Aalto University

Shapes in Action 2020
Sept 8th

Intro & Symmetries

Program schedule for Tue Sept 8th

15:15 Some general instructions

15:30 An approach to symmetries

16:00 Break

16:15 Group work on symmetries in Breakout Rooms

16:45 Wrap up

17:00 Break

17:15 Surface Design & Instructions for the Virtual Workshop on Fri Sept 11th (Laura) & Homework

Course schedule in a nutshell

- **Sept 8th – Oct 13th** (period I) **meetings via Zoom**

  Tue 3:15pm - 6pm
- **Sept 11th – Oct 16th meetings via Zoom**

  Fri 1:15pm – 4pm

Lecturing, working in groups, working individually during the meetings.

Lecturer in Mathematics: Kirsi Peltonen

- Responsible Teacher of the Course
- PhD, Docent in Math
- Geometric Analysis
- Mainstreaming Mathematics
- Visualizations

Photo: Eero Kaarlehto

Lecturer in Arts: Taneli Luotoniemi

- Doctor of Arts
- Mathematics in Arts education
- Artistic research of mathematics
- 4D, tilings, knots

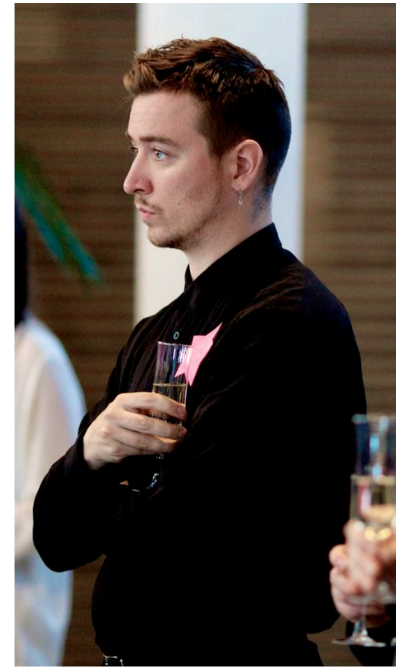


Photo: Eero Kaarlehto

Lecturer in Arts: Laura Isoniemi

- Master of Arts (Textiles)
- Art Pedagogy
- IDBM Pro studies
- Multi-disciplinary freelance designer
- Part time teacher at Aalto
- Tilings, Patterns, Foldings, Knots

Student groups

1. Hanna Arhe (ARTS)
   Khoa Lai (SCI)
   Lassi Malvikko (SCI)
   Tuan Nguyen (SCI)
   Inkeri Rouvinen (SCI)
   Rasmus Ruohola (ELEC)
   Konsta Tiilikainen (SCI)
2. GuTing Huang (SCI)
   Anna Huttunen (SCI)
   Navneet Kumar (ELEC)
   Janne Lehtimäki (ARTS)
   Bruce Nguyen (SCI)
   Niko Nästi (SCI)
   Marja Tiainen (SCI)
3. Daniel Aaltonen (SCI)
   Otso Laasonen (SCI)
   Tomi Monahan (ARTS)
   Hanne Sauer (SCI)
   Elias Seeve (ELEC)
   Thi Tran (SCI)
4. Ilkka Mutanen (ARTS)
   Dat Nguyen (SCI)
   Tuomas Pajunpää (ELEC)
   Ella Palo (SCI)
   Binh Pham (SCI)
   Aiswarya Sudhir (SCI)
   Niilo Summanen (SCI)

ARTS(8), SCI (42), ELEC(7), BIZ(4), ENG(1), CHEM(1)

5. **Georgy Ananov (SCI)**
Khue Nguyen (SCI)
Kasper Niinimäki (BIZ)
Aliisa Perikangas (ARTS)
Tommi Räsänen (CHEM)
Kristian Wasastjerna (SCI)

6. **Duong Le (SCI)**
Lina Siltala-Li (SCI)
Anders Sundstöm (SCI)
Emilia Söderström (ENG)
Severi Virolainen (ARTS)
Selim Virtanen (SCI)

7. **Kristen Barretto (ARTS)**
Vanessa Ihl (SCI)
Wanchote Jiamjitrak (SCI)
Anne Kasterpalu (SCI)
Hieu Nguyen (SCI)
Rainer Ruuskanen (BIZ)

8. **Pouya Adibnezhad (SCI)**
Lauri Lindh (SCI)
Aki Malinen (SCI)
Simo Muraja (SCI)
Viktorija Piaulokaite (ARTS)
Dias Tlekbay (ELEC)

9. **Juho Hassinen (SCI)**
Rajat Kaul (SCI)
Daniel Lammi (BIZ)
Duong Le (SCI)
Tuomas Pajunpää (ELEC)
Meike Rudlaff (SCI)

10. **Simeoni Kangasmaa (SCI)**
Michael McCrea (ELEC)
Iida Nenonen (BIZ)
Pellervo Ruponen (SCI)
Jaakko Takala (SCI)
Viktor Åberg (SCI)

Goals

Mathematics ⟷ Arts

- **Interaction in both ways!**
- **Increase *understanding***
- **Useful tools (or recipes perhaps)**
- **New point of views and connections**
- **Current research perspective**
- **Critical point of view**

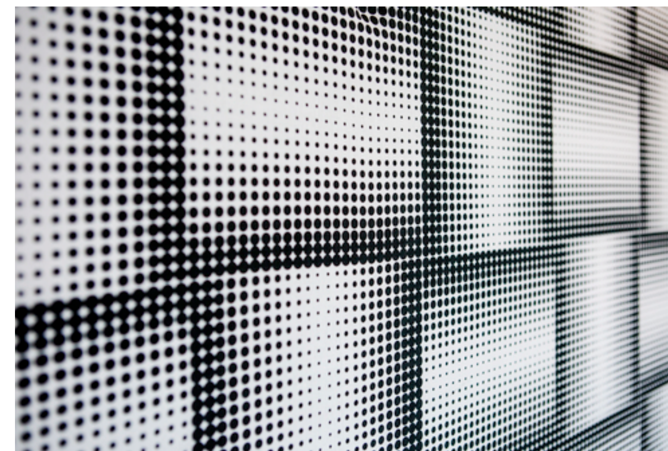


Photo: Päivi Kiuru

Math topics include

- Tilings (symmetries, classification)
- Models in geometry (spherical, Euclidean, hyperbolic)
- Surfaces and orbifolds via symmetries
- Kleinian groups
- Fractals

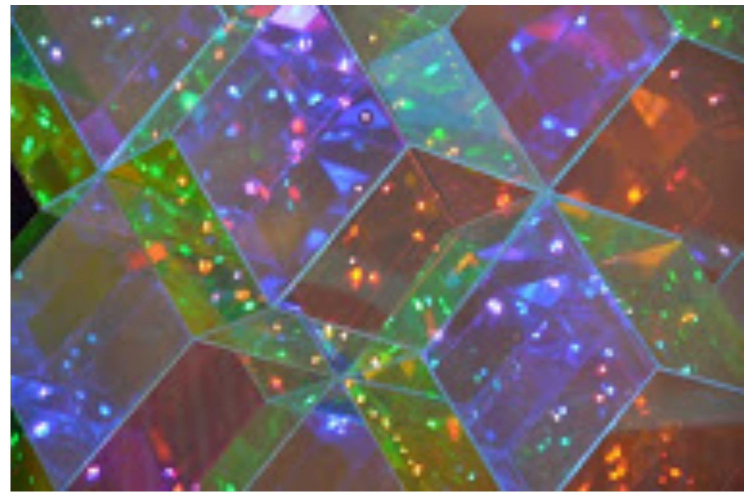


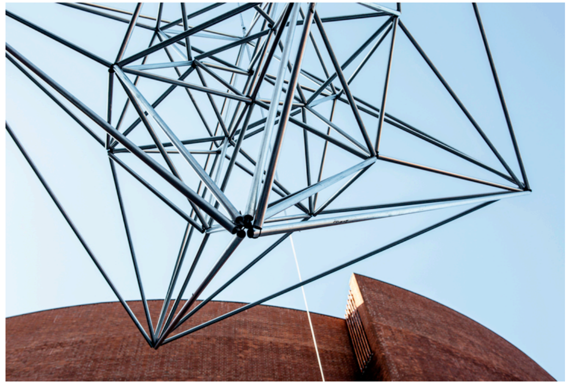
Photo: Viivi Livio

What are you supposed to do?

Active participation during contact meetings via Zoom

Portfolio through MyCourses consisting:

1. **Diary** (=reflections and summary) (1/3)
2. **Weekly exercises and group work** (1/3)
3. **Essay** (1/3)



Some groupwork tasks

Important: *always consult Kirsi before starting essay/projects*

Some literature

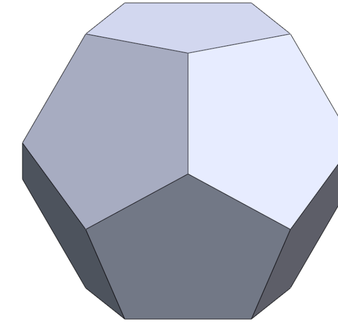
J.H. Conway, H. Burgiel, C. Goodman-Strauss: **The Symmetries of Things**

D. Mumford, C. Series, D. Wright: **Indra's Pearls**

F. A. Farris: **Creating Symmetry**

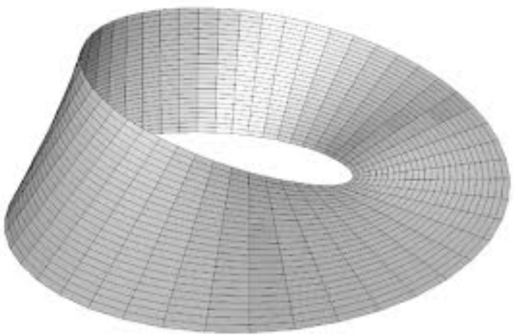
J. Weeks: **Shape of Space**

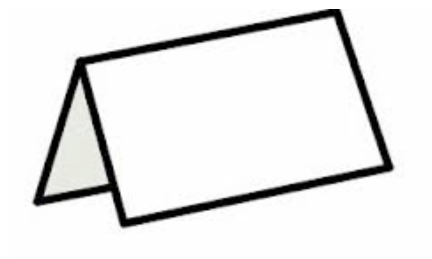
What is *geometry* ?

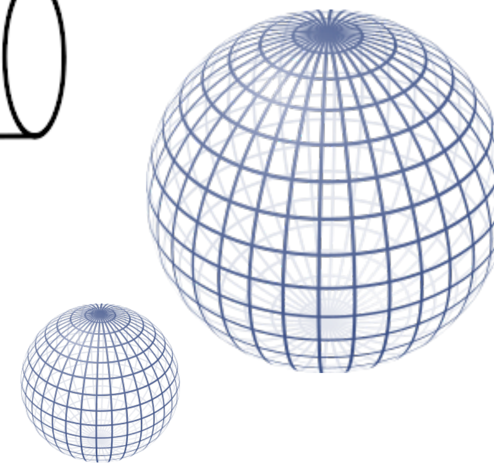


Γεωμετρία : *geo* = earth, *-metron* = measurement

- Operations that preserve distances between points (=isometry)
- Smoothness issues

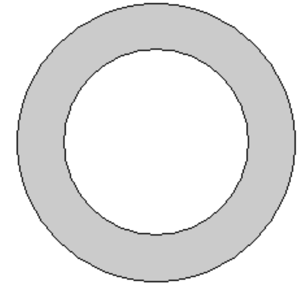


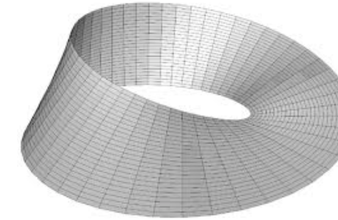




What is *topology?*

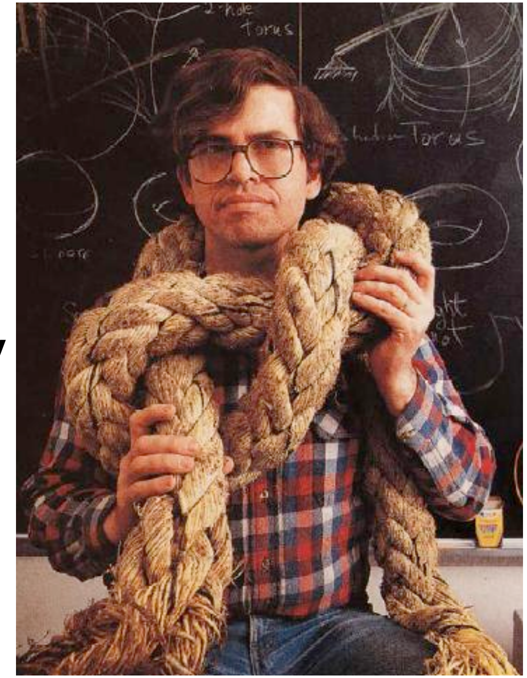
Photo: Henry Segerman





- Continuous operations (in both ways)

Symmetries of planar patterns



Goals for today:

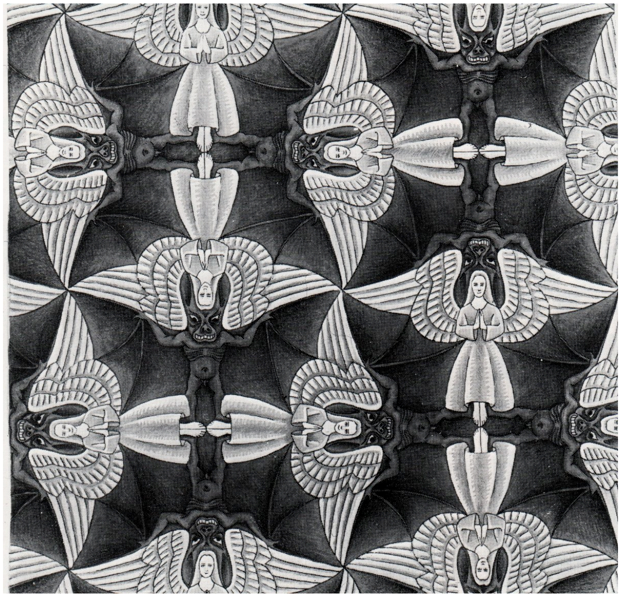
- An introduction to a systematic way to classify (=numerate) patterns
- Focus on concepts
- New insight to an old problem and its earlier solutions

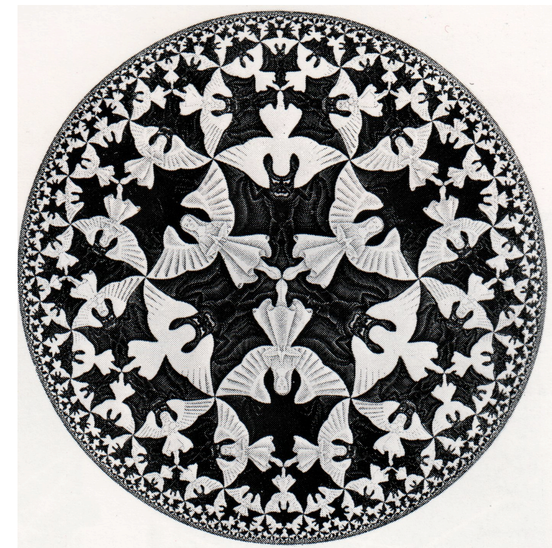
Signature/Orbifold notation due to B. Thurston and J.H. Conway (90´)

What is symmetry?

What is beauty?

Euclidean (=flat), spherical and hyperbolic models of 2D geometry



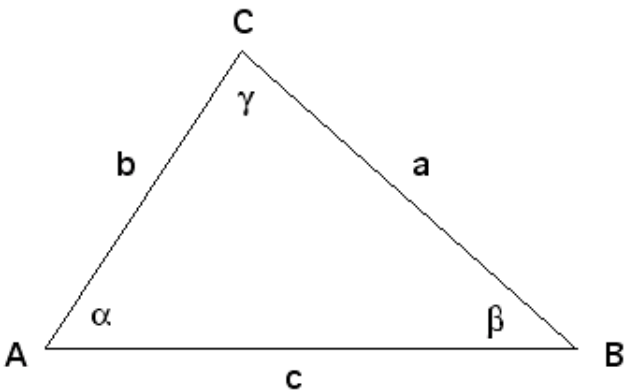


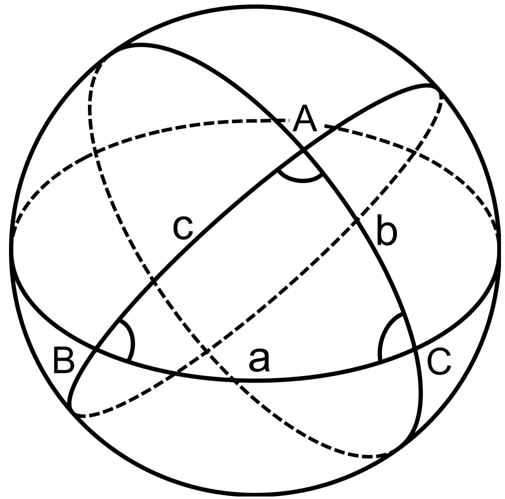
K = 0 (17 types)

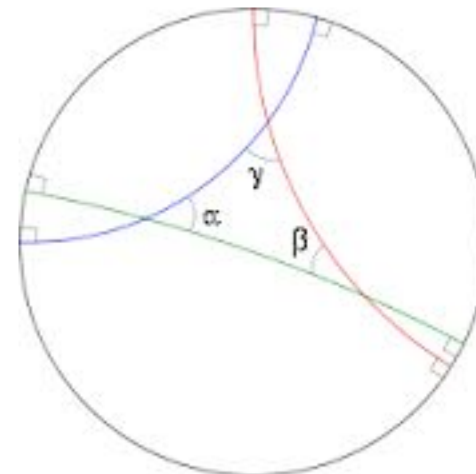
K>0 (14)

K<0 (∞)

Models in terms of angles and triangles







Sum of angles of a triangle = 180°

>180°

<180°

Some history of Euclidean tilings (=Crystallographic groups)

2D Wallpaper groups (17)

- 1890 E.S Federov
- 1924 G. Pólya
- 1992 B. Thurston, J.H. Conway

3D Space groups (219/230)

- 1890 W. Barlow, E.S Federov, A. Schönflies
- 1911 L. Bieberbach
- 1948 H. Zassenhaus (algorithmic approach)
- 1983 T.Hahn (ed.) International tables of crystallography
- 2001 B. Thurston, J.H. Conway

Higher dimensions

D. Hilbert's 18th Problem (1900 ICM Paris) *"Is there in n-dimensional Euclidean space [. . .] only a finite number of essentially different kinds of groups of motions with a [compact] fundamental region"*

- 1910 L. Bieberbach : Only finitely many in all dimensions
- 1948 H. Zassenhaus : Algorithmic approach to all dimensions
- 1980 R. Schwarzenberger

4D (4783)

- 1978 H. Brown, R. Bülow, J. Neubüser (via computers)

5D (222 018)

- 2000 W. Plesken, T. Schultz

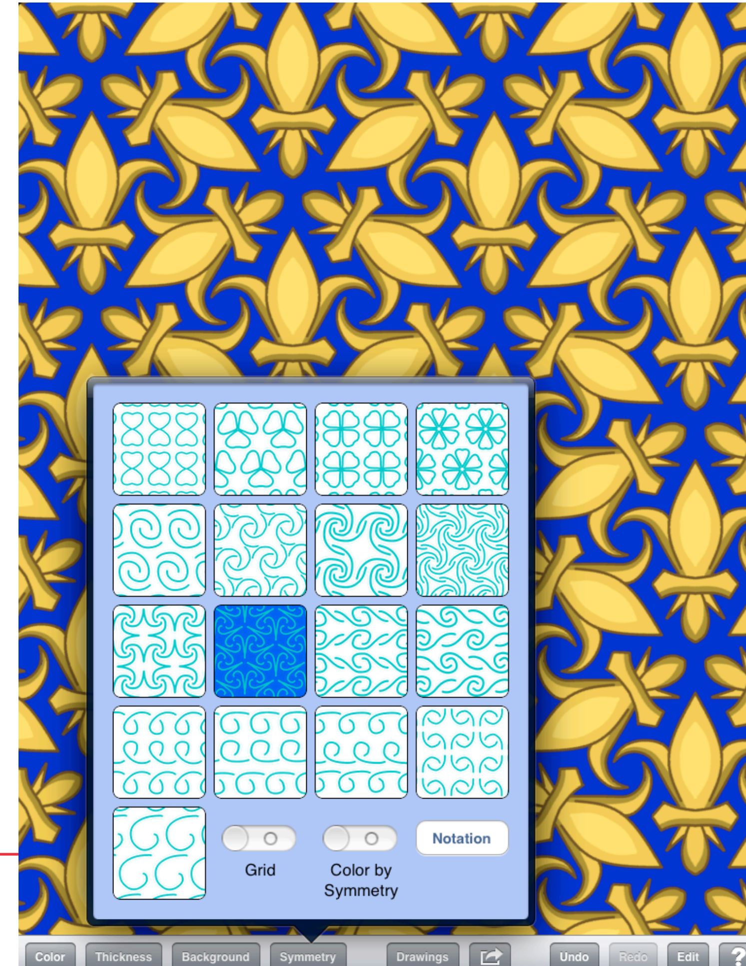
6D (28 927 922)

- 2000 W. Plesken, T. Schultz

Visualizations

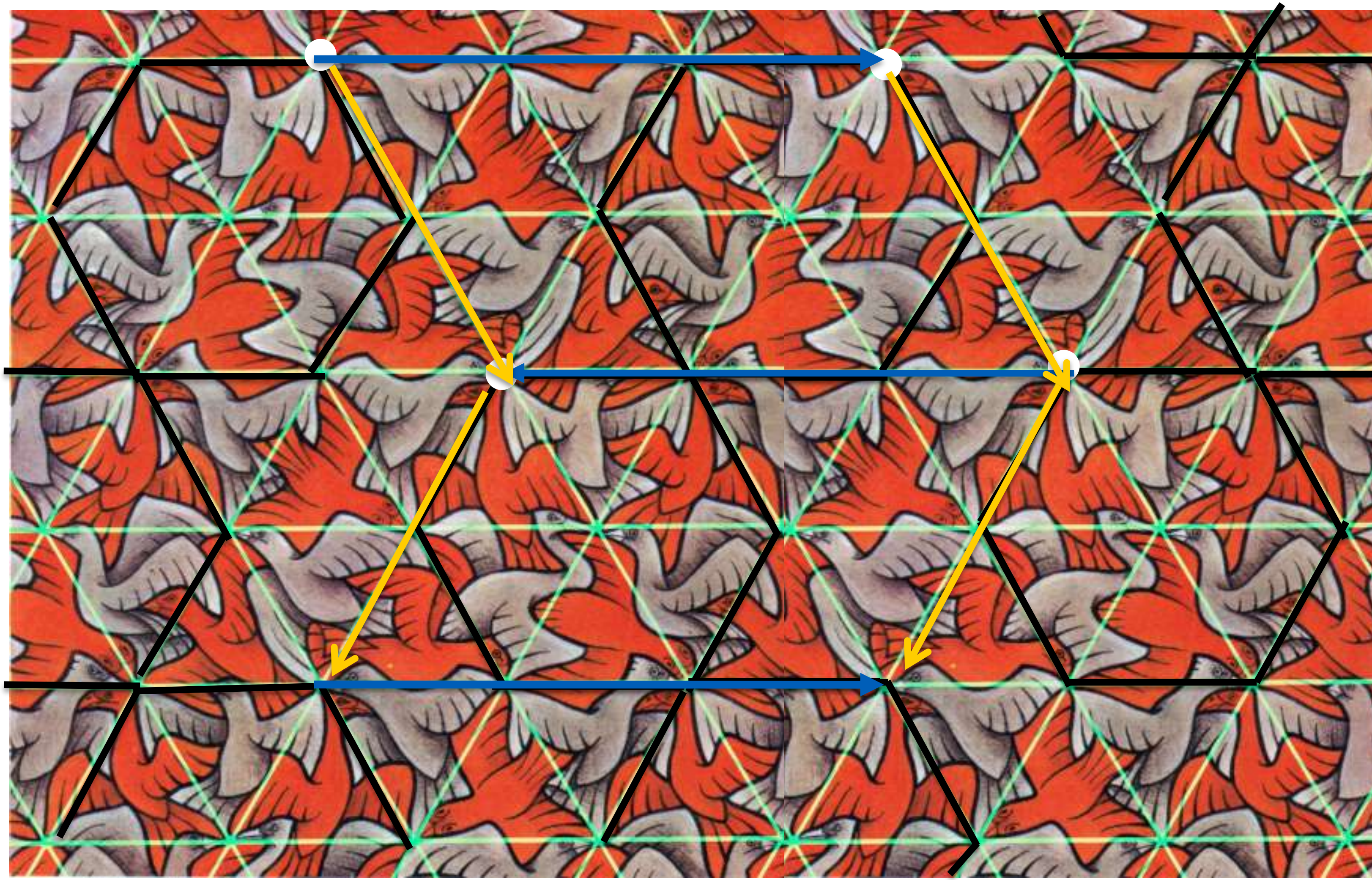
2D Jeff Weeks: KaleidoPaint application for iOS & Android
http://geometrygames.org/

3D http://spacegroup.info/



Goal: fix *meaning* and *notation* for the symmetry

Find a pattern that is repeated (up to reflection)



Would like to know

- What are the essentially different ways to subdivide plane into patterns that are repeated ?
- What is a ‘pattern’ and a ‘repetition’?
- How to distinguish different subdivisions?
- How to get them all?
- How to describe the topological type of ‘patterns’?
- Does one topological type correspond only one subdivision?

First we need some definitions…..

Symmetry (group) : =actions that preserve the given pattern in the given geometry.

Consider today only patterns in the plane.

Given geometry = standard Euclidean plane.

Actions := rigid motions (=*isometries)* of the plane:

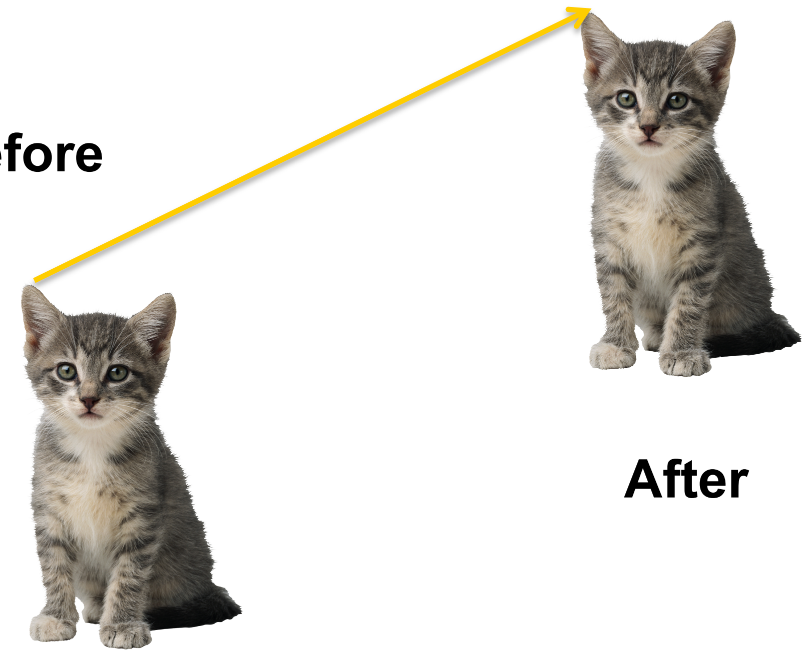
- Translations
- Rotations with respect to a point
- Reflections with respect to a line

And all possible combinations of those.

(Proof: second year linear algebra)

A (pure) translation

Before



After

Note: only ‘ before’ and ‘after’ situations are taken into account. Not the actual way of prforming the motion.

A (pure) rotation of 90 degrees (to the positive direction) wrt the top of the right ear

After

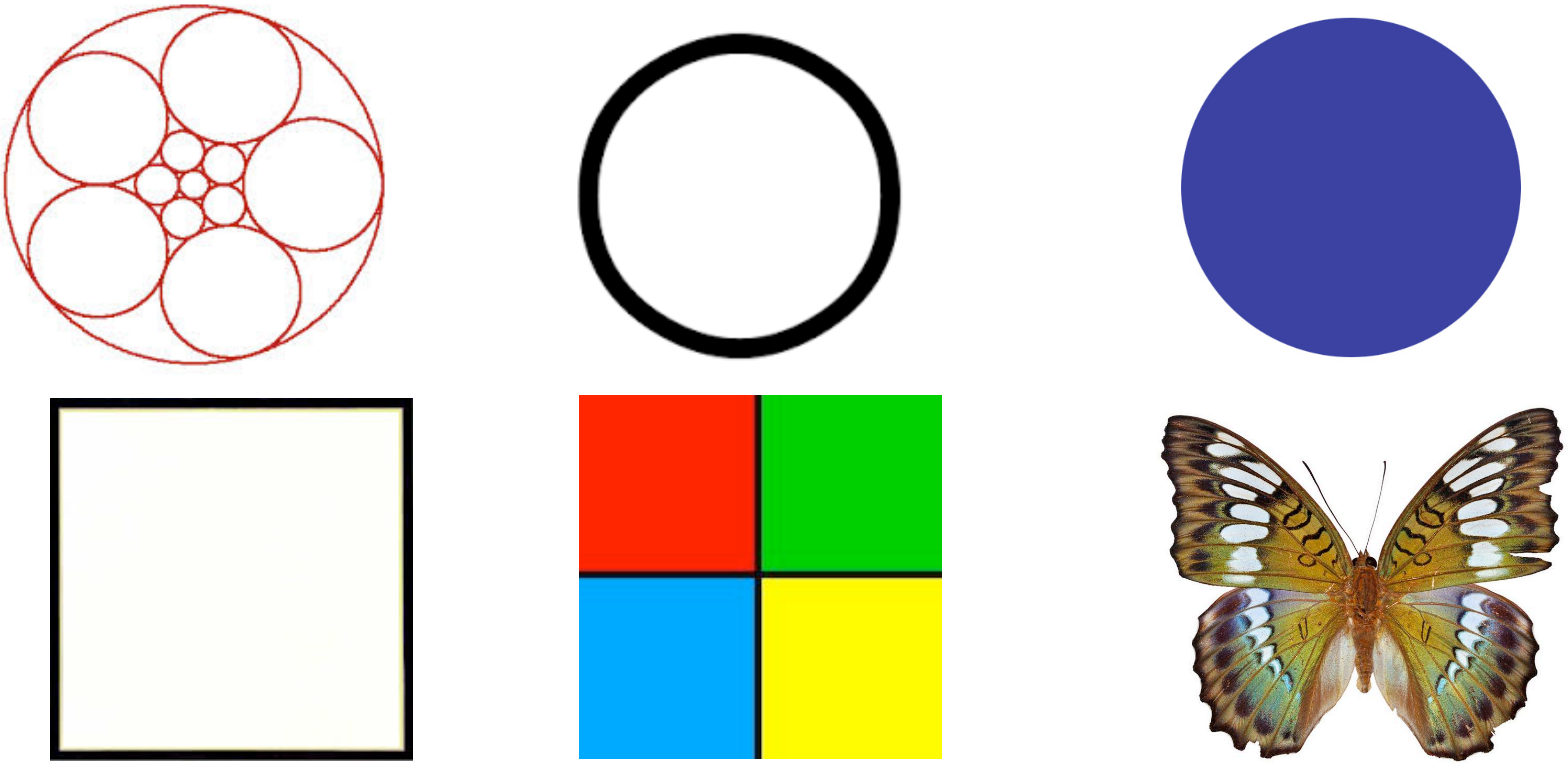
Before

A (pure) reflection wrt a line

Before (or after)

After (or before)

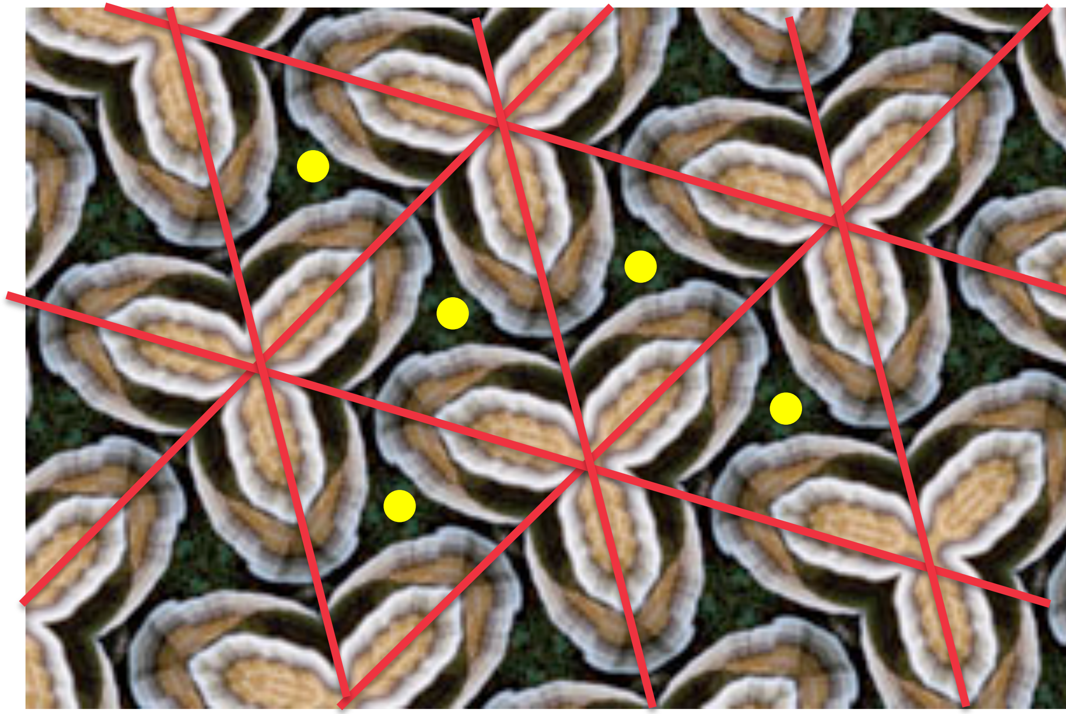
Which pattern is most (least) symmetric?



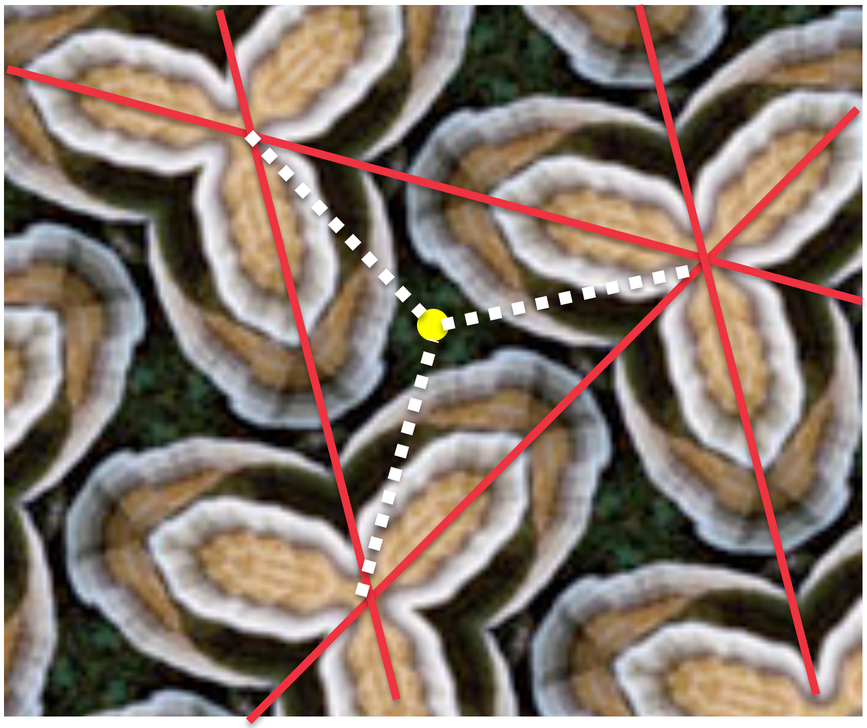
What about infinite (repeating) patterns?

Find reflection lines first (if any)

Then rotation points (if any)



Further subdivisions into identical pieces



3*3

Group work in Zoom breakout rooms

1. **Introduce yourself to your group mates**
2. **What symmetries can you find in the provided pictures ?**

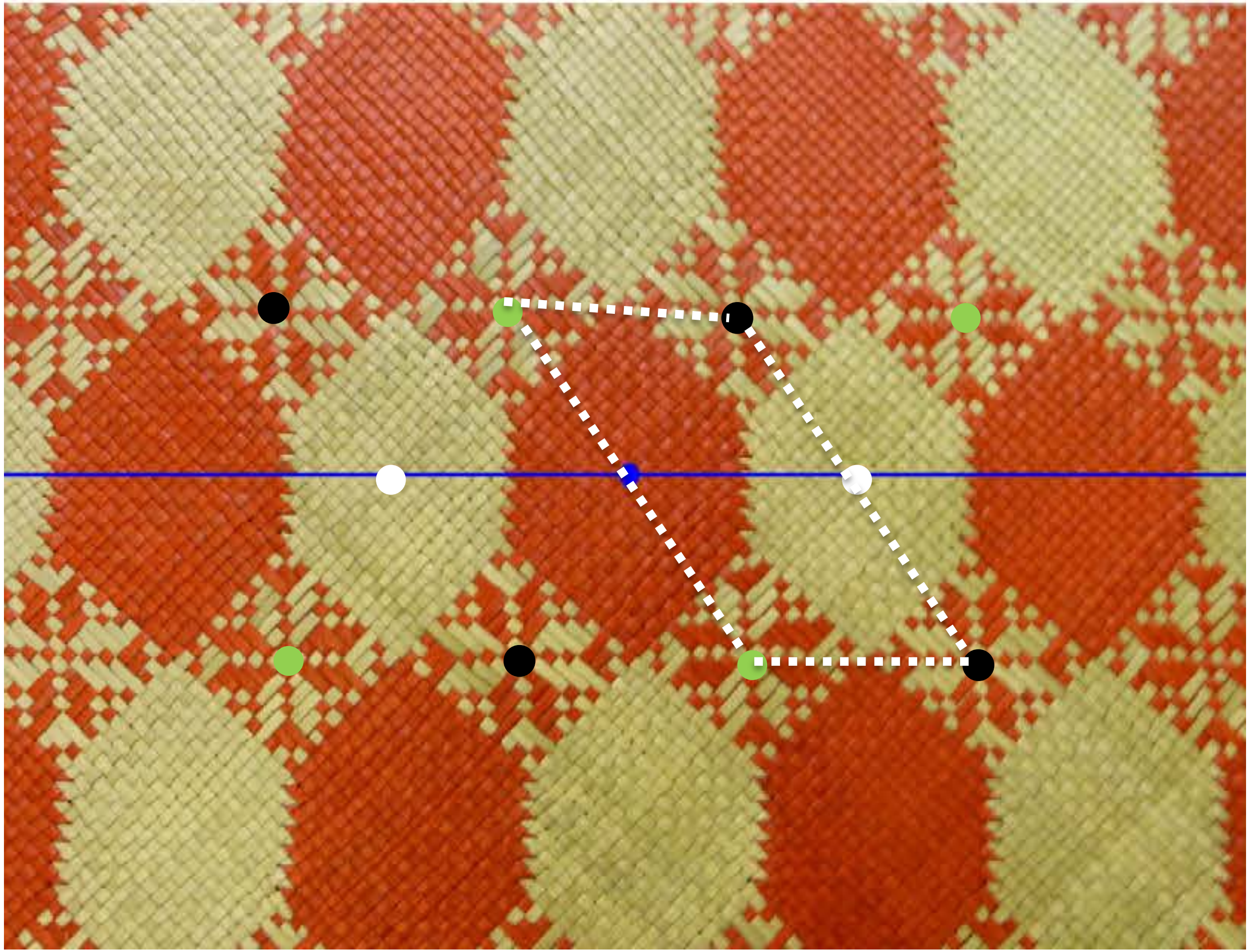
'Materials' section in MyCourses:
https://mycourses.aalto.fi/course/view.php?id=29623&section=7

1. **Are there reflection lines or rotation points ?**
2. **Can you find the smallest possible piece that gives you the whole tessellation when all symmetries are taken into account ?**
3. **Stamping Videos:**

https://www.youtube.com/watch?v=Gpb6nxvabkM

https://www.youtube.com/watch?v=E69KrNrj4KQ (2222)

https://www.youtube.com/watch?v=GlzwVB3_6Y8 (4*2)

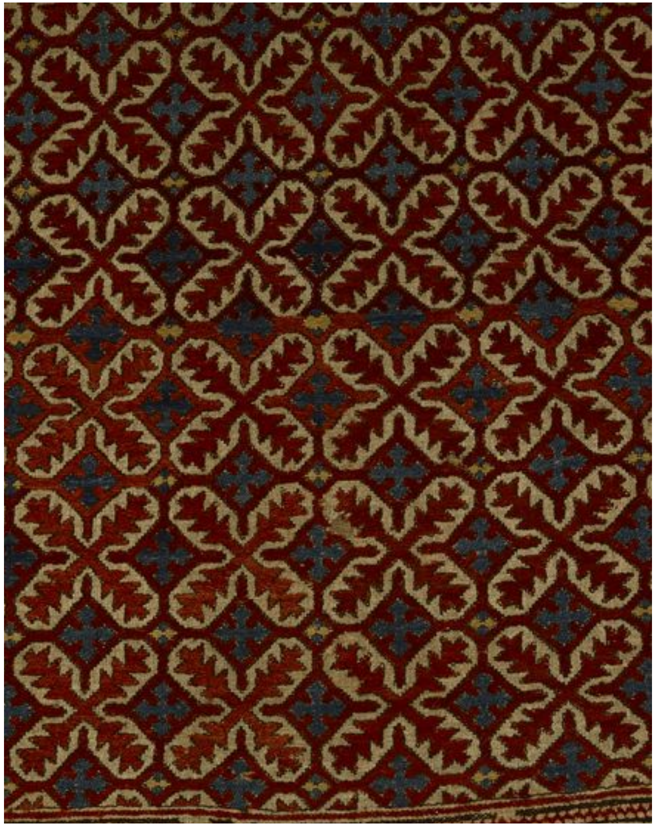


No reflection lines !
Four different rotation points of order 2

2222

Textiles & Weawings from Phillippines Math Intelligencer, vol 36, 2014

Some patterns from Finnish textile tradition



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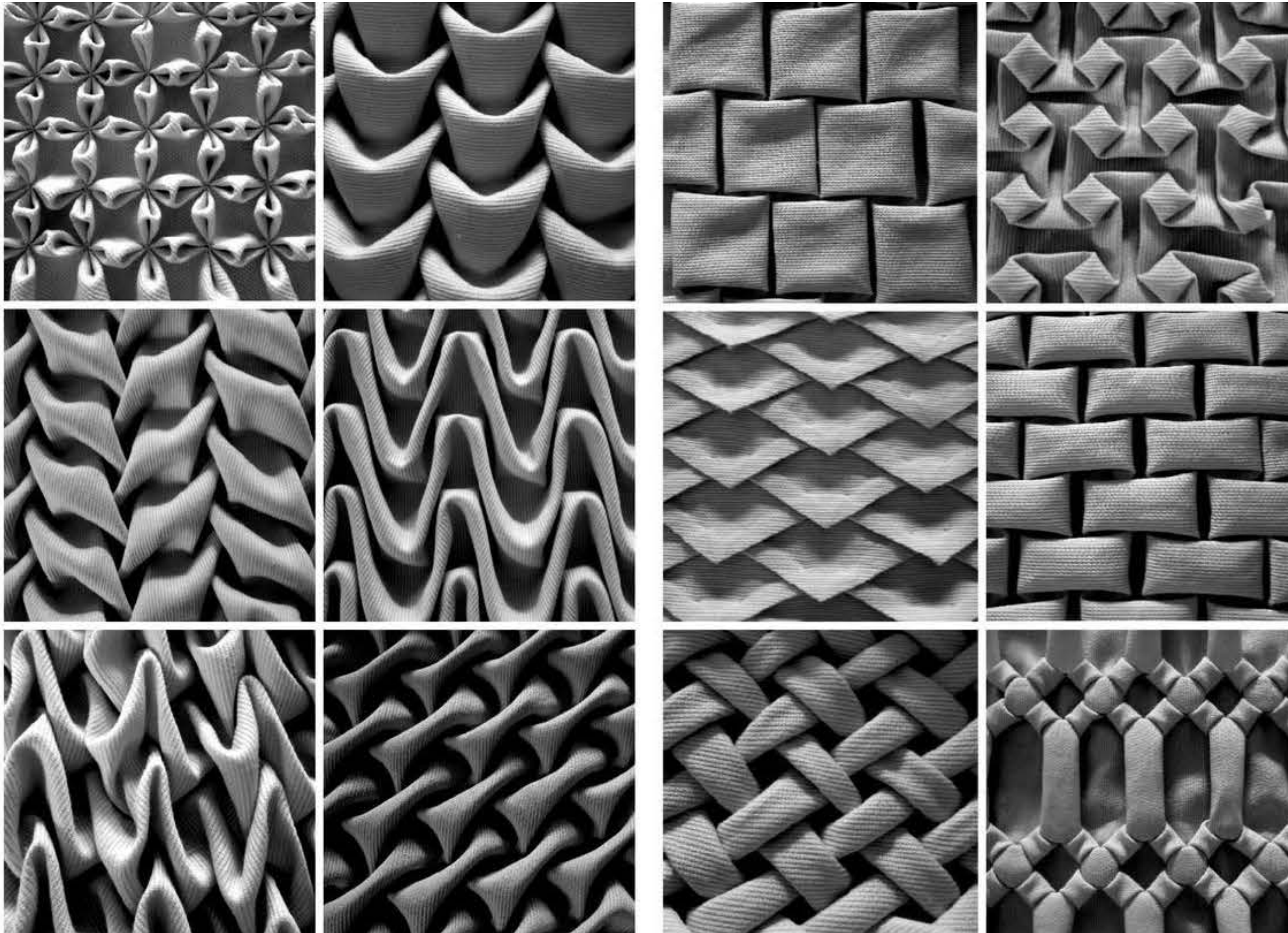
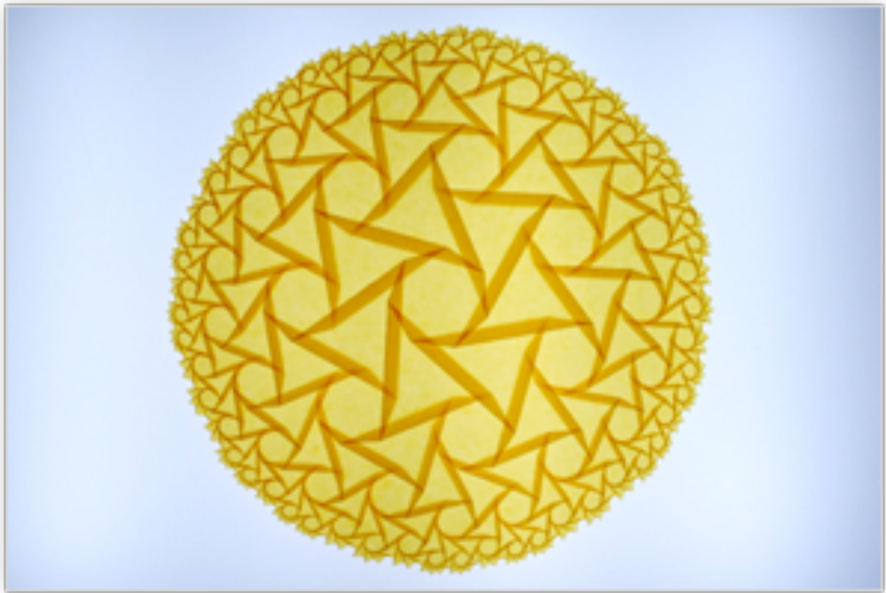
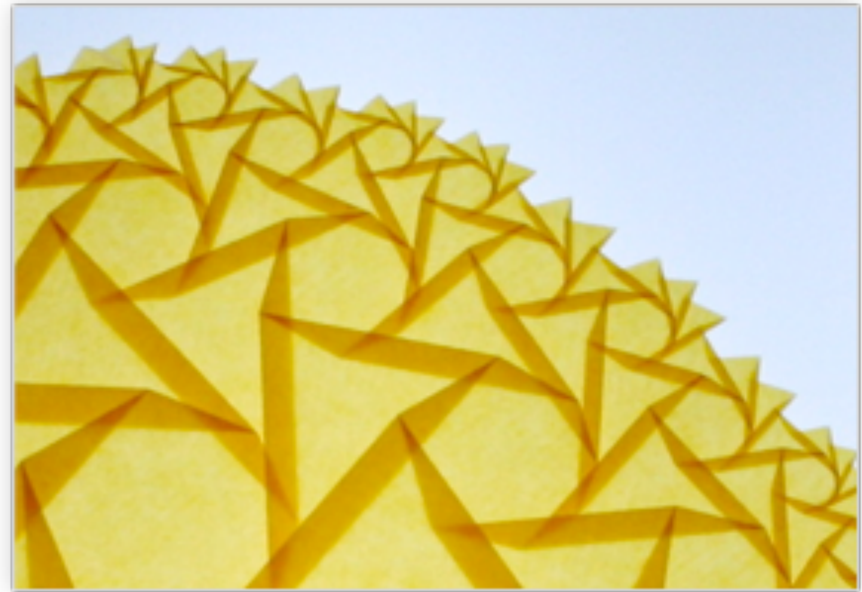
Finnish National museum collection (Finna)







Homework

1. **Reflections** through MyCourses by Sept 11th
2. **By Sept 15th through MyCourses**

Analyse the provided images. Are there

- reflection lines ? Do they intersect?
- rotation points ? How many different ?
- mirror images without reflection lines ?

3. *Observe* your surroundings. Can you find repeating patterns from your everyday life? Fabrics, brick walls, cloths etc. Take 3-4 photos and include them into MyCourses. You can also use other pictures you find interesting.