# Exercise 0 – Solutions

**8.9.2020**

## #1 Binomial distribution, expected value

See the related Excel sheet for concrete calculations.

As there are 13 spades in the deck, the probability of getting a spade on a single draw is 13/52=1/4=0.25. Let $X$ denote the number of spades obtained from the five draws. Now $X$ follows the binomial distribution with the parameters $n$ =5 and $p$ =0.25, i.e., $X \sim Bin(5, 0.25)$. The probability density function of X is given by $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, \dots, n$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

a) The probability of getting exactly three spades is $P(X = 3) \approx 0.088$.
b) The probability of getting at most one spade is $P(X \leq 1) = P(X = 0) + P(X = 1) \approx 0.63$.
c) The expected profit is obtained by $\sum_{k=0}^{5} P(X = k) * (Profit|X = k) \approx 6.81$.

The amount you would be willing to pay to participate in this game depends on your risk attitude that will be discussed later in the course. As the expected profit from the game is 6.81 €, you could expect to gain some money if the participation fee to the game is smaller than 6.81 € and lose money if the fee is greater than that.

## #2 Conditional probability, law of total probability, Bayes' theorem

See the related Excel sheet for concrete calculations.

The info given can be represented mathematically as follows: $P(vaccination = true) = 0.95$, $P(disease = true|vaccination = false) = 0.07$, $P(disease = true|vaccination = true) = 0.004$.

a) $P(disease = true) = P(disease = true \,|\, vaccination = true) * P(vaccination = true) + P(disease = true \,|\, vaccination = false) * P(vaccination = false) \approx 0.0073$.
b) We can apply Bayes' theorem: $P(A|B) = \frac{P(B|A)*P(A)}{P(B)}$. Hence,
$P(vaccination = false|disease = true) = \frac{P(disease=true|vaccination=false)*P(vaccination=false)}{P(disease=true)} \approx 0.48$.
c) If only 50% of people have been vaccinated, it applies $P(disease = true) \approx 0.037$ and $P(vaccination = false|disease = true) \approx 0.95$.

## #3 Monte Carlo simulation using Excel

Parts a)-c), see the related Excel sheet.

d) The average length, standard deviation, and the share of rejected nails correspond well to their theoretical values.

## #4 Monte Carlo simulation using Matlab

See the related Matlab file.