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3. Computer exercises

Logarithmic transformations

Time series are often studied after logarithmizations. Intuitive reasoning for logarithmization:

- Relative changes of the values of variables are often more interesting than the absolute changes.

Technical reasoning for logarithmization:

- If the time series involves an exponential trend, then the trend can be linearized by logarithmizing the time series.

- If the variance (or seasonal variation) of the time series increases together with the level of the time series, then it might be possible to standardize the variance by logarithmization.

Logartihmization and relative changes
If the value x_0 of a variable x changes $p\%$, then the new value x_1 is

$$x_1 = \left(1 + \frac{p}{100}\right) x_0$$

Logarithmization yields:

$$\log(x_1) = \log(x_0) + \log\left(1 + \frac{p}{100}\right) \approx \log(x_0) + \frac{p}{100}$$

Hence, a relative change in the level of a time series is after logaritmization (almost) independent of the level, and depends (almost) only on the percentual change p.

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Demo exercises

3.1 Study the following time series.

File (.txt)	Variable	Description	Interval	Length
INTEL	Intel_Close Intel_Volume	Intel stock price Intel stock volume	Exchange day	$n = 20$
SUNSPOT	Spots	Number of sun spots	1 year	$n = 215$
MLCO2	MLCO2	Carbon dioxide measurements from the Mauna Loa volcano	1 month	$n = 216$
SALES	Sales	Sales volume of a wholesaler	1 month	$n = 144$
PASSENGERS	Passengers	Number of airline passengers on inter- national routes in USA	1 month	$n = 144$

Solution. Trajectories of the corresponding time series are presented on the following pages.

```
INTEL <- read.table("INTEL.txt",header=T)
SUNSPOT <- read.table("SUNSPOT.txt",header=T,row.names=1)
MLCO2 <- read.table("MLCO2.txt",header=T,row.names=1)
SALES <- read.table("SALES.txt",header=T)
PASSENGERS <- read.table("PASSENGERS.txt",header=T,row.names=4)

Intel_Close <- ts(INTEL$Intel_Close)
Intel_Volume <- ts(INTEL$Intel_Volume)
Spots <- ts(SUNSPOT,start=1749)
Mlco2 <- ts(MLCO2$MLCO2,frequency=12)
Sales <- ts(SALES$Sales,frequency=12)
Passengers <- ts(PASSENGERS$Passengers)
```

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INTEL: Intel_Close Intel stock price in New York stock exchange at the end of the trading day. The time frame is four weeks.
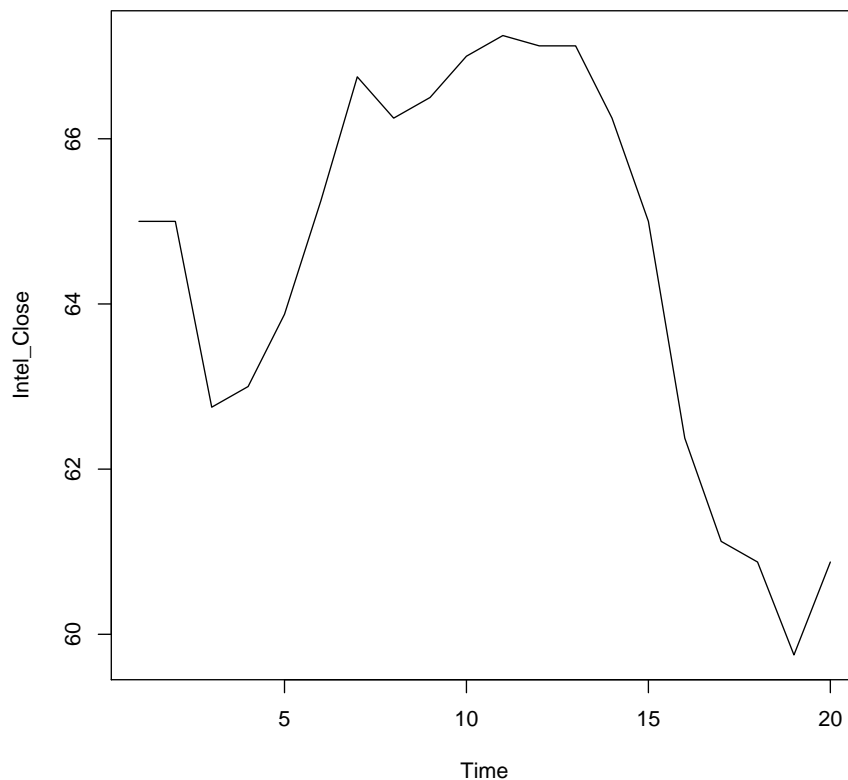
```
plot(Intel_Close)
```


Figure 1: Plot of Intel_Close time series.

Trend:

- No clear trend and the level of the time series alters.

Seasonality:

- No seasonality.

Stationarity:

- Due to the small number of observations, it is hard to say anything about stationarity. However, the mean of the time series does not seem to be constant, which indicates that the time series might not be stationary. This time series is considered with more detail in Problem 3.3.

INTEL: Intel_Volume The daily volume of Intel stocks traded in New York stock exchange during a four week time period.
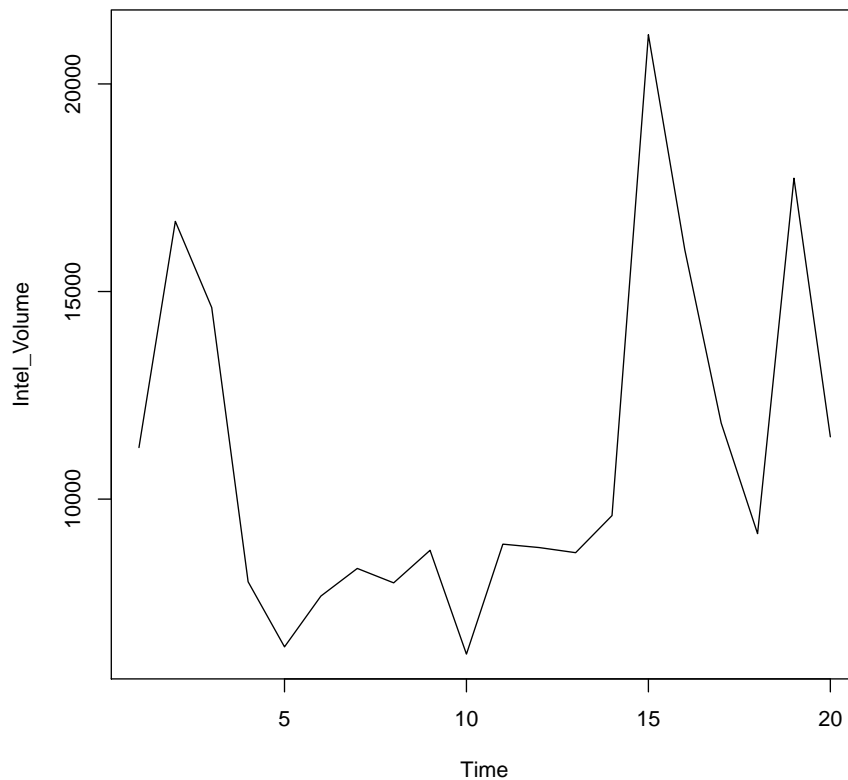
```
plot(Intel_Volume)
```


Figure 2: Plot of Intel_Volume time series.

Trend:

- No clear trend and the level of the time series alters.

Seasonality:

- No seasonality.

Stationarity:

- Due to the small number of observations, it is hard to say anything about stationarity. However, the mean of the time series does not seem to be constant, which indicates that the time series might not be stationary.

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SUNSPOT: Spots
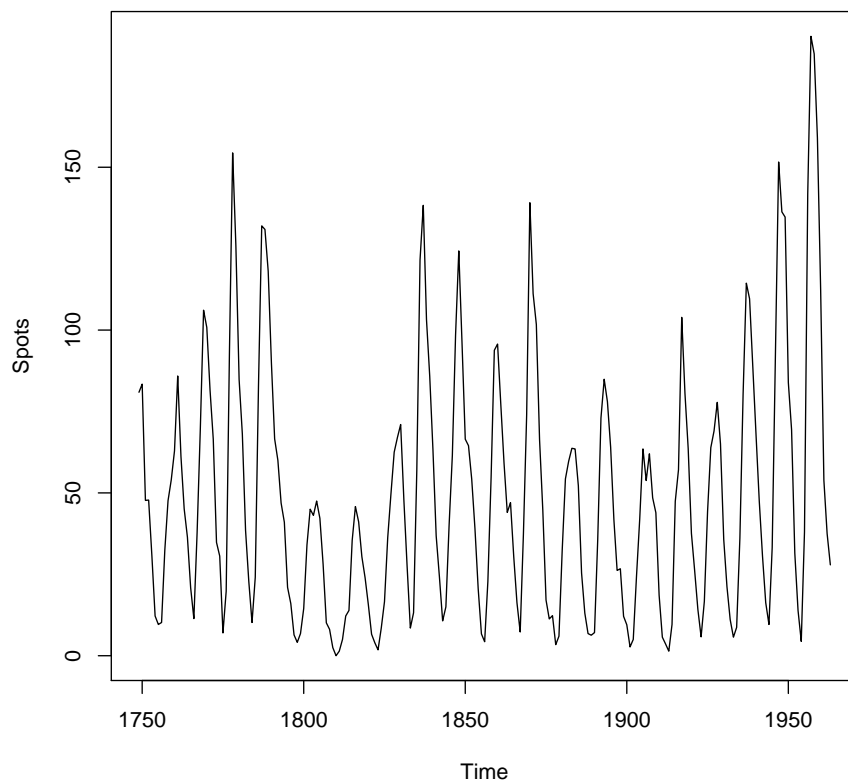Number of yearly sun spots.

```
plot(Spots)
```


Figure 3: Plot of Spots time series.

Trend:

- No trend.

Seasonality:

- Seasonal component with period that is approximately 11 years. The amplitude of the time series alters.

Stationarity:

- Time series does not seem to be stationary, since it clearly has a seasonal component.

MLCO2: MLCO2
Carbon dioxide measurements of the Mauna Loa volcano (Hawaii).
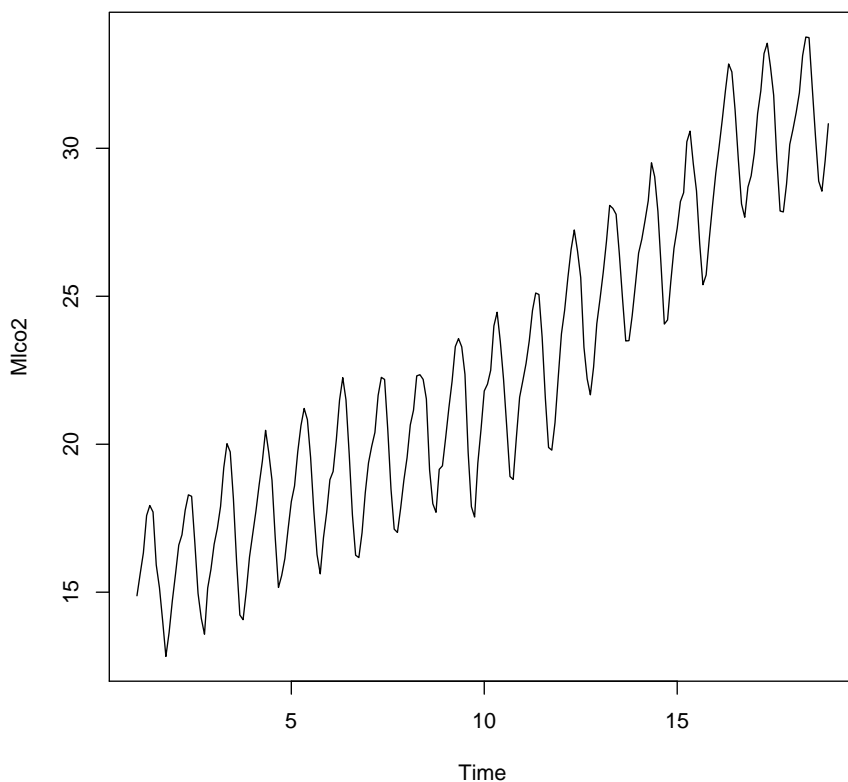
```
plot(Mlco2)
```


Figure 4: Plot of MLCO2 time series.

Trend:

- Linear upward trend.

Seasonality:

- Quite regular seasonality with a period of 12 months. Amplitude of the seasonal component stays constant.

Stationarity:

- The time series does not seem to be stationary, since it has an upward trend and a seasonal component.

SALES: Sales
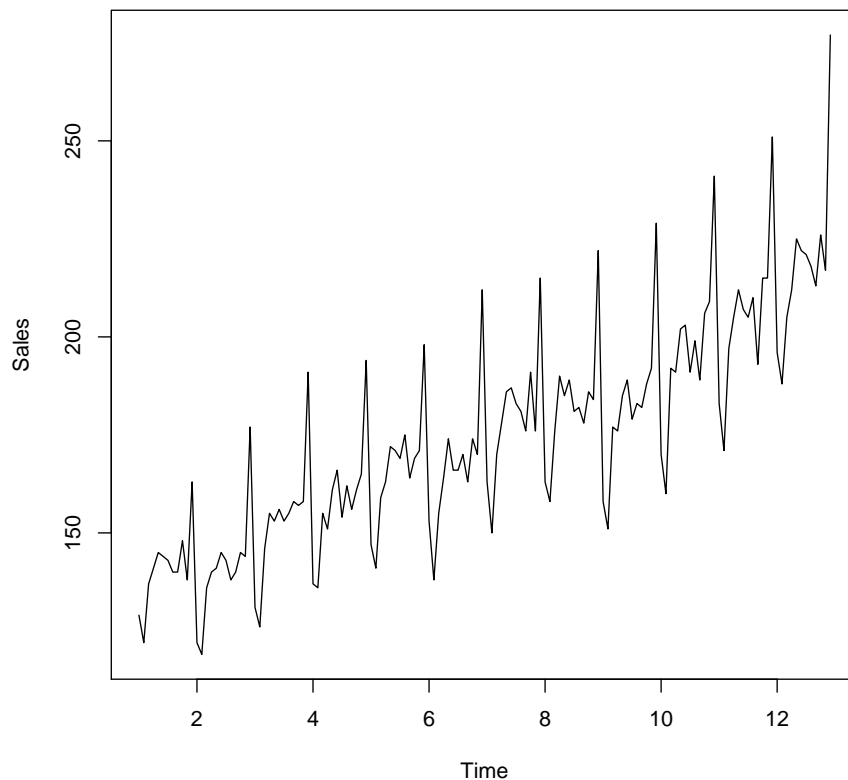Monthly sales volume of a wholesaler.

```
plot(Sales)
```


Figure 5: Plot of Sales time series.

Trend:

- Upward trend.

Seasonality:

- Quite regular seasonality with a period of 12 months. The amplitude of the seasonal component increases with the level of the time series.

Stationarity:

- The time series does not seem to be stationary, since it has an upward trend and a seasonal component.

PASSENGERS: Passengers
Number of airline passengers on international routes in the USA.
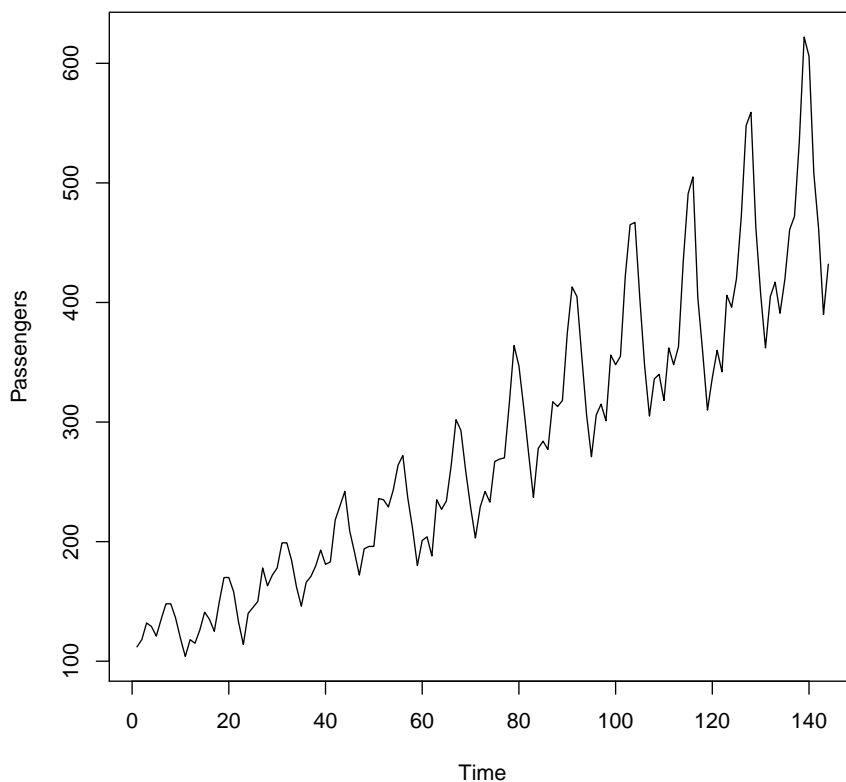
```
plot(Passengers)
```


Figure 6: Plot of Passengers time series.

Trend:

- Upward slightly curvilinear trend.

Seasonality:

- Quite regular seasonality with a period of 12 months. The amplitude of the seasonal component increases with the level of the time series.

Stationarity:

- The time series does not seem to be stationary, since it has an upward trend and a seasonal component.

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3.2 The file `PASSENGERS.txt` contains a time series named Passengers. Plot the time series by using both linear and logarithmic scales on the y-axis. Compare the plots.

File (.txt)	Variable	Description	Interval	Length
PASSENGERS	Passengers	Number of airline passengers on international routes in the USA	1 month	$n = 144$

Solution.

```
PASSENGERS <- read.table("PASSENGERS.txt",header=T,sep="\t")
# Note that the data has been separated with tabulator
names(PASSENGERS)
```

The following command sets the years correctly:

```
PASS2 <- ts(PASSENGERS$Passengers,start=1949,frequency=12)

par(mfrow=c(1,2),mar=c(2.5,2.5,1.5,1.5))
# with par() we can draw both time series in the same plot.

plot(PASS2,main="Passengers")
plot(log(PASS2),main="Log(Passengers)")

 dev.off()
# dev.off() returns the default settings for function par()
```

Figure 7 illustrates, that the amplitude of the seasonal component of the original time series increases together with the level of the time series (left figure). When logarithmic scale is used, the amplitude is almost constant (right figure). On the other hand, the (slight) curvature of the trend of the original time series gets (slightly) overcompensated with logarithmic scale.

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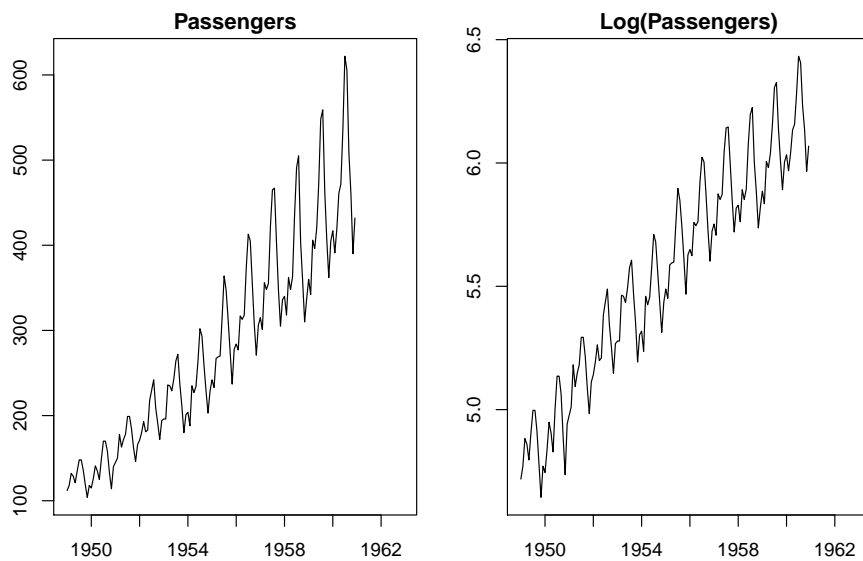Aalto University   Exercise 3.

Figure 7: Plot of Passengers time series with linear and logarithmic scales.

3.3 Study the following time series by estimating their autocorrelation and partial autocorrelation functions.

File (.txt)	Variable	Description	Time interval	Length
INTEL	Intel_Close	Intel stock price	Exchange day	$n = 20$
SUNSPOT	Spots	Number of sun spots	1 year	$n = 215$

Solution.

Intel_Close

The autocorrelation function and the partial autocorrelation function are estimated as follows.

```
par(mfrow=c(1,2))
acf(Intel_Close)
pacf(Intel_Close)
```

The blue lines in Figure 8 indicate statistical significance with 5% level of significance (is correlation significant with a particular lag). Correlations that lie inside the blue lines can be considered as noise with a 5% level of significance. The levels of the blue lines are given by

```
qnorm((1 + 0.95)/2)/sqrt(length(Intel_Close))
-qnorm((1 + 0.95)/2)/sqrt(length(Intel_Close))
```
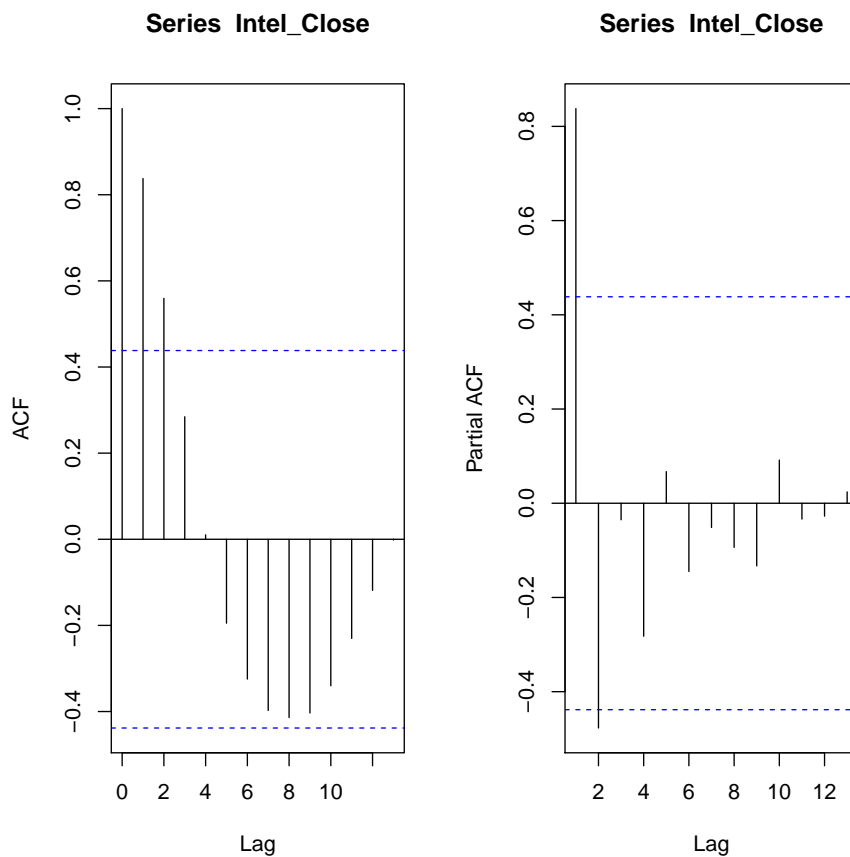
Series Intel_Close   **Series Intel_Close**

Figure 8: Auto- and partial autocorrelation functions of Intel_Close time series.

Comments:

(1) Time series **Intel-Close** could be stationary based on the figures and hence, it does
    not require differencing. The level of the time series changes, but the behavior is
    calm locally. We cannot see a monotonic trend or a visible seasonal component.

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Spots

Based on Exercise 3.1, the time series Spots is not stationary. Next, we study the characteristics of the auto- and partial autocorrelation functions of a non-stationary time series.

```
par(mfrow=c(1,2))
acf(Spots,lag.max=50)
pacf(Spots,lag.max=50)
```
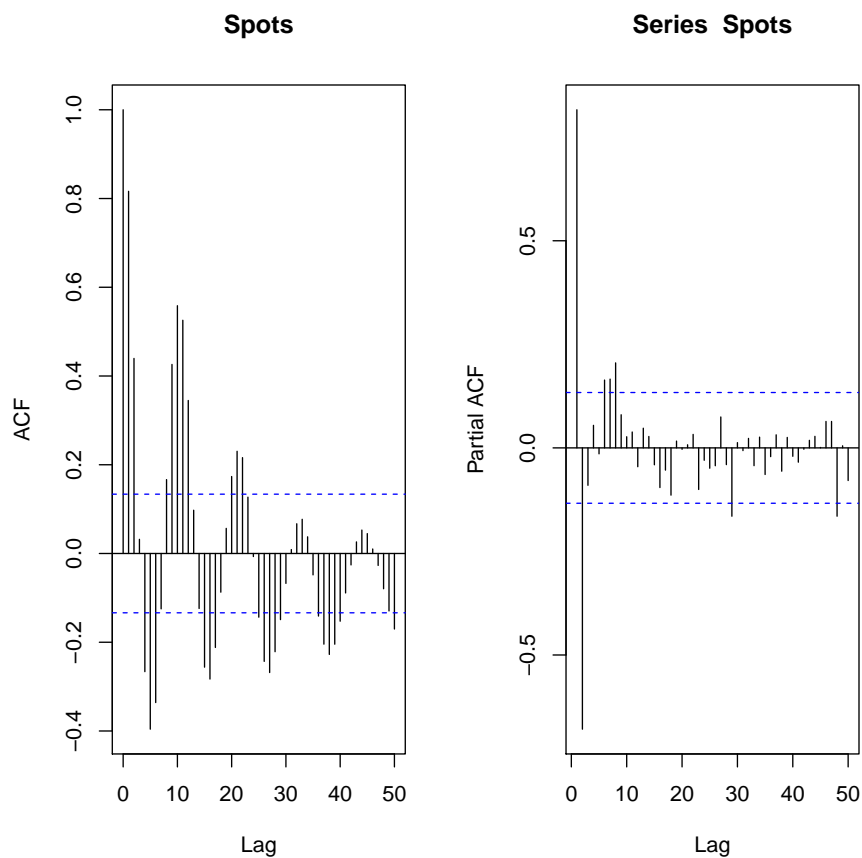

Figure 9: Auto- and partial autocorrelation functions of Spots time series.

Comments:

(1) Note that the seasonality is clearly visible in the autocorrelation function. By enlarging the figure, we can see that the period of the season seems to be about 11 years.

Homework

3.4 Consider the time series **Sales** from the file `SALES.txt`. Apply differencing, seasonal differencing and logarithmic transformations to remove the trend, the seasonality and the increasing variance. Which difference operations did you apply? Visualize both the original and the transformed time series. **Hint:** Difference operators are given by the function `diff` in R.

File	Variable	Description	Interval	Length
SALES.txt	Sales	Sales volume of a wholesaler	1 month	$n = 144$