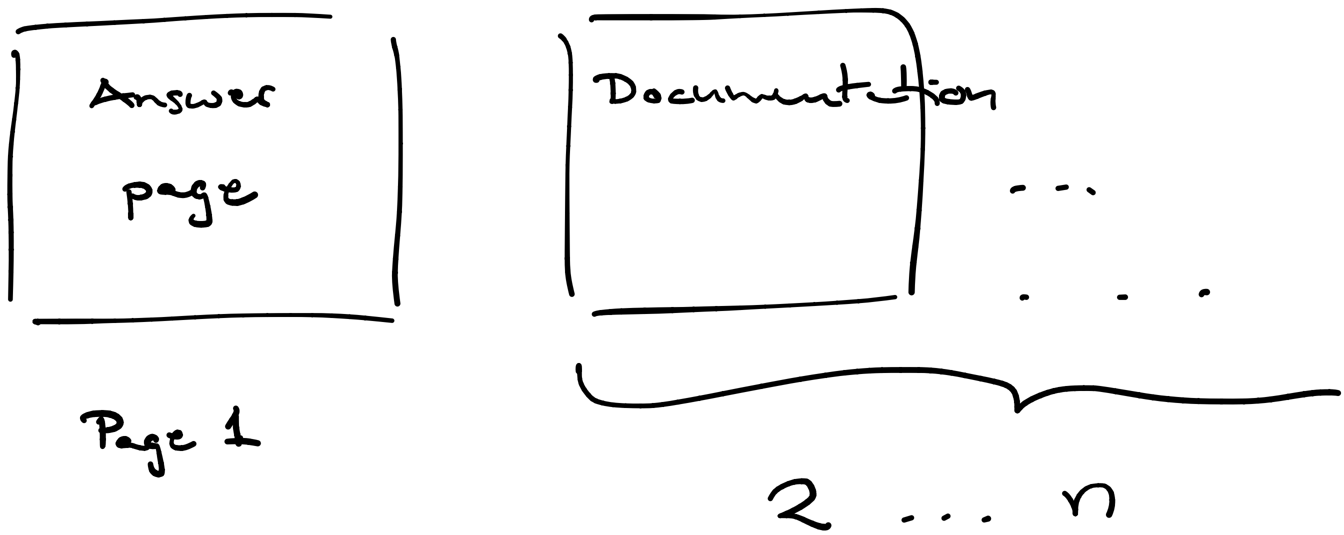
HOME EXAM: Submission guidelines



Impossible problems ?

DO NOT WORRY !

→ Just indicate the error and e.g. add the missing absolute values.

DIFFERENT TYPES OF EXAMS

COURSE EVALUATION (electronic)
→ bonus points

EXAMPLE $\quad \dfrac{dy}{dx} + \dfrac{y}{x} = 1, \quad x > 0$

$$p(x) = \frac{1}{x}, \quad \mu(x) = \int p(x)\,dx = \int \frac{dx}{x} = \ln x$$

$$e^{\mu(x)} = x$$

So

$$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$$

$$= x\left(\frac{dy}{dx} + \frac{y}{x}\right) = x$$

$$\Rightarrow \quad xy = \int x\,dx = \frac{1}{2}x^2 + C$$

$$\Rightarrow \quad y = \frac{1}{x}\left(\frac{1}{2}x^2 + C\right) = \frac{x}{2} + \frac{C}{x}$$

Alternative: $K = K(x)$; $\quad \dfrac{dy}{dx} + \dfrac{y}{x} = 0$

$$\Rightarrow \quad y = Ke^{-\mu(x)}$$

$$= \frac{K}{x}$$

We get: $\quad \dfrac{1}{x}K'(x) - \cancel{\dfrac{1}{x^2}K(x)} + \cancel{\dfrac{1}{x^2}K(x)} = 1$

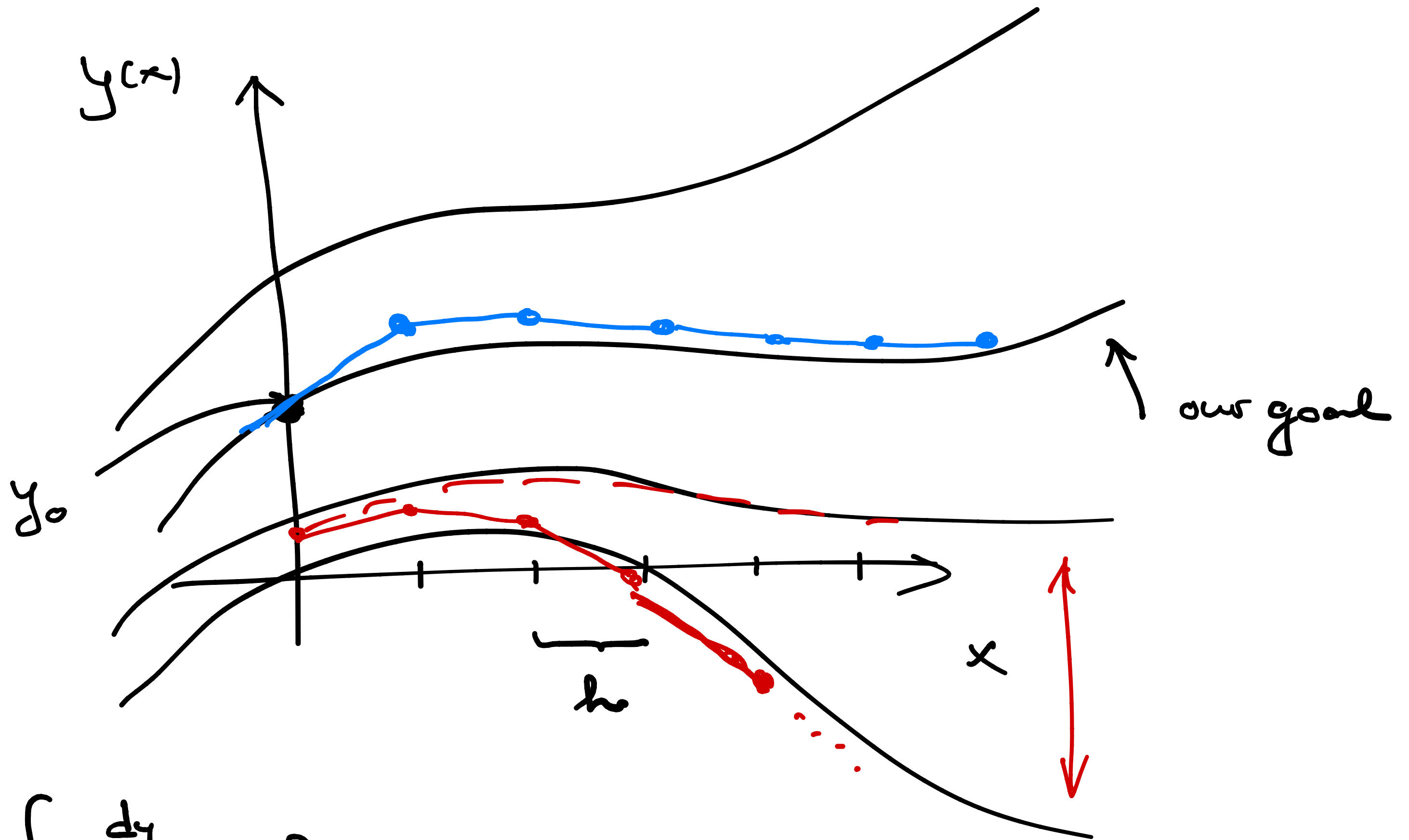
$$K'(x) = x \quad \Rightarrow \quad K(x) = \frac{1}{2}x^2 + C$$

Q: $L(y + y_h) = q(x)$

$$\begin{array}{ccc} y(x) & \xrightarrow{\qquad L \qquad} & q(x) \\ + & & + \\ y_h(x) & \xrightarrow{\qquad L \qquad} & 0 \end{array}$$

$$\begin{aligned} D(x^2 + x) &= D(x^2) + D(x) \\ &= 2x + 1 \\ D(x^2 + 1) &= D(x^2) + D(1) \\ &= 2x + 0 = 2x \end{aligned}$$

NUMERICAL SOLUTION OF ODES



$$\begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

Definition Euler's Method (Explicit)

$$x_{n+1} = x_n + h \; ; \quad y_{n+1} = y_n + h f(x_n, y_n)$$

Example $\dfrac{dy}{dx} = x - y \; , \quad y(0) = 1$

Interval: $[0,1]$, $h = 1/5$; $y(x) = x - 1 + 2e^{-x}$

Euler : $x_0 = 0$, $y_0 = 1$; $x_n = \frac{n}{5}$

$$y_{n+1} = y_n + \frac{1}{5}\underbrace{(x_n - y_n)}_{f(x_n, y_n)}$$

At $x_n = 1$; Error $e_n = y(x_n) - y_n$

~ 0.08

Definition Modified Euler's Method

$$x_{n+1} = x_n + h$$

$$u_{n+1} = y_n + h f(x_n, y_n)$$

$$y_{n+1} = y_n + h\left(\frac{f(x_n, y_n) + f(x_{n+1}, u_{n+1})}{2}\right)$$

"Predictor - Corrector" - method

$\downarrow$ u_{n+1} (Predictor) $\quad$ $\downarrow$ y_{n+1} (Corrector)

Definition Euler's Method (Implicit)

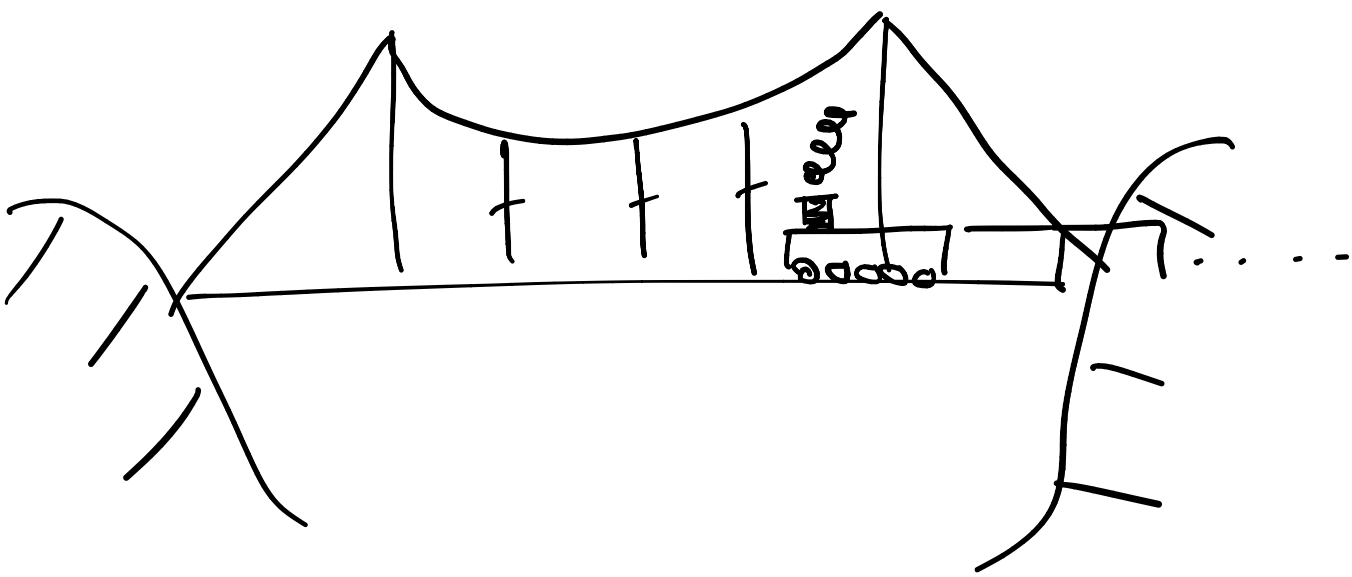
$$x_{n+1} = x_n + h \;;\; y_{n+1} = y_n + h f(x_{n+1}, y_{n+1})$$

Notice: Every step requires a solution of an equation.

Rule of thumb:

As $h \to 0$ Euler's method becomes convergent.

Conversely, Implicit Euler is stable for all h.



2^{nd} ORDER ODEs

$$\phi(x, y, y', y'') = 0 \qquad \text{(implicit)}$$

$$y'' = f(x, y, y') \qquad \text{(explicit)}$$

Solution : $y = \varphi(x, C_1, C_2)$

Initial value problem :

$$y(x_0) = y_0 \; , \; y'(x_0) = p_0$$

Boundary value problem :

$$y(x_1) = y_1 \; , \; y(x_2) = y_2$$

<u>Theorem</u> $\quad y'' = f(x, y, y')$

$$\begin{cases} y' = z \\ z' = f(x, y, z) \end{cases}$$

There are equivalent !