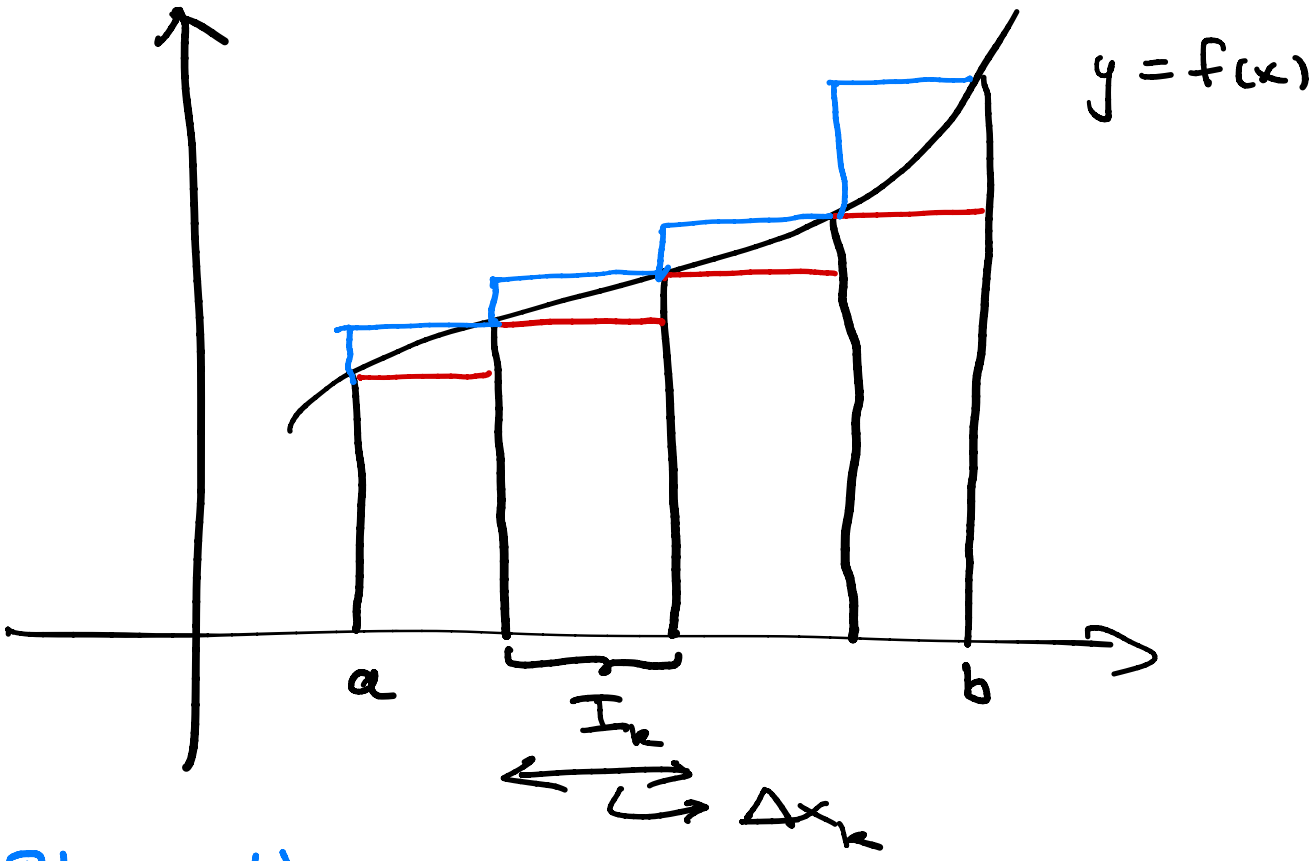
$$\lim_{x \to 0} \frac{2\sin x - \sin 2x}{2e^x - 2 - 2x - x^2}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \Theta(x^7)$$

$$e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + \Theta(x^{n+1})$$

$$\lim_{x \to 0} \frac{2\left(x - \frac{x^3}{3!}\right) - \left(2x - \frac{2^3 x^3}{3!}\right)}{2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) - 2 - 2x - x^2}$$

$$= \lim_{x \to 0} \frac{-\frac{x^3}{3} + \frac{4x^3}{3}}{\frac{x^3}{3}} = 3$$



Blue : Upper sum

Red : Lower sum

f(x) is continuous : sup is max
inf is min

Here : The integral is the area
under the graph of the curve.