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Lecture 1:
What exactly is a plasma?

Today’s menu

- Saha equation and definition of plasma
- Debye length & plasma sheath
- Plasma frequency
- Concept of quasineutrality
- Plasma parameter
- Weakly and strongly coupled plasmas
- Examples of plasma

Different states of matter

Consider H_2O:

- $T < 0°C$ ➔ ice = solid state
- $0°C < T < 100°C$ ➔ water = liquid state
- $T > 100°C$ ➔ vapor = gaseous state

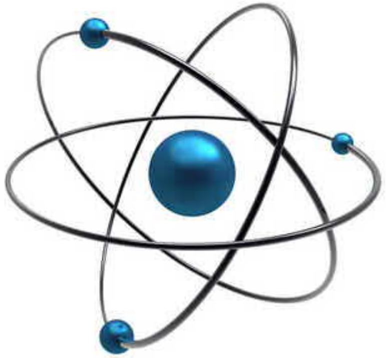
Moving from one state to another happens via *phase transitions* where energy is either released or absorbed by the system: *latent heat*

Is 3 states of matter the best we can do?

What happens if we further heat the system = pump energy into it?

i.e., is there a possibility of moving yet to another, *qualitatively different* state of matter?

What can happen to a material that has already been broken to its basic constituents, i.e., atoms?



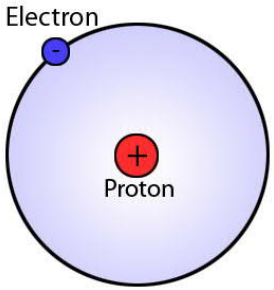
… atoms are *not* basic constituents of matter…

So let’s go a step deeper in …

Qualitative considerations

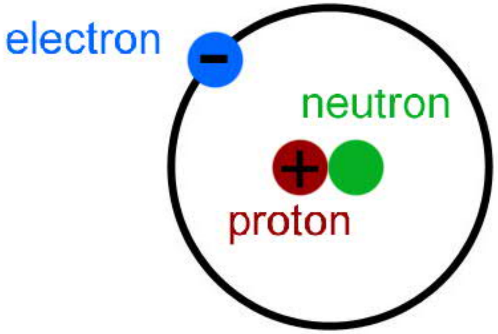
From gas to plasma

- In each state of matter, the constituents are bound together with different interactions that are broken by additional energy introduced to the system.
- In gas, there are no binding interactions between the constituents – or,what we have so far *considered* basic consitutents !
- Consider the simpliest element, hydrogen:



Atom = e + ion

Ionization energy = 13.6 eV



The gas would thus need to be heated to 16 000 K to rip off the electrons …

The Saha equation

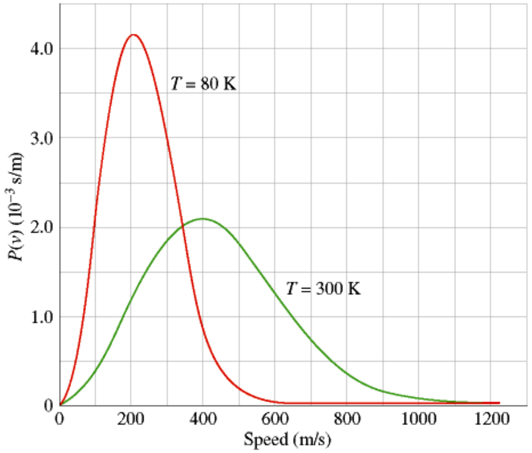
- If the temperature is not far above that corresponding to ionization energy, the competing process, *recombination*, makes the matter consist of both neutral and charged particles, i.e., be *partially ionized*.
- The degree of ionization is given by the Saha equation:

$$\frac{n_i}{n_n} \approx 3 \cdot 10^{27} T^{3/2} n_i^{-1} e^{-U_i/T}$$

where U_i = *ionization energy* and $[T]$ = eV.

The physics of Saha

- Ionization requires strong head-on collision
- Velocity distribution in a gas = Maxwellian
  - ➔ # of particles with $E_{kin} > U_i$ depends exponentially on T
- Recombination rate depends on # of electrons $n_e \sim n_i$
  - ➔ $1/n_i$ dependence due to recombination



➔ n_i starts to rapidly increase when $T \rightarrow U_i$, but is limited by 'itself', i.e., by recombination

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Different ’gases’

Usual air (mostly nitrogen) in room temperature, T = $20°C$:

- $20°C \sim 0{,}03$ eV:
- $U_i(N) \sim 14{,}5$ eV ➔ $n_i/n_n \sim 10^{-122} \sim 0$
- $n_n \sim 3 \times 10^{25}$ m^{-3}

Lagoon nebula *(ESO)*

Interstellar *plasma*:

- $T \sim 10 - 20°K \sim 0{,}002$ eV
- $n_n \sim 1$ cm^{-3}

➔ (thermal) ionization is rare, but recombination is even rarer!!! ➔ ***plasma***

Note: on the unit of temperature in plasma physics

For units of temperature, eV is the natural one because it is the *energy* that is relevant, not temperature as we experience it

- Ionization energies
- Maxwellian distribution

Conversions:

- $1\ \mathrm{eV} \approx 1.6 \cdot 10^{-19}\ \mathrm{J}$
- $k_B \approx 1.4 \cdot 10^{-23}\ \mathrm{J/K} \approx 8.6 \cdot 10^{-5}\ \mathrm{eV/K}$

Thus we shall replace $k_B T$ by just T – and understand that it is in eV

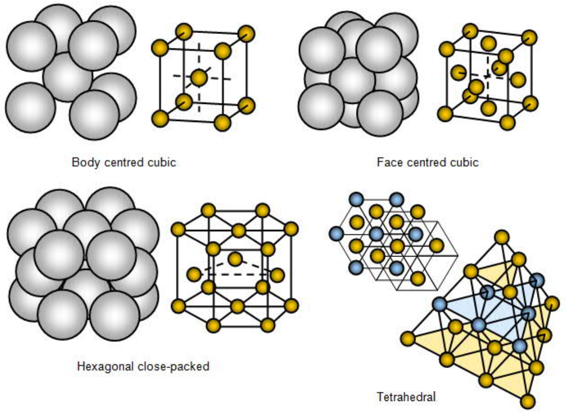
Intuitive look at plasma as a state of matter

Why consider plasma as a separate state of matter?

Isn’t it just one kind of gas?

What distinguishes different states of matter:

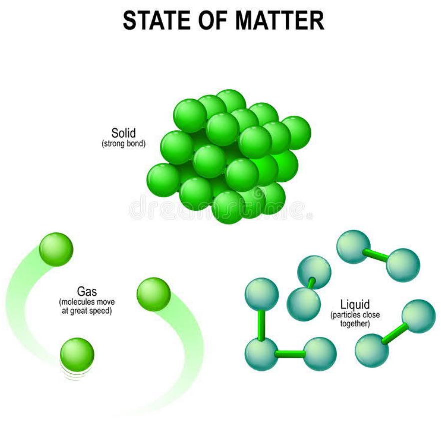
nature of interactions !!!

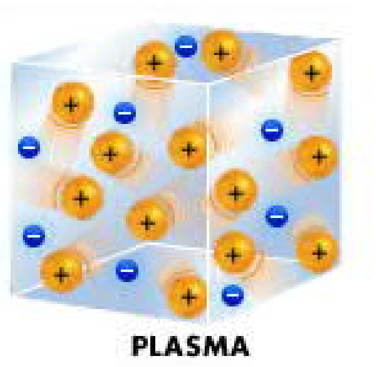


1. Solid:
   - Fixed structure due to strong bonds = interactions between nearest neighbors: strong means $E_{kin} << U_{bond}$

... Fluids ...

2. Liquid:
   - $E_{kin} \rightarrow U_{bond}$ ➔ some mobility but still sticking together
3. Gas:
   - $E_{kin} > U_{bond}$ ➔ independent constituents, interactions via head-on collisions
4. Plasma:
   - $E_{kin} > U_i$ ➔ Charged particles ➔ Coulomb interactions with infinite range, 1/r





➔ In this sense plasma is more fundamentally different from the other states of matter than those are from each other!

The concept of a *fluid*

Why then is it common to lump plasmas together with liquids, gases and call them *fluids?*

Consider and attractive ’girl’ in a pub.

If you enter the pub after her, you probably won’t notice her – she is surrounded by other ’men’ ➔ she is ***shielded***.

The same happens in plasmas: free charges are shielded, $\Phi_{Coulomb} \propto \frac{e^{-\frac{r}{\lambda_D}}}{r}$

➔ In some conditions the plasma can behave as a regular gas

NOTE: *do not sneer at people in the past…*

See how far the ancient Greeks got without advanced math and modern measuring instruments …

➔ *Do not under-estimate the power of thinking !*

Earth

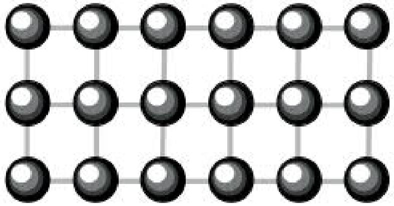
Water

Air

Fire

Earth

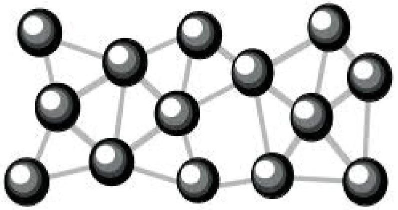
- Solid
- $T < 0$C



Water

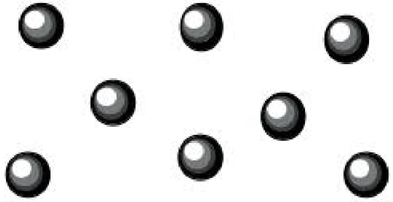
- Liquid

0C $< T < 100$C



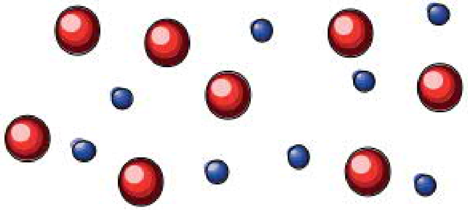
Air

- Gas
- $T > 100$C



Fire

- Plasma!
- $T > 13$ eV



When is a ’*partially ionized gas*’ not a gas but a plasma?

The definition of a plasma is not given as a critical number for the Saha equation but, rather, in a more complicate manner:

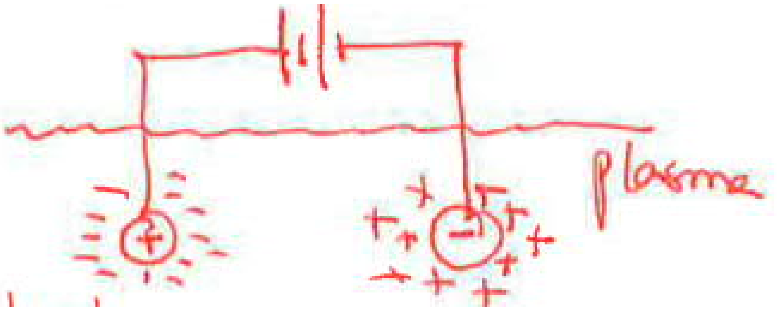
”A plasma is a ***quasineutral*** *gas of charged particles which exhibits* ***collective behaviour***”

- *Collective behaviour* = motions that depend not only on the local conditions but also on the state of the plasma in remote regions
- *Quasineutrality:* over-all neutrality allowing local charge non-uniformities

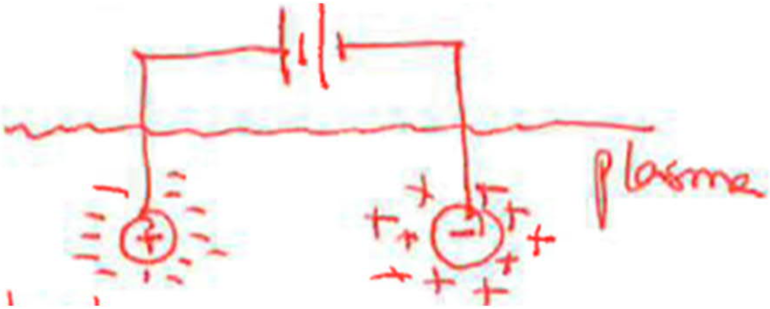
Getting more quantitative …

Debye length

- We already saw what happens to an attractive girl in a pub.
- The ’shielding distance’ λ_D, is called the *Debye length.*
- This shielding is also important in *plasma diagnostics*, e.g. when measuring something with metal probes inserted to plasma.

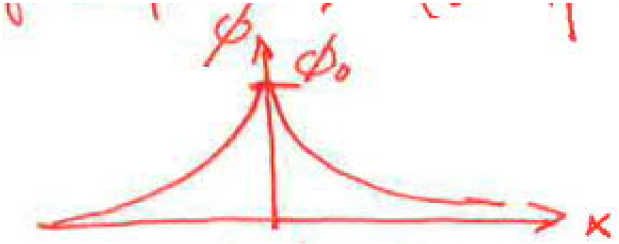


From Debye length to plasma sheath



- Assume ’cold’ plasma:
  - Here ’cold’ means no thermal motion
  - ➔ Shielding charges just sit there
  - ➔ Perfect shielding
- $T \neq 0$
  - Allow thermal motion
  - ➔ Potentials of the order of $k_B T$ can leak into the plasma
  - ➔ $\boldsymbol{E} \neq 0$ within the sheath region …

Width of the sheath region?



- For simplicity, take a 1D situation
- $m_i / m_e \sim 2000$

  ➔ assume ions fixed, electrons mobile
- Poisson equation: $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \leftrightarrow -\frac{d^2\Phi}{d^2x} = \frac{1}{\epsilon_0} e(n_i - n_e)$
- Electron velocity distribution when electrostatic potential Φ is present:

$$f(x, v) = Ae^{-E_{tot}/T} = Ae^{-(\frac{1}{2}mv^2 - e\Phi(x))/T}$$

- Infinitely far from the probe Φ=0 ➔ $n_e(\infty) = A\int_{-\infty}^{\infty} e^{-(\frac{1}{2}mv^2)/T} dv = n_i \equiv n_0$

Sheath = Debye!

So we have $n_e(x) = n_0 e^{e\Phi(x)/T}$

'Far enough' from the plate (finding the range of electric field): $\frac{e\Phi(x)}{T} \ll 1$

$$\varepsilon_0 \frac{d^2\Phi}{d^2x} \approx en_0\left(1 + \frac{e\Phi(x)}{T} + \frac{1}{2}\left(\frac{e\Phi(x)}{T}\right)^2 + \ldots - 1\right) ;\ \text{Taylor expansion}$$

Keep only the 1st order:

$$\varepsilon_0 \frac{d^2\Phi}{d^2x} \approx en_0 \frac{e\Phi(x)}{T} \Rightarrow \Phi(x) \approx \Phi_0 e^{-\frac{x}{\lambda_D}}$$

Where $\lambda_D^2 = \frac{\varepsilon_0 T}{e^2 n_0}$ is the ***Debye length***(*) – and the extent of the ***plasma sheath***

(*) *(also obtained for the girl-in-the-pub: HW)*

Observations on Debye length/sheath

$$\lambda_D = \sqrt{\frac{\varepsilon_0 T}{e^2 n_0}}$$

- Debye length/sheath is large when
  - temperature is high ➔ thermal motion allows for large excursions
  - Density is small ➔ need large distance to accumulate the enough electrons to cause the shielding
- Debye length/sheath is small when
  - Reverse the above arguments

Usefulness of Debye length

Charge imbalances thus occur only in the scale of λ_D

- *A collection of charged particles behave like a plasma only if* $\lambda_D \ll L$, where *L* is the size of the plasma/scale of the phenomenon

➔ Any local charge concentrations and/or external potentials are shielded out within $\lambda_D \ll L$

➔ Bulk of the plasma is free of large scale potential differences:

$$\nabla^2\Phi = \frac{\rho}{\varepsilon_0} \approx 0 \rightarrow n_e \approx n_i$$; difference typically of the order 10^{-6}

This common density $n_e \approx n_i \equiv n_0$ is called the ***plasma density***

The concept of quasineutrality

Plasma is ***quasineutral***, which means that

Plasma is neutral enough to assume $\boldsymbol{n_e \approx n_i \equiv n_0}$ but not so neutral as to eliminate all electromagnetic forces

This can be satisfied when $0 < \lambda_D \ll L$: then potentials $\Phi \sim T$ can easily be introduced by small charge imbalances

Weakly and strongly coupled plasmas

Criterion for a plasma includes the size of the plasma… inconvenient
➔ Let's look at a collection of charged particles in a different way:

- Inter-particle distance : $r_d = n_0^{-1/3}$
- 'interaction' distance = distance of closest approach, r_C :

$$\frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0 r_C} \rightarrow r_C = \frac{e^2}{4\pi\varepsilon_0 T} \quad ; < E_{kin} > = T$$

  - $\frac{r_d}{r_C} \ll 1$: particles closer than r_C of each other ➔ continuous strong interaction

  ➔ Strongly coupled plasma

  - $\frac{r_d}{r_C} \gg 1$: only occasional (strong) interaction, r_C has some relevance

  ➔ Weakly coupled plasma (dominated by small-angle Coulomb scattering ...)

Plasma parameter

Let us introduce a new parameter,

$$\Lambda \equiv \frac{1}{\sqrt{4\pi}} \left(\frac{r_d}{r_C}\right)^{3/2}$$

You will show that this can also be written as

$$\Lambda = \frac{4}{3} n_0 \pi \lambda_D^3,$$

i.e., Λ gives the # of particles in a Debye sphere!

- Weakly coupled plasma: $\Lambda \gg 1$, *'genuine'* plasma
- Strongly coupled plasma: $\Lambda \ll 1$, resembles liquids

➔ ***size-independent plasma criteria:*** $\boldsymbol{\Lambda \gg 1}$

As if this wasn’t enough…

Recall the definition of plasma: two things are required

1. Quasineutrality (which we just addressed), and
2. collective phenomena...

Phenomenologically, what sets a plasma apart from the other states of matter is its ability to generate and sustain *collective phenomena.*

Example of a collective phenomenon

- Move a slab of electrons by δx.

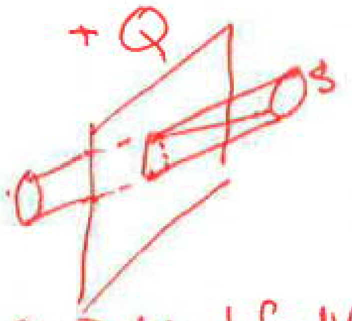
➔ At the faces of the 'deprived' region there will be a surface charge +σ (right) and -σ (left): $\sigma = en_0\delta x$

- Use Gauss' law to obtain E_x : $E_x = \sigma / \varepsilon_0$

➔ Restoring force for each electron in the slab:

$$m_e\frac{d^2\delta x}{dt^2} = -eE_x \leftrightarrow \frac{d^2\delta x}{dt^2} = -\frac{e^2n_0}{m_e\varepsilon_0}\delta x$$

➔ $\delta x = \delta x_0 \sin \omega_p t$ or $\delta x_0 \cos \omega_p t$



$$\int \nabla\cdot\vec{E}\, dV = \frac{1}{\varepsilon_0}\int \rho\, dV$$

$$\int \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0} q$$

$$\Rightarrow \bar{E} = \frac{Q}{\varepsilon_0 S} = \frac{\sigma}{\varepsilon_0}$$

Plasma responds by oscillating at ***plasma frequency*** $\boldsymbol{\omega_p^2 \equiv \frac{e^2n_0}{m_e\varepsilon_0}}$

Yet another requirement for plasma…

Plasma oscillations (= collective phenomenon) is inhibited (= screwed up) if *collision frequency* is higher than the plasma frequency

➔ *For a collection of charges to be called a plasma, the collisions have to occur on a time scale slower than* ${\omega_p}^{-1}$.

Otherwise the dynamics is collision dominated and no collective phenomena can occur due to randomization by collisions.

This is why, for instance, the ionized gas in a jet exhaust is *not* a plasma.

Prerequisites to be called a plasma

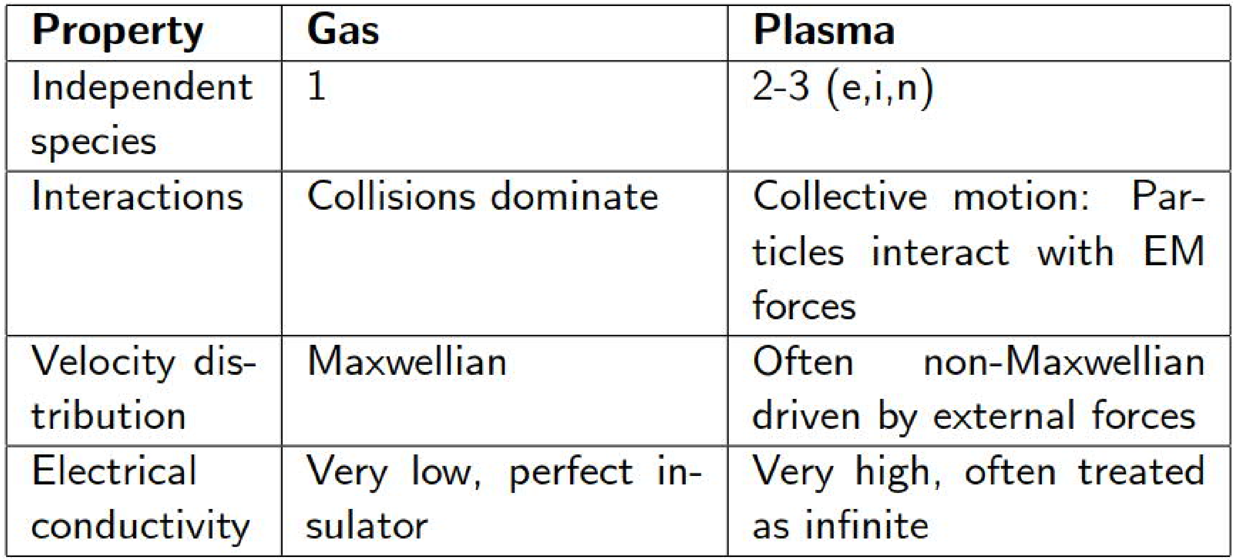
1. $\lambda_D \ll L$
2. $\Lambda \gg 1$
3. $\omega_p \tau_{coll} \gg 1$

Distinguishing features of plasma

- Electrically conductive, can generate electrical currents and magnetic fields
- Responds strongly to electromagnetic fields
- Each particle influences *simultaneously* many nearby particles leading to collective behaviour

➔ ***Plasma is very different from a regular gas***

Gas vs plasma in a nutshell



Property	Gas	Plasma
Independent species	1	2-3 (e,i,n)
Interactions	Collisions dominate	Collective motion: Particles interact with EM forces
Velocity distribution	Maxwellian	Often non-Maxwellian driven by external forces
Electrical conductivity	Very low, perfect insulator	Very high, often treated as infinite

Examples of plasmas

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99% of the universe …

- lightning
- welding torch
- plasma ball
- fusion plasmas
- magnetosphere around Earth
- solar wind
- sun & other stars
- interstellar space