

**Problem Set 1 (Due Sep 18, 2020)**

1. In the lecture notes, I included a link to historical data on global measures of poverty, life expectancy, education etc. on Our World in Data. This is a wonderful site for finding current and historical data on all types of social statistics across different countries. Another excellent source for data is Gapminder. This exercise invites you to get first experience on using such data sources. You will get much more practice of this type in Principles of Empirical Analysis in Period III, but it is important to become familiar with typical graphical methods of displaying data.
  - (a) On the tools page Gapminder tools, you see a cross tabulation of current incomes and life expectancies across different countries. If income and life expectancy tend to vary in the same direction (i.e., life expectancy tends to be higher in countries with a higher income), we say that income and life expectancy are positively correlated. If they vary in the opposite direction, we say that they are negatively correlated. By just looking at the picture, can you see if life expectancy and income are positively correlated?
  - (b) By clicking on the variable on each axis, you can create different bubble diagrams. Keep the variable on the x-axis income and find a variable on the y-axis that is negatively correlated with income.
  - (c) How would you describe the relationship between population size and income across the different countries?
  - (d) Click on trends and select China, USA and Finland as the displayed countries (from the right panel). Note that the scale on the y-axis is logarithmic. This means that moving up  $k$  steps on the y-axis represents multiplying the original value of  $y$  by  $2^k$ . This means that a line with a positive slope indicates exponential income growth at a constant rate. You can see concretely the

effect of different scales by clicking the variable on top of the y-axis and changing the y-axis to linear. Which of the three countries has had a growth rate that has been close to constant over time? Can you find explanations to the periods of negative growth rates (periods of decrease in the log scale)?

2. This exercise is designed to let you see what the feasible set can look like in concrete choice situations. The context is that of allocating a monetary budget of  $w$  euros between gigabytes of data per month of data transfer on a smartphone (x-axis) and all other consumption (on y-axis).
  - (a) The simplest pricing scheme allows you to buy any number  $x$  of GB per month at a fixed price of  $p$  euros per GB (prices and GB are in monthly terms). How many GB can you buy if you use your entire budget on GB? Of course, you may choose not to buy any GB at all and in this case, you can use  $w$  on other consumption. Draw the set of feasible choices of  $(x, y)$ , i.e. combinations of  $x$  GB of data transfer and  $y$  euros of other consumption such that you do not exceed your total budget for the case where  $w = 10$  and  $p = 2$ .
  - (b) To see the effect of a price change, draw the feasible set when  $w = 10$  and  $p = 4$ . To see the effect of a wealth change, draw the feasible set for  $w = 20$  and  $p = 2$ .
  - (c) Consider another plan that allows you to buy  $x$  GB of data at a lower price  $p' < p$  if you pay a fixed (monthly) payment of  $f$  euros. Draw the feasible set for this plan for  $f = 3$  and  $p' = 1$ .
  - (d) Often you are given the choice between alternative plans. What is the feasible set if you can choose between the plans in part a) and c)? (Hint: for each level  $x$  of data transfer, which plan delivers  $x$  at the least cost?)
  - (e) In Finland, it is common to offer schemes with only a fixed payment: you can choose any  $x$  that you want without any extra charge if you pay a higher fixed fee  $f^* > f$ . Draw the feasible set of this plan for  $f^* = 8$  and draw the feasible set when you can choose between all three alternatives.

- (f) Another commonly observed plan is where you pay a fixed fee  $f''$  and get up to  $x''$  GB per month for free and you pay  $p'' > p$  for any additional GB. Draw the feasible set for this scheme when  $x = 6$ ,  $f'' = 6$ ,  $p'' = 4$ . Is there some amount of data consumption  $x$  such that this plan is the best choice?
3. In order to move towards choice from such feasible sets, we start by forming the indifference curves of a consumer in this market.
- (a) In the  $(x, y)$  -coordinate system of the previous question, think about the shape that indifference curves should take. In particular, do you think that MRS between GB and other consumption becomes lower as you increase the number of GB's? If so, draw indifference curves reflecting this.
- (b) Suppose that because of time constraints, you will never use more than 100GB per month. What is your MRS at any point  $(x, y)$  with  $x > 100$ ?
- (c) For the feasible set of Problem 2.a), determine graphically the optimal choice for the consumer with the indifference curves from part a) of this question. How can you express the MRT in this problem?
- (d) Consider plans from parts a) and c) of the previous question. Is it possible that a consumer is indifferent between the two plans?
- (e) We say that Ann likes data more than Bob if the MRS of Ann (denoted by  $MRS_A$ ) is higher than the MRS of Bob ( $MRS_B$ ) at all  $(x, y)$ . Show by drawing the picture for the plan in 2.a) that at optimum, Ann chooses a higher  $x$  than Bob. (Hint: draw the picture for Ann's optimal choice and consider Bob's indifference curve through Ann's optimal consumption).
- (f) (Extra Credit) This is an open ended question and you will have to wait until at least next year to get the tools to do the analysis. You may still want to make an educated guess of why an operator might want to let consumers choose from such a set rather than just offering one plan.
4. The last problem in this problem set concerns feasible lifetime consumptions. The choice is how to allocate a lifetime budget between

consumption as student (on the x-axis) and consumption when working (y- axis). When studying, Cecilia receives a student benefit to  $b$  and when working, she receives a wage income  $w$ . This means that the point  $(b, w)$  in the coordinate system is a feasible consumption pair.

- (a) If Cecilia cannot borrow or save, she can only consume up to  $b$  when student and up to  $w$  when old. If she can save at an interest rate  $r$ , she can transform  $\Delta x$  units of foregone consumption when student into  $(1 + r)\Delta$  units of consumption when working. Draw the set of feasible consumptions when Cecilia can save but not borrow for numerical values  $b = 5, w = 20r = .1$ . What is the MRT for Cecilia in this problem?
  - (b) Draw the feasible set for the case where Cecilia can also borrow at the same interest rate  $r$ .
  - (c) Suppose next that Cecilia has symmetric preferences over consumptions when young and when old. This means that her  $MRS = 1$  at any point where  $x = y$ , i.e. when her consumption when student and when working are the same. Is it optimal for Cecilia to consume the same amounts when student and when working?
  - (d) When should Cecilia consume more? Can you decide from the information thus far if Cecilia should borrow or save?
5. For those of you itching to get your hands dirty with computations, here is a simple case to consider. In the context of Problem 4. Suppose that the  $MRS$  of Cecilia takes the form  $MRS$  at point  $(x, y)$  is equal to  $\frac{y}{x}$ . In this case,  $MRS = MRT$  gives the optimal solution to the problem. Express the amount of consumption when working  $y$  as a function of the borrowing  $(x - b)$  and equate  $MRS$  to  $MRT$  to solve for optimal consumption when student  $x^*$  and compute the resulting optimal  $y^*$ . For this specification of  $MRS$ , is it optimal to save or borrow?