Aalto University
School of Engineering

Power Electronics

ELEC-E8412 Power Electronics, 5 ECTS

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Course Objectives

At the end of this course, you will be able to:

generally analyse the uncontrolled and controlled half-wave rectifiers with different loads and apply the power computation concepts from the previous chapter to these circuits.

Half-Wave Rectifiers

A **rectifier** converts **AC** to **DC**. The half-wave rectifier is used most often in **low-power** applications.

1. The Uncontrolled Half-Wave Rectifiers:

Assuming you have an AC source, and you are going to supply a DC load. For this process, you need an AC/DC rectifier.

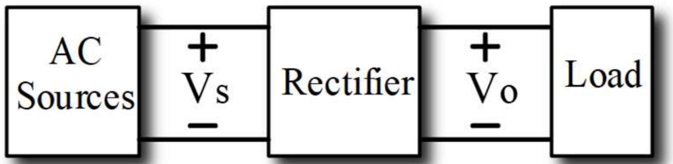
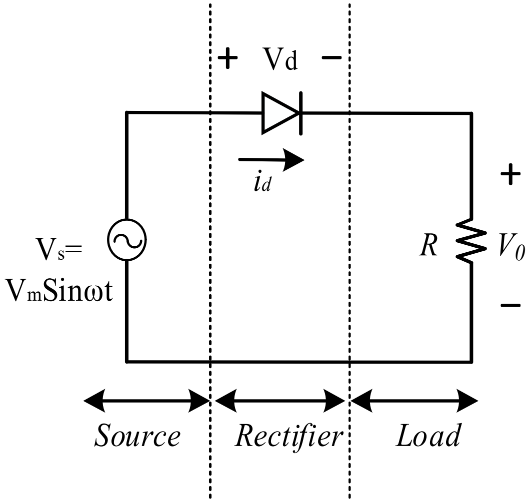
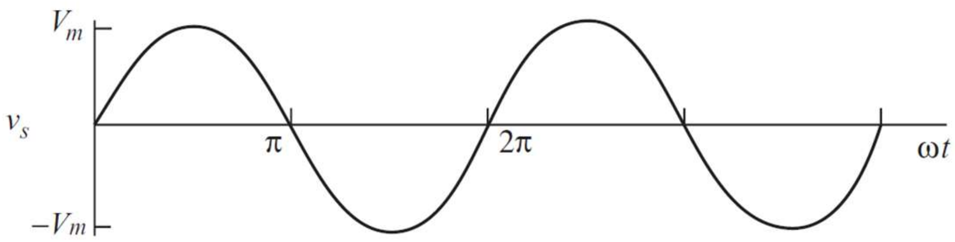


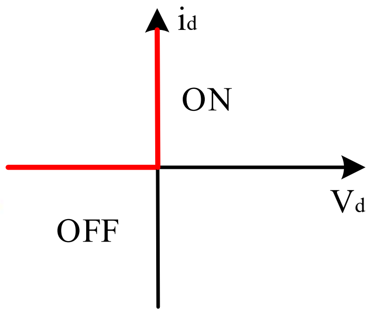
Figure 3-1

- Vo is purely DC or has a specified DC component
- AC source could be sinusoidal, could be a voltage source

A. The Uncontrolled Half-Wave Rectifiers with resistive load:



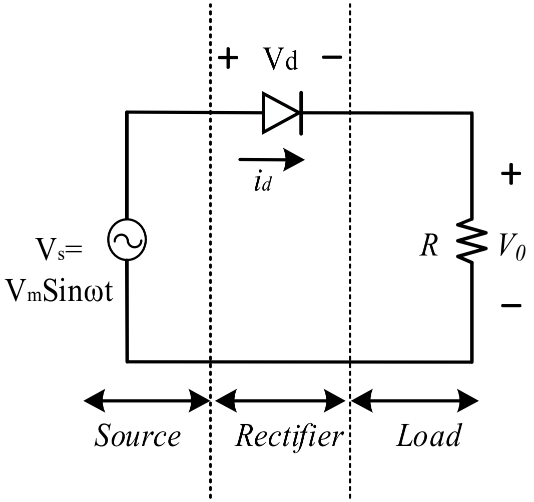


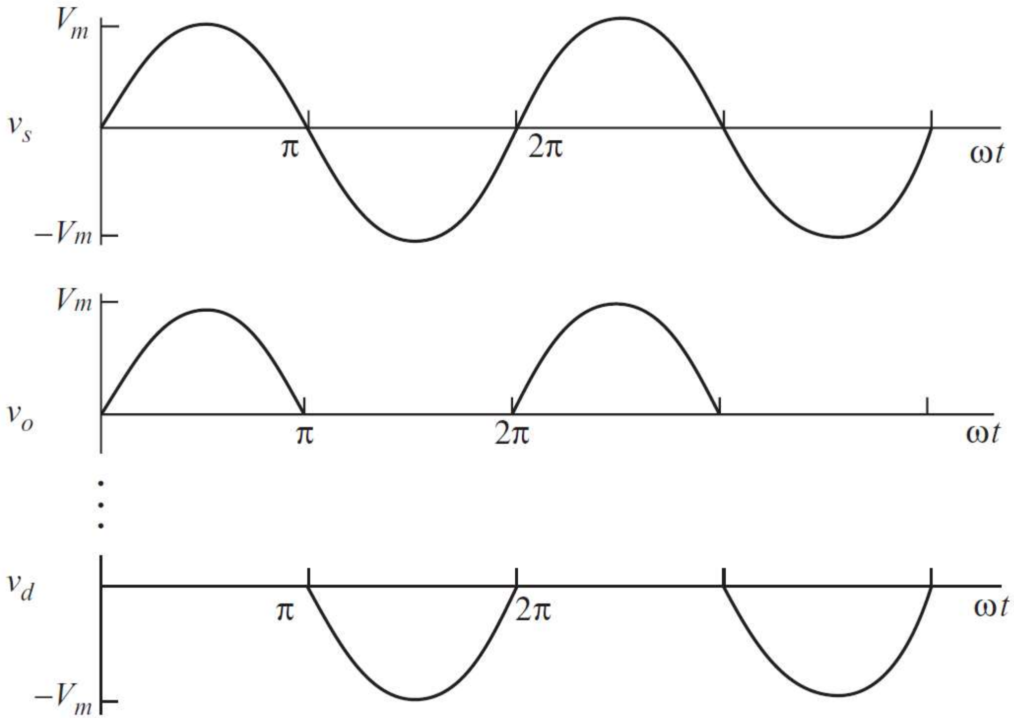


$$
\begin{cases}
0 < \omega t < \pi \quad \text{assume} \quad \text{D is OFF} \quad \Rightarrow \quad i_d = 0 \quad \rightarrow \quad V_0 = 0 \quad \rightarrow \quad V_s = V_d > 0 \\
0 < \omega t < \pi \quad \text{assume} \quad \text{D is ON} \quad \Rightarrow \quad V_d = 0 \quad \rightarrow \quad V_0 = V_s \quad \rightarrow \quad i_d = \dfrac{V_s}{R} > 0
\end{cases}
$$

×Wrong assumption

√ Right assumption





$$\begin{cases} \pi < \omega t < 2\pi \quad \text{assume} \quad \text{D is ON} \\ \Rightarrow \mathrm{V_d}=0 \rightarrow \mathrm{V_0}=\mathrm{V_{in}} \rightarrow \mathrm{i_d}=\frac{V_{in}}{R}<0 \end{cases}$$

×Wrong assumption

$$\begin{cases} \pi < \omega t < 2\pi \quad \text{assume} \quad \text{D is OFF} \\ \Rightarrow \mathrm{i_d}=0 \rightarrow \mathrm{V_0}=0 \rightarrow \mathrm{V_d}=V_{in}<0 \end{cases}$$

√ Right assumption

$$<V_o> = V_{avr} = \frac{1}{2\pi}\int_0^{2\pi} V_0 \, d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} V_m \,\mathrm{Sin}(\omega t)\, d(\omega t) + \overbrace{\frac{1}{2\pi}\int_\pi^{2\pi} 0 \, d(\omega t)}^{0} = \frac{V_m}{\pi}$$

← The average value is not zero

$$<i_d> = \frac{V_m}{R\pi}$$

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_0^2 \, d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \,\mathrm{Sin}(\omega t)]^2 \, d(\omega t)} = \sqrt{\frac{1}{2\pi}\frac{V_m^2 \pi}{2}} = \frac{V_m}{2}$$

$$I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_{orms}}{R} = \frac{V_m}{2R}$$

$$P_{out} = \frac{V_{o-rms}^2}{R}$$

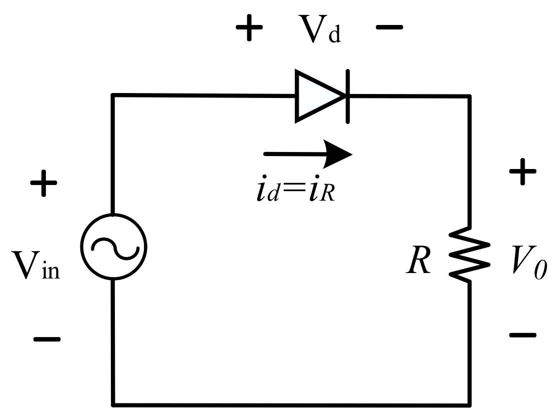
Example: If $V_{in}(t)$=170Sin (366t) and R=12 Ω, determine:

(a) The average value of the load current

(b) The rms value of the load current

(c) The apparent power supplied by the source

(d) Power factor of the circuit



Solution:

(a) $$< i_o > = < i_R > = < i_d > = \frac{V_m}{R\pi} = \frac{170}{\pi \times 12} = 4.51 \quad (A)$$

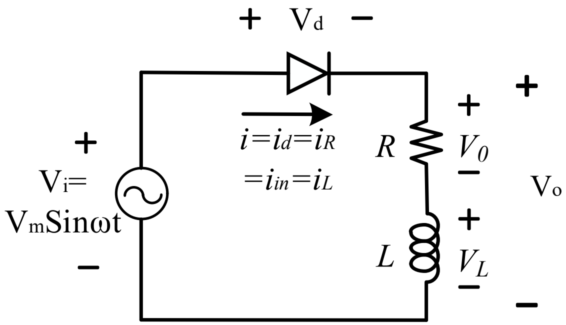
(b) $$I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_m}{2R} = \frac{170}{2 \times 12} = 7.08 \quad (A)$$

(c) $$\text{apparent power} = V_{in-rms}.I_{in-rms} = \frac{170}{\sqrt{2}} \times 7.08 = 851.47$$

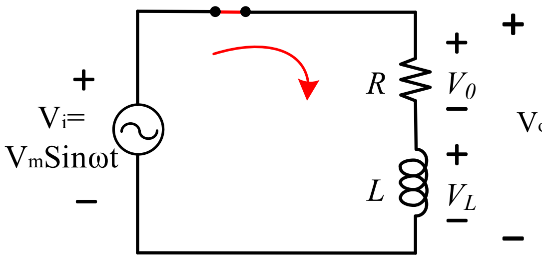
(d) $$\text{pf} = \frac{\text{average power}}{\text{apparent power}} = \frac{P_{out}}{V_{in-rms}.I_{in-rms}} = \frac{R \times I_{o-rms}^2}{V_{in-rms}.I_{in-rms}} = \frac{12 \times (7.08)^2}{\frac{170}{\sqrt{2}} \times 7.08} = \frac{\sqrt{2}}{2} = 0.707$$

B. The Uncontrolled Half-Wave Rectifiers with R-L load:

Example: Providing power to the field winding of a DC machine or armature winding of a DC machine (winding has internal resistance R and self-inductance L).



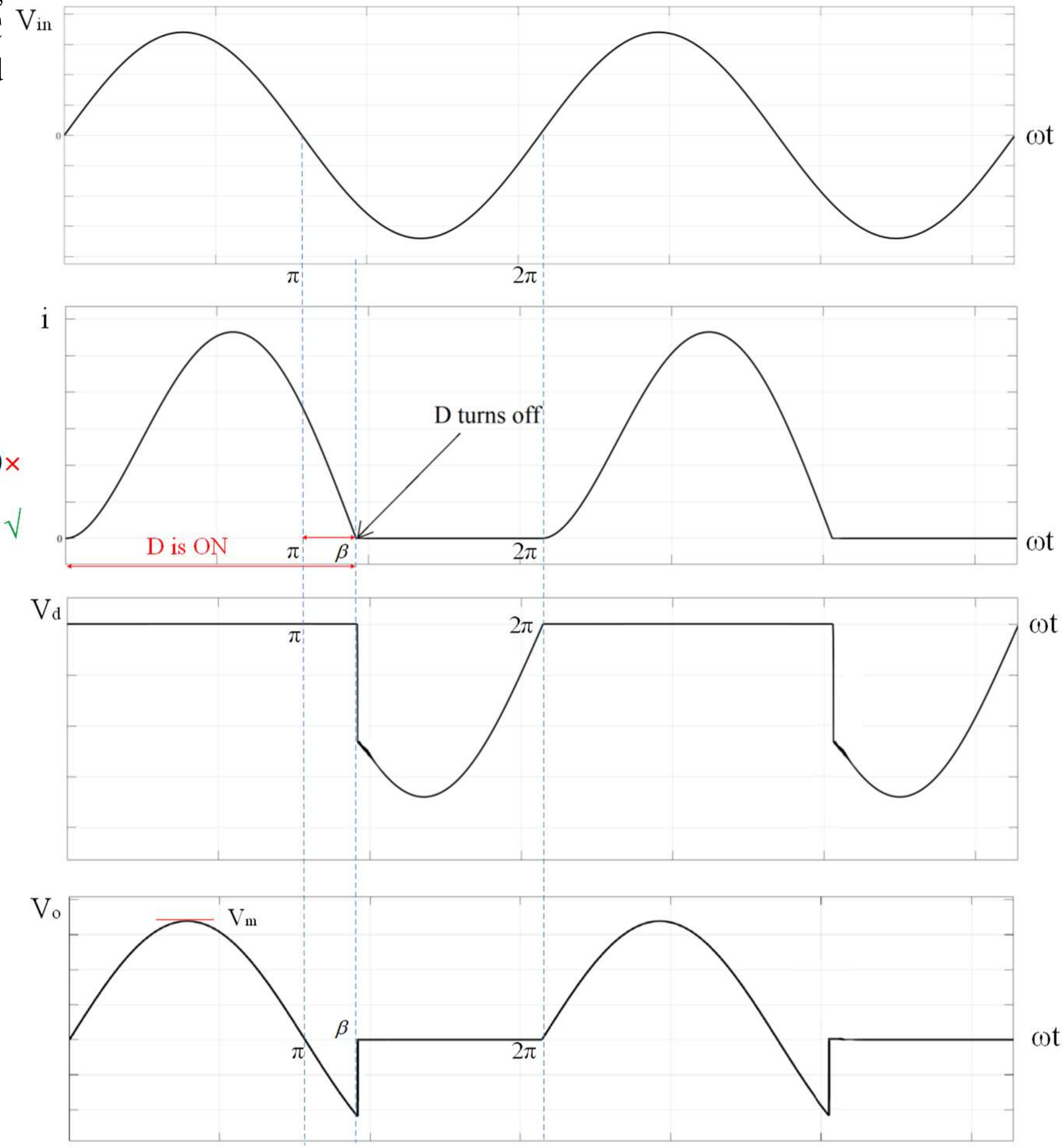
$$\begin{cases} 0<\omega t<\pi \quad \text{assume} \quad \text{D is OFF} \quad \Rightarrow V_{in}=V_d>0 \times \\ 0<\omega t<\pi \quad \text{assume D is ON} \Rightarrow V_d=0 \rightarrow V_0=V_{in} \quad \surd \end{cases}$$

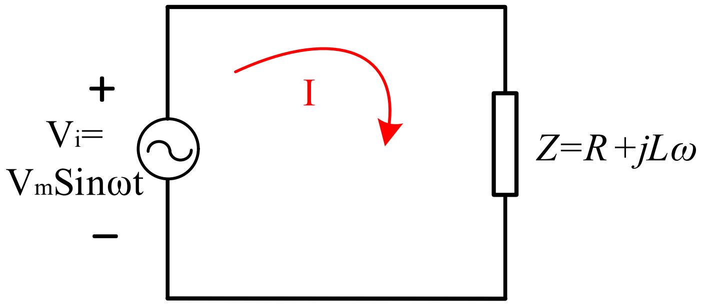


$$V_{in}=V_R+V_L \Rightarrow V_m \,\mathrm{Sin}\,\omega t = Ri + L\frac{di}{dt}$$

$$\Rightarrow i(t)=\underbrace{\frac{V_m}{\sqrt{R^2+(L\omega)^2}}\mathrm{Sin}(\omega t-\theta)}_{steady-state\ response}+\underbrace{Ae^{-t/\frac{L}{R}}}_{natural\ response}$$

$$\theta=\tan^{-1}\left(\frac{L\omega}{R}\right)>0$$





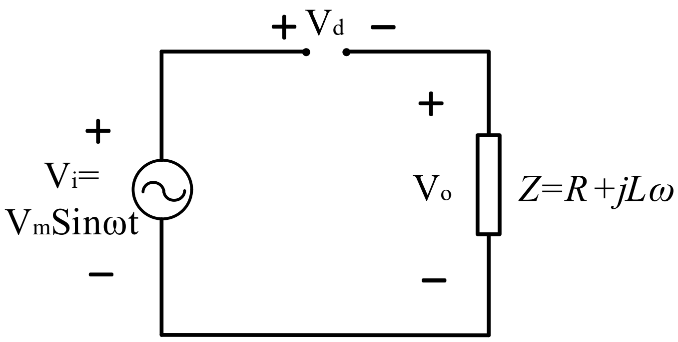
$$0 = Ri + L\frac{di}{dt} \longrightarrow i(t) = Ae^{-t/\frac{L}{R}}$$

$$I = \frac{V_m}{|Z|}\text{Sin}(\omega t - \measuredangle Z)$$

$A = ?$ initial condition $i(0) = 0 \;\Rightarrow\; 0 = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\text{Sin}(-\theta) + A \;\Rightarrow\; A = \frac{V_m \text{Sin}\,\theta}{\sqrt{R^2 + (L\omega)^2}}$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\left[\text{Sin}(\omega t - \theta) + \text{Sin}\,\theta e^{-t/\frac{L}{R}}\right]$$

$$\begin{cases} \text{D is ON} \quad \text{for} \quad 0 < \omega t < \beta & \beta > \pi \\ \text{D is OFF} \quad \text{for} \quad \beta < \omega t < 2\pi & \text{because } i(t) \text{ tends to become negative} \end{cases}$$



D is OFF

D is OFF $\Rightarrow i_d = 0 \Rightarrow V_R = V_L = V_o = 0 \Rightarrow V_{in} = V_d$

when $\omega t = \beta \;\Rightarrow\; i(\omega t = \beta) = 0$

$$i(\beta) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\left[\text{Sin}(\beta - \theta) + \text{Sin}\,\theta e^{-\beta/\frac{L\omega}{R}}\right] = 0$$

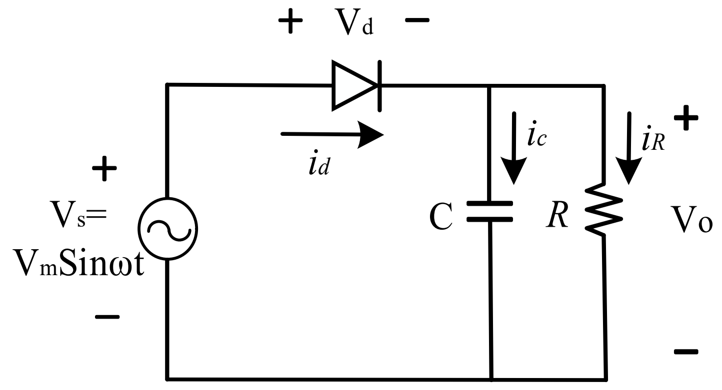
* There is no closed-form sloution to find the β and it must be calculated by software programing like MATLAB but the value of β is always as $\pi \le \beta \le 2\pi$

* β is called as conduction angle or turn off angle of diode.

C. The Uncontrolled Half-Wave Rectifiers with R-C load:

$$D \text{ is } ON \;\Rightarrow\; i_d > 0 \text{ or } i_d = i_C + i_R > 0$$

$$\text{if } D \text{ is } ON \;\Rightarrow\; V_{in} = V_C = V_o = V_m \,\text{Sin}(\omega t)$$



Half-wave rectifier with RC load

$$i_C + i_R > 0$$

$$C\frac{dV_C}{dt} + \frac{V_0}{R} > 0$$

$$C\frac{d(V_m \text{Sin}(\omega t))}{dt} + \frac{V_m \text{Sin}(\omega t)}{R} > 0 \;\rightarrow\; C\omega V_m \text{Cos}(\omega t) + \frac{V_m}{R}\text{Sin}(\omega t) > 0$$

θ is the time at which the diode is going to turn off because its current turns to get negative.

Until ωt=π/2 both terms are positive. After ωt>π/2 the first term get negative and D turns off at ωt=Θ. So, the capacitor is getting discharged through the resistor.

$$C\omega V_m \text{Cos}\,\theta + \frac{V_m}{R}\text{Sin}\,\theta = 0$$

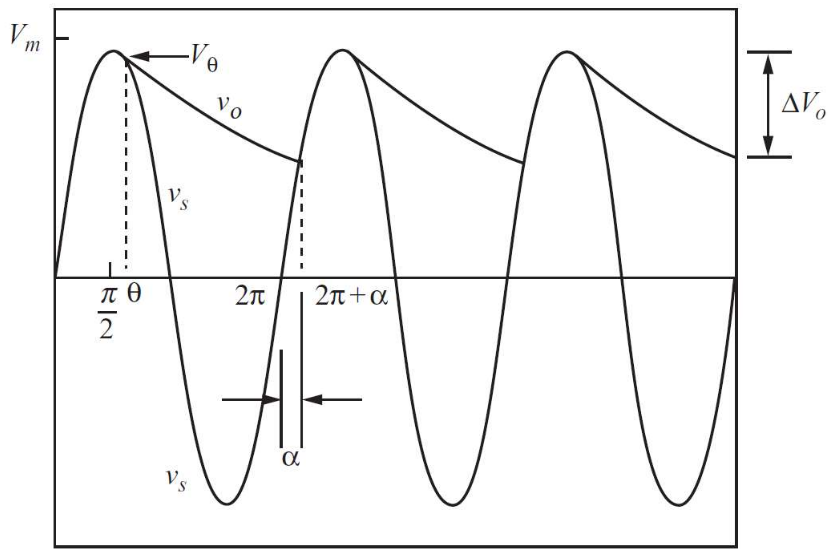
$$\tan\theta = -RC\omega \;\rightarrow\; \theta = \tan^{-1}(-RC\omega) = \pi - \tan^{-1}(RC\omega)$$

$$RC\omega > 0 \;\rightarrow\; 0 < \tan^{-1}(RC\omega) < \frac{\pi}{2} \;\Rightarrow\; \frac{\pi}{2} < \theta < \pi$$

$$\text{if } RC\omega >> 0 \;\Rightarrow\; \tan^{-1}(RC\omega) \rightarrow \frac{\pi^-}{2} \Rightarrow\; \theta \rightarrow \frac{\pi^+}{2}$$

In practical circuits where the time constant is large,

$$\theta \approx \frac{\pi}{2} \text{ and } V_m \;\text{Sin}\,\theta \approx V_m$$



Input and output voltages

D turns OFF:

In this mode capacitor C discharges through resistor R.

if D is OFF: $i_C + i_R = 0 \rightarrow C\frac{dV_C}{dt} + \frac{V_C}{R} = 0$

$$\frac{dV_C}{V_C} + \frac{dt}{RC} = 0 \rightarrow \frac{dV_C}{V_C} = -\frac{dt}{RC}$$

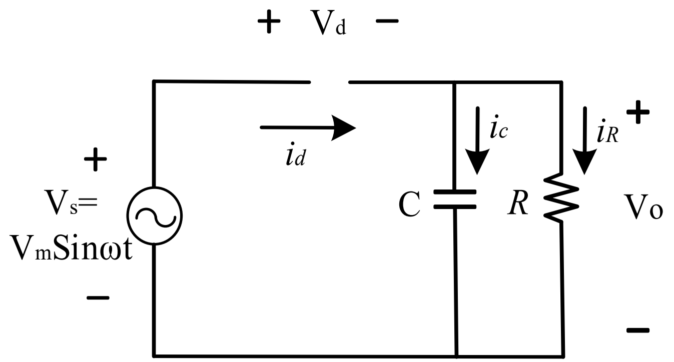
$$\int_{V_\theta}^{V_C} \frac{dV_C}{Vc} = -\int_{\theta}^{\omega t} \frac{dt}{RC} \rightarrow \int_{V_m \sin\theta}^{V_C} \frac{dV_C}{Vc} = -\frac{1}{\omega}\int_{\theta}^{\omega t} \frac{d\omega t}{RC} \rightarrow Ln\frac{V_C}{V_\theta} = -(\frac{\omega t - \theta}{\omega RC})$$

$$V_C = v_o = V_m \sin\theta e^{-(\omega t - \theta)/\omega RC}$$

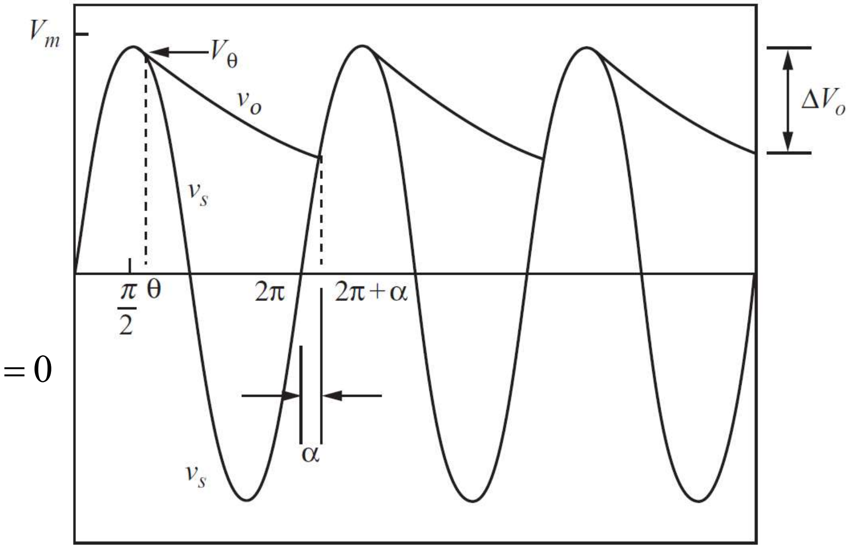
for $\omega t = 2\pi + \alpha \;\rightarrow\; V_C = V_m \sin\theta e^{-(2\pi+\alpha-\theta)/\omega RC}$

if D turns ON at $\omega t = 2\pi + \alpha \Rightarrow V_m \sin(2\pi+\alpha) - V_m \sin\theta e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$

$$\underbrace{{}^*\sin\alpha - \sin\theta e^{-(2\pi+\alpha-\theta)/\omega RC} = 0}_{\text{No closed form solution for } \alpha \text{ must be solved numerically}}$$



Half-wave rectifier with RC load



Input and output voltages

$$V_o(\omega t) = V_C(\omega t) = \begin{cases} V_m \sin\theta e^{-(\omega t-\theta)/\omega RC} & \theta \le \omega t \le 2\pi+\alpha \text{ (diode is OFF)} \\ V_m \sin\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \text{ (diode is ON)} \end{cases}$$

$$i_C = C\frac{dV_C}{dt} = \begin{cases} -\frac{V_m \sin\theta e^{-(\omega t-\theta)/\omega RC}}{R} & \text{diode is OFF} \\ C\omega V_m Cos\,\omega t & \text{diode is ON} \end{cases}$$

where $\theta = \pi - \tan^{-1}(RC\omega)$ and α is the numerical solution of *.

What is the peak to peak ripple of output voltage (ΔV_o)?

The maximum output voltage is V_m. The minimum output voltage occurs at $\omega t=2\pi+\alpha$, which can be computed from $V_m \sin\alpha$. Then, the peak-to-peak ripple of output voltage can be expressed as:

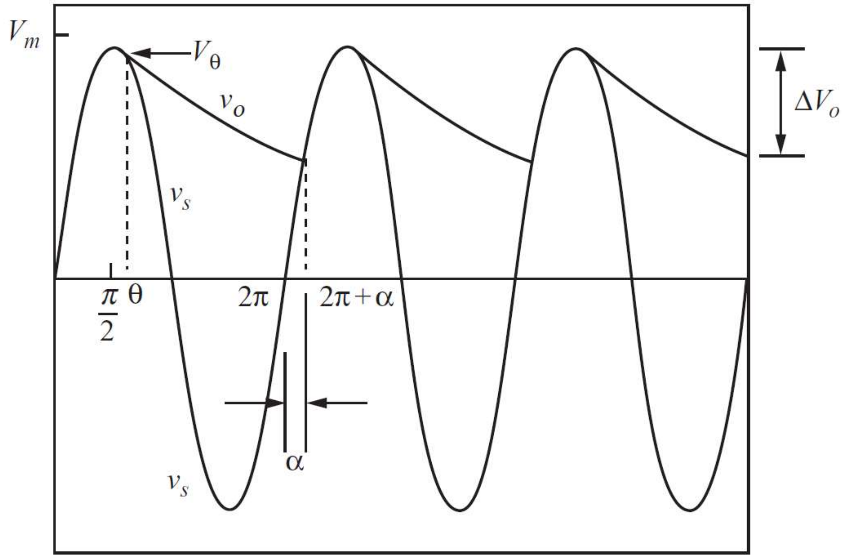
$$\Delta V_0 = V_m - V_m \sin\alpha = V_m(1-\sin\alpha)$$

It is not easy to use this equation if is not existed. For this reason we need to consider some approximations.

The peak-to-peak ripple is approximately:

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \left(\frac{V_m}{fRC}\right)$$

*This approximation is valid if: $\begin{cases} \omega RC \gg 1 \\ or \\ RC \gg T \end{cases}$



Example: The half-wave rectifier with RC load has a 120-V rms source at 60 Hz, R= 500 (Ω), and C=100 (μF). Determine:

a. an expression for output voltage

b. the peak-to-peak voltage variation on the output

c. an expression for capacitor current

d. the peak diode current

e. the value of C such that ΔV_o is 1 percent of V_m.

Solution:

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7\ V$$

$$\omega RC = (2\pi \times 60)(500)(10)^{-6} = 18.85 \text{ rad}$$

The angle θ is determined as $\rightarrow \theta = \pi - \tan^{-1}(RC\omega) = \pi - \tan^{-1}(18.85) = 1.62 \text{ rad} = 93^o$

The angle α is determined as $\rightarrow \sin\alpha - \sin\theta e^{-(2\pi+\alpha-\theta)/\omega RC} = 0 \rightarrow \sin\alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62)/18.85} = 0$

yielding: α=0.843 rad=48^o

a. Output voltage is expressed as:

$$V_o(\omega t) = \begin{cases} V_m \sin\theta e^{-(\omega t-\theta)/\omega RC} & \theta \le \omega t \le 2\pi+\alpha \\ V_m \sin\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases} \Rightarrow \begin{cases} 169.5e^{-(\omega t-1.62)/18.85} & \theta \le \omega t \le 2\pi+\alpha \\ 169.7\sin\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases}$$

b. Peak-to-peak output voltage can be expressed as:

$$\Delta V_0 = V_m(1-\sin\alpha) = 169.7(1-\sin 0.843) = 43\ V$$

c. The capacitor current is determined as:

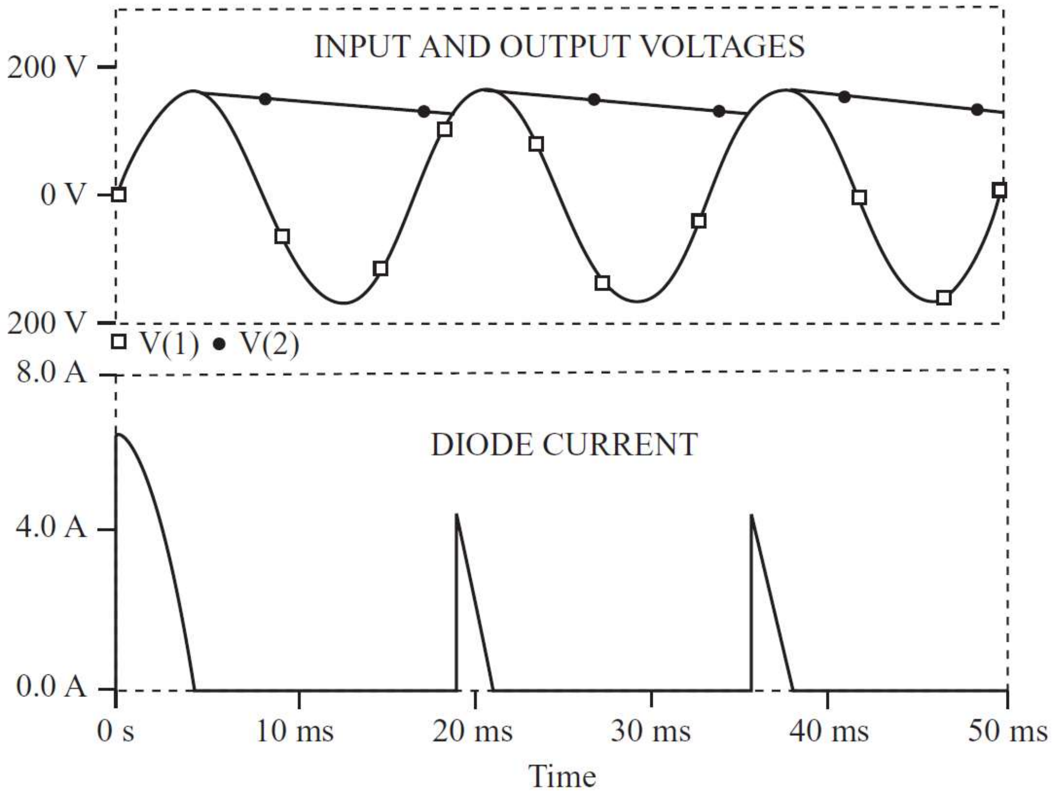
$$i_C(\omega t) = \begin{cases} -\dfrac{V_m \sin\theta e^{-(\omega t-\theta)/\omega RC}}{R} & \theta \le \omega t \le 2\pi+\alpha \\ C\omega V_m Cos\,\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases} \Rightarrow \begin{cases} -0.339e^{-(\omega t-1.62)/18.85}\ \text{A} & \theta \le \omega t \le 2\pi+\alpha \\ 6.4Cos(\omega t)\ \text{A} & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases}$$

d. Peak diode current is determined as:

$$i_{d,peak} = V_m(C\omega\cos\alpha + \frac{\sin\alpha}{R}) = \sqrt{2}(120)\left[377(10)^{-4}\cos 0.843 + \frac{\sin 0.843}{500}\right] = 4.26 + 0.34 = 4.50\ \text{A}$$

e. For $\Delta V_o = 0.01\ V_m$, C can be calculated as:

$$C \approx (\frac{V_m}{fR(\Delta V_o)}) = (\frac{V_m}{60\times 500\times 0.01V_m}) = \frac{1}{300}F = 3333\mu F$$

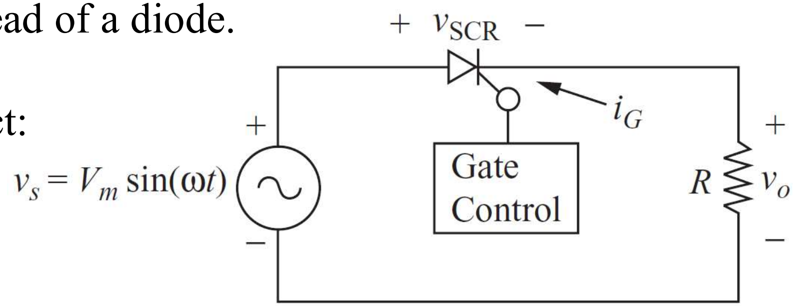


2. The Controlled Half-Wave Rectifier:

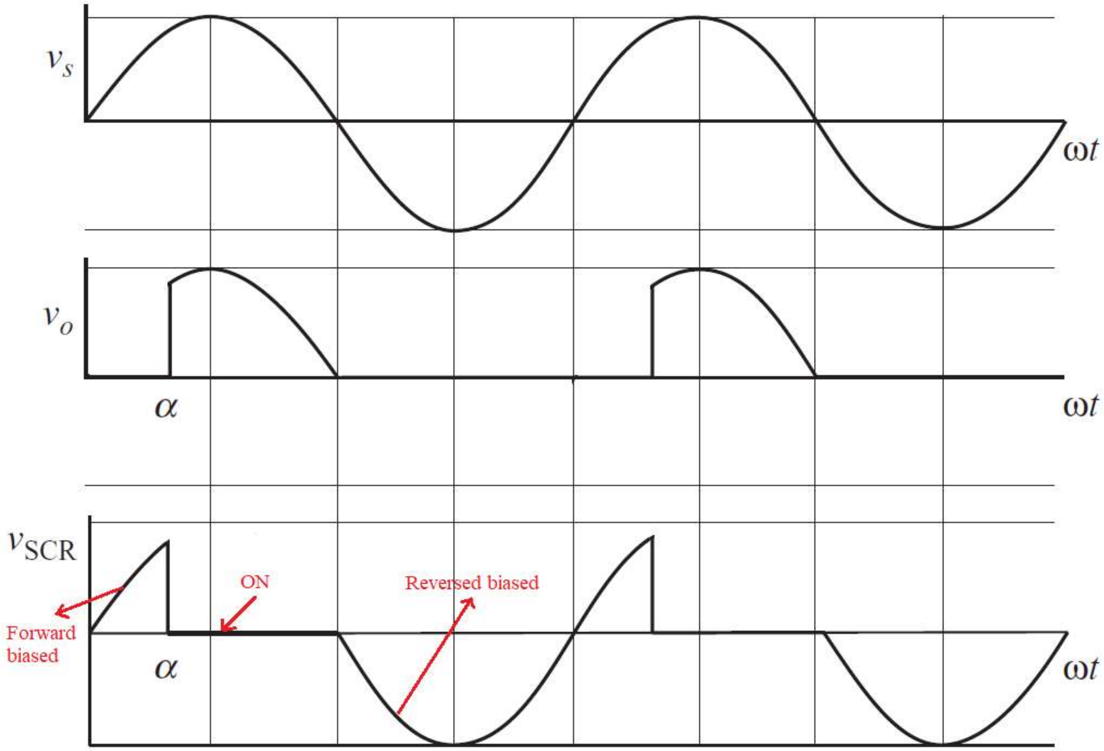
The half-wave rectifiers analyzed previously are classified as uncontrolled rectifiers. Away to control the output of a half-wave rectifier is to use an SCR instead of a diode.

Two conditions must be met before the SCR can conduct:

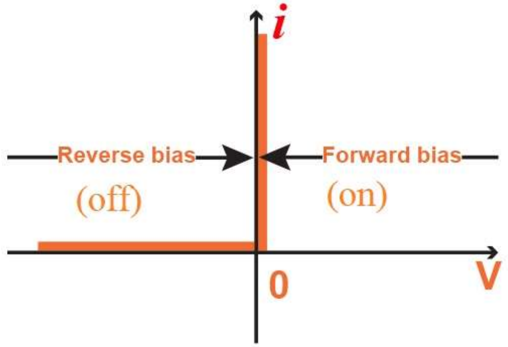
1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.



A basic controlled rectifier



Voltage waveforms



$0<\omega t<\alpha$, SCR is forward blocking (OFF)

$\alpha <\omega t<\pi$, SCR is ON

$\pi <\omega t< 2\pi$, SCR is reversed biased

@ $\omega t=\alpha$, the SCR is triggered and starts conducting

@ $\omega t= \pi$, the SCR current reduces to zero and it stops conducting

α is called the delay angle

The average (DC) voltage across the load resistor can be calculated as:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

The power absorbed by the resistor is $V_{rms}^2 / 2$, where the rms voltage across the resistor is computed from

$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[V_o(\omega t)\right]^2 d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \operatorname{Sin}^2(\omega t) d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \times (\frac{1-\operatorname{Cos}2\omega t}{2}) d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{(\frac{\pi-\alpha}{\pi}) + (\frac{\operatorname{Sin}2\alpha - \operatorname{Sin}2\pi}{2\pi})} = \frac{V_m}{2}\sqrt{(1-\frac{\alpha}{\pi}) + (\frac{\operatorname{Sin}2\alpha}{2\pi})} \qquad \alpha \text{ in rad}$$

$$\text{Power Factor (pf)} = \frac{\text{average power}}{\text{apparent input power}} = \frac{P_{in} = P_{out}}{V_{in,rms} \times I_{rms}} = \frac{V_{rms}^2 / R}{V_{in,rms} \times V_{o,rms}/R} = \frac{V_{o,rms}}{V_{in,rms}}$$

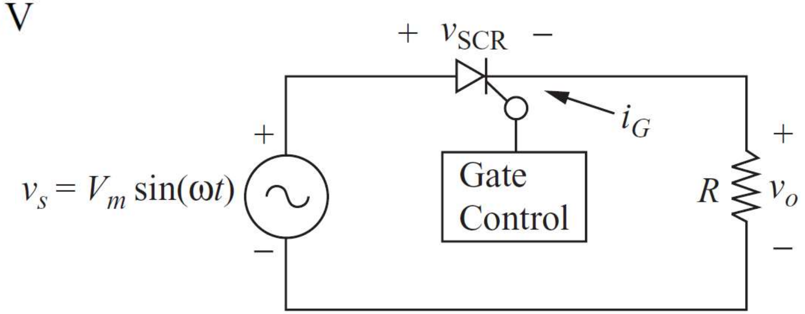
$$= \frac{\frac{Vm}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\operatorname{Sin}2\alpha}{2\pi}}}{\frac{Vm}{\sqrt{2}}} = \sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\operatorname{Sin}2\alpha}{4\pi}} \qquad \alpha \text{ in rad}$$

Example: Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

Solution: In an uncontrolled half-wave rectifier, the average voltage will be $V_m/\pi = (120\sqrt{2})/\pi = 54$ V.

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right] = \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

$$V_{rms} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$



Load power is:

$$P_R = \frac{V_{rms}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is:

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

Questions and comments are most welcome!