ELEC-E8116 Model-based control systems (5 cr.)

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Lectures on **Wednesdays at 14.15 – 16.00 Zoom**

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Exercises on **Thursdays at 14.15 – 16.00 Zoom**

Books (related to course topics) :

1. Glad, Ljung: Control Theory, (multivariable and nonlinear methods), Taylor and Francis, 2000. Textbook of the course.
2. Skogestad, Postlethwaite: Multivariable Feedback Control, (Analysis and Design), Wiley, 2005.
3. Kirk: Optimal Control Theory, An Introduction, Dover, 2004.

Optional: Material on model predictive control (MPC)

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Passing the course

- **Two intermediate exams or one full exam are needed to pass the course.** The intermediate exams have 3 problems each, max 15+15 = 30 points. The full exam has five problems, max. 30 points altogether.

- First intermediate exam on Thursday 22.10.2020, **14:00**-16:00. The second intermediate exam on Thursday 10.12.2020, **14:00**-16:00. (How the examinations are arranged will be decided later). Intermediate exams cannot be repeated. The first full exam is on Tuesday, 15.12.2020, **13:00** – 16:00. You can participate to all above exams without separate registration (the registration to the course is sufficient). To all later exams you have to register.

- **Six homework problems are given during the course. You must do and leave for evaluation at least three homework problems. However,** to get more homework points it is reasonable to do as many homework exercises as possible, preferably them all. The homework results are scaled to give a maximum of 6 points to be added to the exam result. The final grade is determined based on the sum of two intermediate exams (or full exam) and the homework points. The grade evaluation is done based on 0-36 points. 18 points is always enough to pass.

- It is believed that all lectures and exercise hours are arranged as distant learning (Zoom). If that changes, announcement is given in MyCourses. In other aspects also it is good to follow course information in MyCourses regularly.

- Note that there are regular lectures and exercise hours in the course. Homework problems are published separately in MyCourses (Assignments). They must be done and returned in MyCourses according to a given time-table. Help to the homework problems can be received by contacting the assistant or lecturer.

- The homework points and intermediate exam results remain valid, until the course lectures start again (Fall 2021).

- Lecture slides and problems with solutions appear on the course pages in the MyCourses portal.

- Use the web-oodi to register yourself to the course. It is not necessary to register to the exams 22.10, 10.12 and 15.12. You must however register to the next full exams after that. (E.g. the full exam 11.1.2021)

Prerequisites to the course

- Firm knowledge of basic control theory of continuous time systems (e.g. ELEC-C1230 Control Engineering).
- A knowledge on digital control is desirable, but not absolutely necessary (e.g. ELEC-E8101 Digital and Optimal control).
- If you have not studied optimal control at all (LQ theory), you have to study a bit harder here.

Main idea of the course

- Classical control theory: SISO-systems, linear or linearized system models
- Extension to multivariable (MIMO) systems
- Performance and limitations of control
- Uncertainty and robustness,
- IMC-control,
- LQ and LQG control
- Optimal control
- Basics of Robust control
- Optional: Introduction to Model Predictive Control

Basics....

- Basics in continuous control theory, e.g. Dorf, Bishop: ”Modern Control Systems”.
- Basics in discrete time control theory,

  e.g. Åström, Wittenmark: ”Computer-Controlled Systems, theory and design”, Franklin, Powell, Workman: ”Digital Control of Dynamic Systems”.
- one more book about control:

  (Glad, Ljung: Control Theory)
- Wang: Model Predictive Control System Design and Implementation Using MATLAB

Example 1

Multivariable system:

$$y_1 = \frac{2}{s+1} u_1 + \frac{3}{s+2} u_2$$

$$y_2 = \frac{1}{s+1} u_1 + \frac{1}{s+1} u_2$$

Two inputs, two outputs. General multivariable system.
MIMO = multiple inputs, multiple outputs
Interconnections between the two loops
”Pairing problem”

Let $u_1 = \frac{K_1(s+1)}{s}\left(r_1 - y_1\right)$

PI-controller (control 1 – output 1)

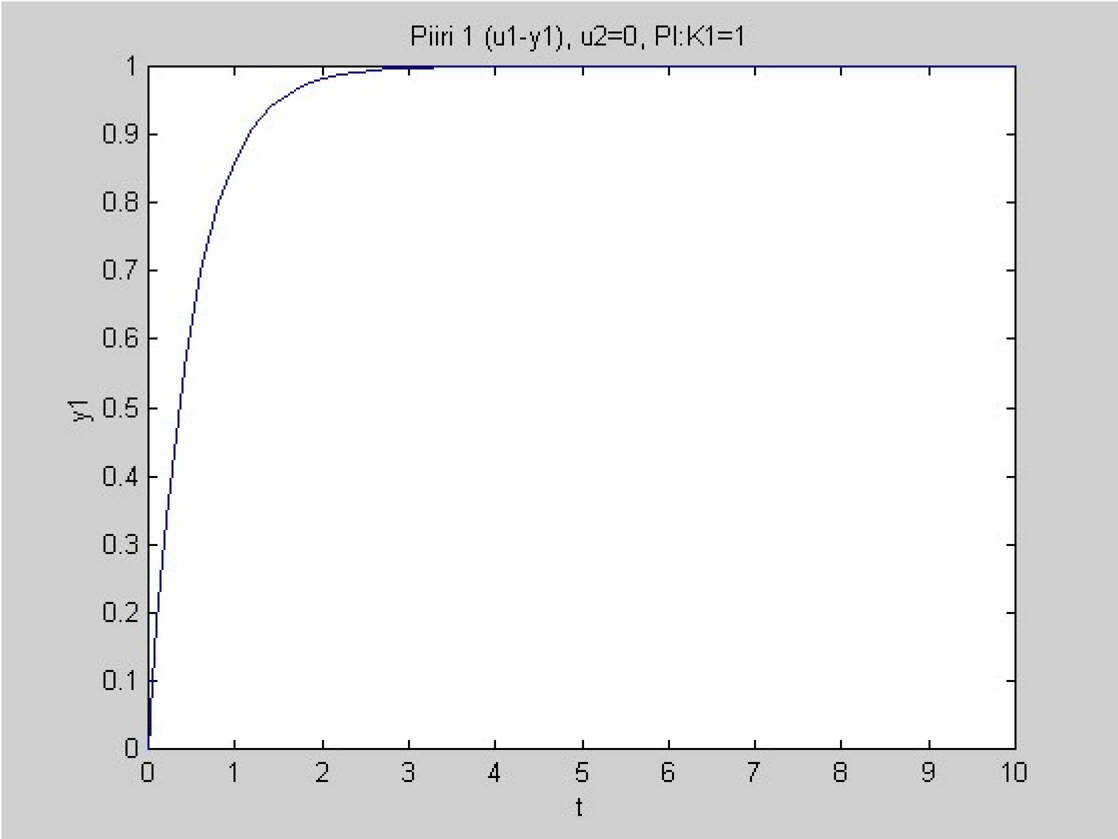
Transfer function, when u2=0

$$\frac{2K_1}{s+2K_1}$$

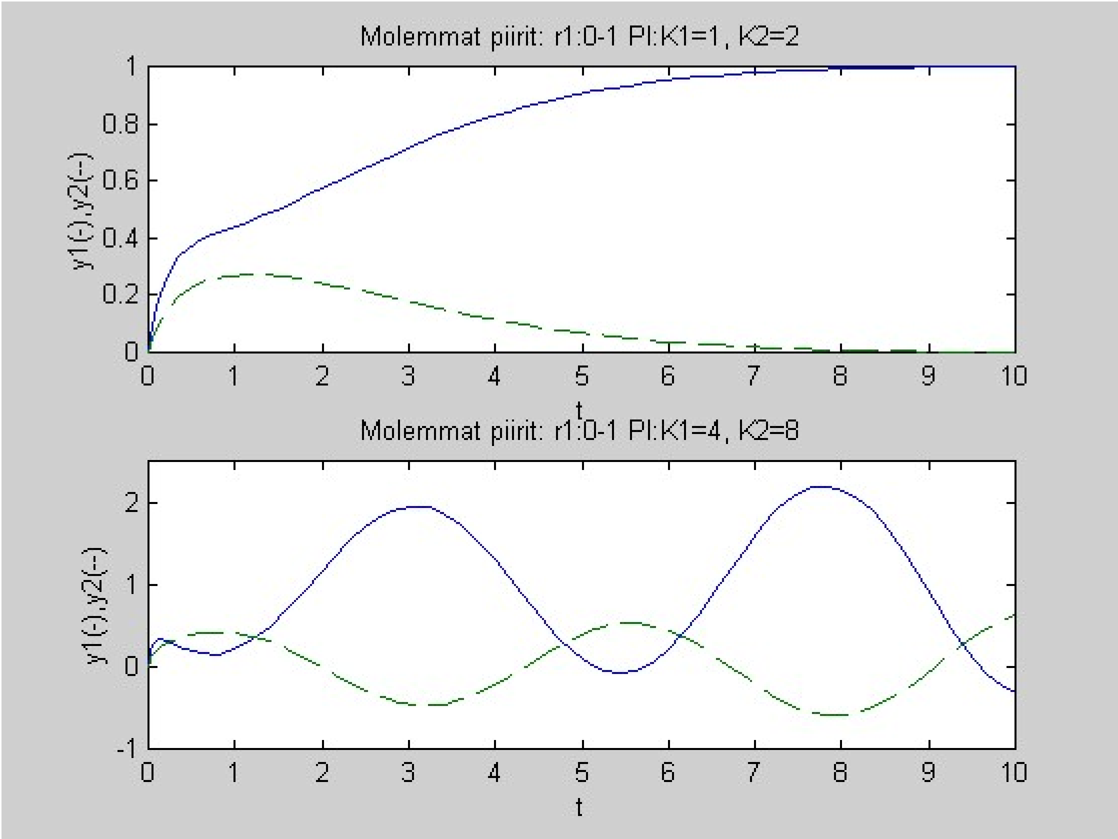
Correspondingly (control 2 – output2) $u_2 = \frac{K_2(s+1)}{s}\left(r_2 - y2\right)$

Transfer function, when u1=0

$$\frac{K_2}{s+K_2}$$



The result is as good as expected.



The good response is lost, when both controllers are operating, Reason? Interconnections? Analysis?

But we can always change the pairing
(control 1 – output 2), (control 2 – output1)

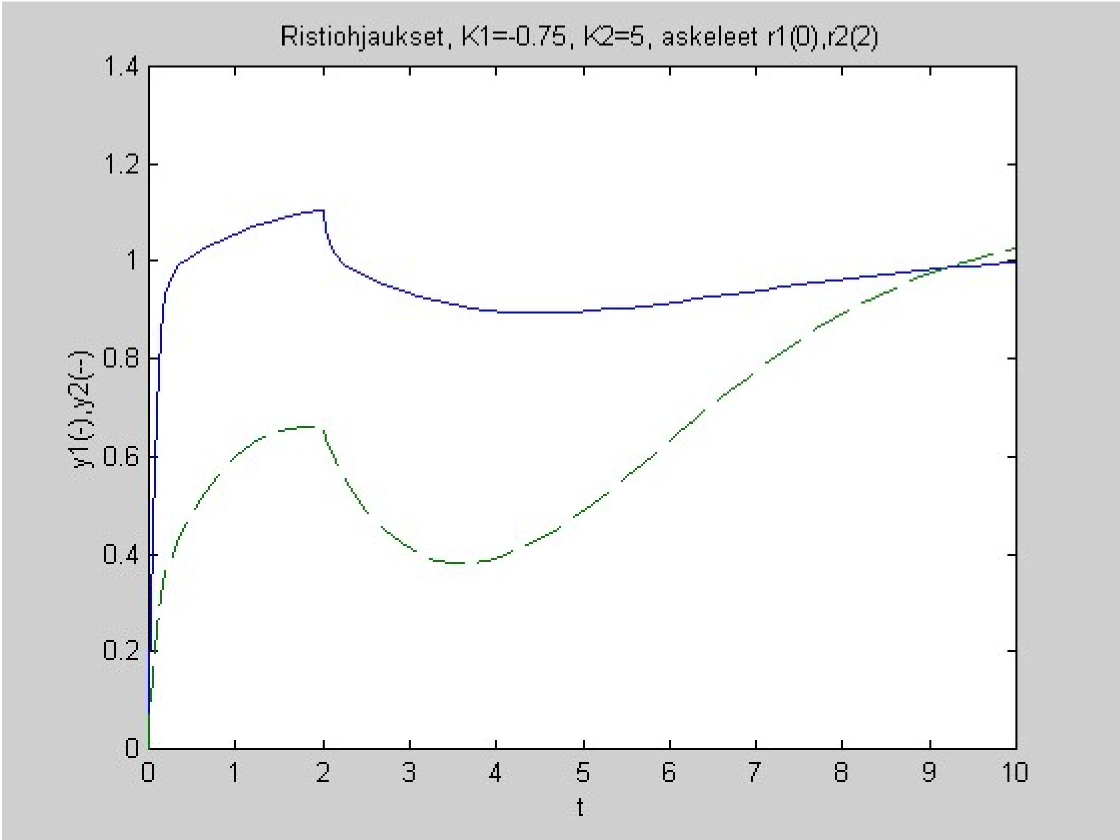
$$u_1 = K_1 \frac{s+1}{s}(r_2 - y_2) \qquad u_2 = K_2 \frac{s+2}{s}(r_1 - y_1)$$

This leads to the transfer function (ref. 1 – output1):

$$\frac{K_2(3s^2 + (3+K_1)s - K_1)}{s^3 + (1 + K_1 + 3K_2)s^2 + (K_1 + 3K_2 + K_1K_2)s - K_1K_2}$$

In order to be stable, the coefficients in the denominator must be positive; K1 or K2 must be negative.

Difficult to tune.



Analysis will later show that the difficulties are because the system has a RHP-zero (zero in the right half plane).

Example 2

Control signal limitations

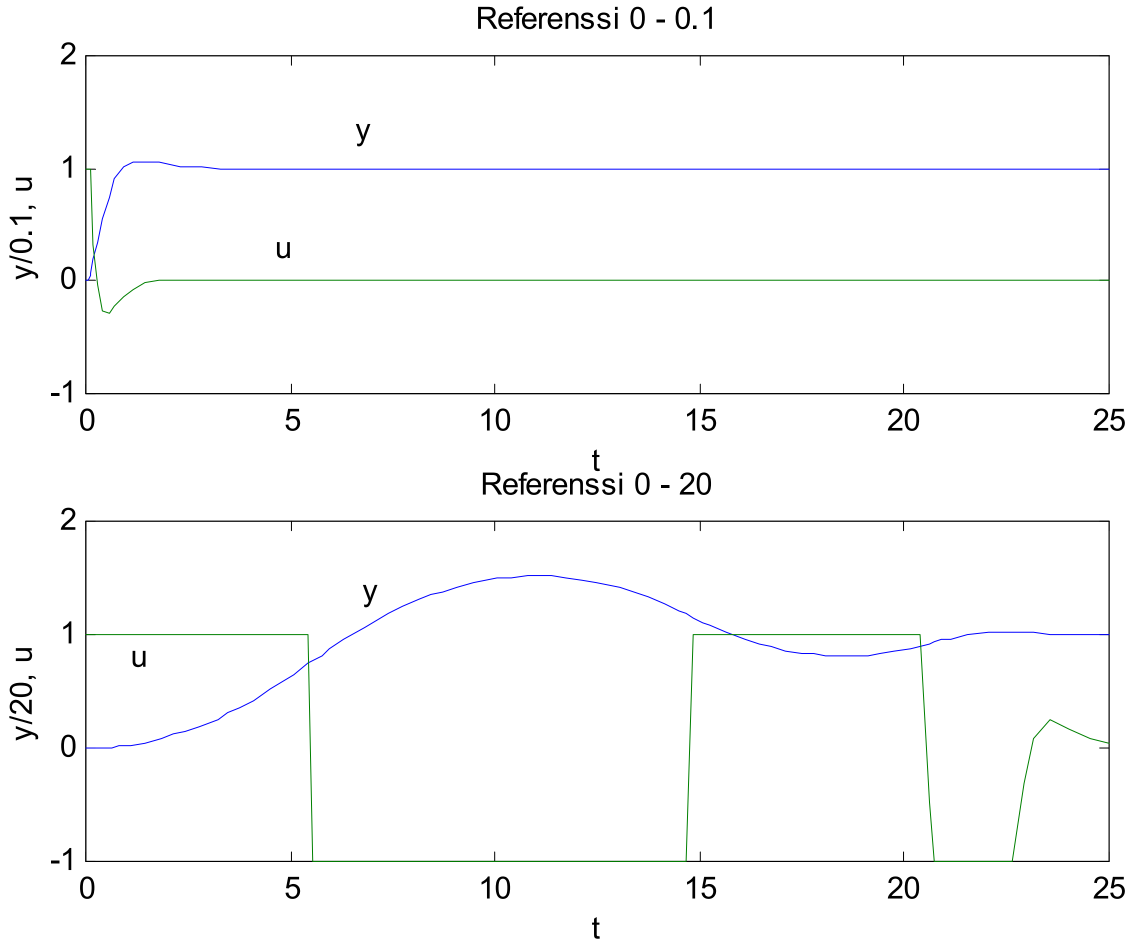
Double integrator $\ddot{y}(t) = u(t)$

in which the control is limited as $|u(t)| \leq 1$

Control law $u = F(r - y)$ $F(s) = \frac{30(s+1)}{s+10}$

Open loop transfer function $\frac{30(s+1)}{s^2(s+10)}$

Cross-over frequency 3 rad/s, phase margin 55 degrees.



The output deteriorates clearly because of the control signal limitation.

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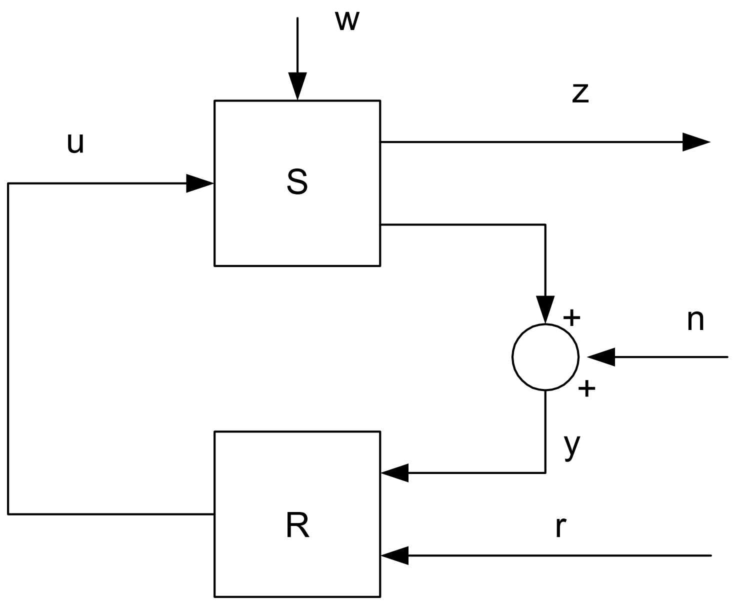
The control problem



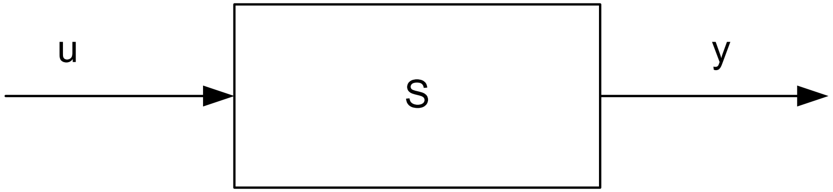
”Given a system S and measurements y. Determine a control u such that the controlled variable z follows the reference (set point) r as close as possible irrespective of process disturbances w and measurement disturbances n.”

In order to design the controller (R), the system (S) and disturbances w must be described (modelling, identification).

On the other hand, there must be different design methods and approaches for different model classes (continuous/discrete, SISO/MIMO, linear/nonlinear).

Modeling, classification of models, and analysis and synthesis methods of a wide application area are needed.

System

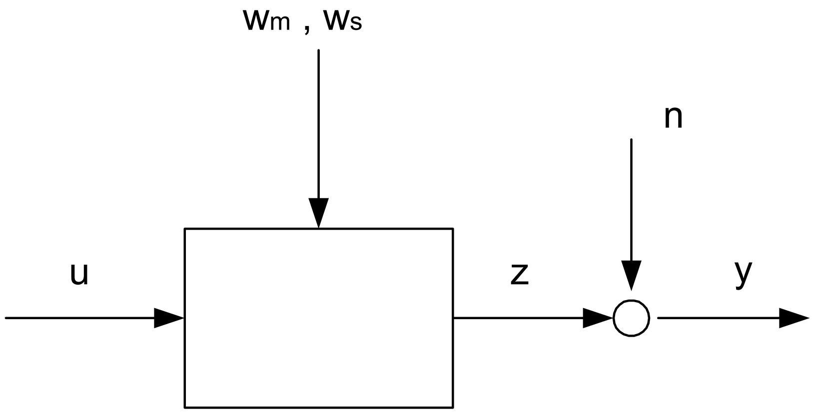


$$u(t), \quad -\infty < t < \infty \qquad\qquad y(t), \quad -\infty < t < \infty$$

$$u \xrightarrow{S} y$$

$$y = S(u)$$

causal/non-causal, static/dynamic, continuous/discrete, SISO/MIMO, time-invariant/time-varying, linear/nonlinear.



$$y = z + n$$

$$z = S(u, w_m, w_s)$$

”The regulator must be such that it compensates measurable disturbances, and the effect of non-measurable disturbances is as small as possible.”

Signal and system ”sizes”

If the signal z is a n-dimensional vector, its ”size” at time t can be defined as the Euclidean vector norm

$$|z(t)|^2 = \sum_{j=1}^{n} z_j^2(t) = z^T(t)z(t)$$

The size of the whole signal can be measured e.g. by

$$\|z\|_\infty = \sup_t |z(t)|$$ H_∞ -norm, ”infinity”-norm

$$\|z\|_2^2 = \int_{-\infty}^{\infty} |z(t)|^2 \, dt$$ H_2 -norm, 2-norm

$$\|z\|_2^2 = \sum_{t=-\infty}^{\infty} |z(t)|^2$$

discrete-time 2-norm

Gain:

The matrix equation

$$y = Ax$$

can be understood as a system, in which the matrix *A* maps the input *x* to the output *y.*

The operator (or matrix) *A* norm is defined as the largest possible gain, as *x* changes

$$|A| = \sup_{x \neq 0} \frac{|\mathrm{y}|}{|\mathrm{x}|} = \sup_{x \neq 0} \frac{|\mathrm{Ax}|}{|\mathrm{x}|}$$

More generally: consider the system *S*

$$y = S(u)$$

The **gain** of the system is defined as

$$\|S\| = \sup_u \frac{\|y\|_2}{\|u\|_2} = \sup_u \frac{\|S(u)\|_2}{\|u\|_2}$$

in which *u* covers all possible signals with a finite 2-norm. The gain can be infinite, too.

For a cascade connection of systems

$$\|S_1(S_2)\| \le \|S_1\| \cdot \|S_2\|$$

Ex. $y(t) = f(u(t))$ static nonlinear system

in which $|f(x)| \leq K \cdot |x|$

and where the equality holds for some $x = x^*$

We obtain

$$\|y\|_2^2 = \int_{-\infty}^{\infty} \left(f(u(t))\right)^2 dt \leq \int_{-\infty}^{\infty} K^2 \left(u(t)\right)^2 dt = K^2 \|u\|_2^2$$

The gain is then smaller or equal to *K*. But choose

$u(t) \equiv x^*$ so that the gain is *K*.

Ex. Integrator

Consider the system $y(t) = \int_0^t u(\tau) d\tau$

and choose the input

$$u(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

which has the 2-norm equal to 1. The output is identically 1, when $t \geq 1$

The 2-norm is infinite and the gain of the integrator is thus infinite.

Ex. Linear SISO system

$$Y(i\omega) = G(i\omega)U(i\omega)$$

By Parseval's equation it follows

$$\|y\|_2^2 = \int_{-\infty}^{\infty} y^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega|^2 d\omega$$

Assume that $|G(i\omega| \le K$

and that the equality holds for some ω^*

Then it follows

$$\|y\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \le K^2 \|u\|_2^2$$

The gain is smaller or equal to *K* and in fact this value can be approached arbitrarily close. The gain is then

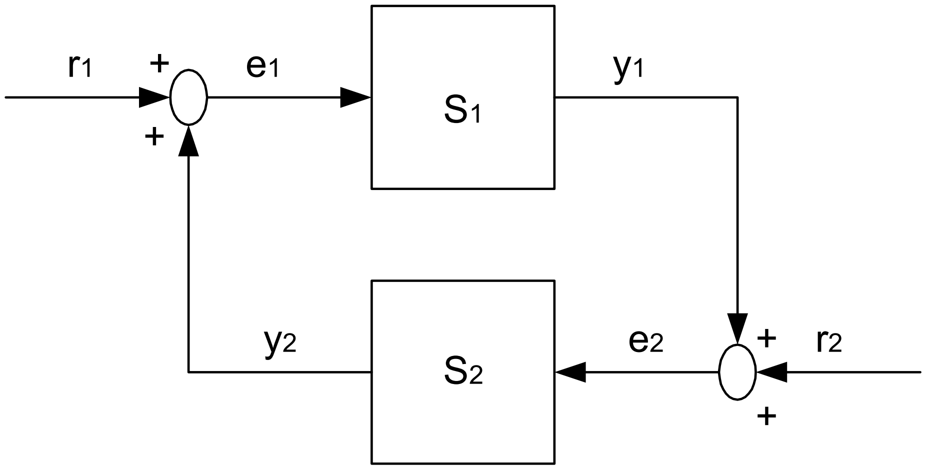
$$\|G\| = \sup_{\omega} |G(i\omega)|$$

which is denoted as $\|G\|_\infty$ (H-infinity norm)

The system norm is the same as the matrix norm of $G(i\omega)$

Stability and the ”Small Gain Theorem”

The system is called **input-output stable** (BIBO-stable), if it has a finite gain.



Inputs: r1, r2 Outputs: e1, e2, y1, y2

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”Small Gain Theorem”: The closed loop system is BIBO
stable, if the product of the system gains is smaller than1.

$$\|S_2\| \cdot \|S_1\| < 1$$

If S1 and S2 are linear, a weaker condition follows

$$\|S_1 S_2\| < 1$$

”Proof”: Writing the system equations

$$e_1 = r_1 + S_2(r_2 + y_1)$$
$$y_1 = S_1(e_1)$$

gives by the triangle inequality

$$\|e_1\| \leq \|r_1\| + \|S_2\|\left(\|r_2\| + \|S_1\|\|e_1\|\right)$$

and

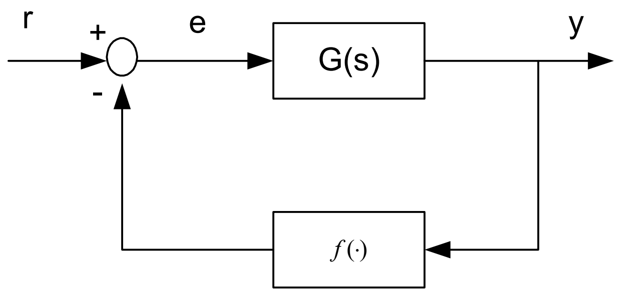
$$\|e_1\| \leq \frac{\|r_1\| + \|S_2\|\|r_2\|}{1 - \|S_2\|\|S_1\|}$$

The gain from r1 and r2 to the output e1 is finite.
Other cases correspondingly.

Obs. It does not matter, whether the feedbacks are positive or negative (the norms of S and –S are the same).

The result is ”conservative” ?

Ex. Nonlinear static feedback

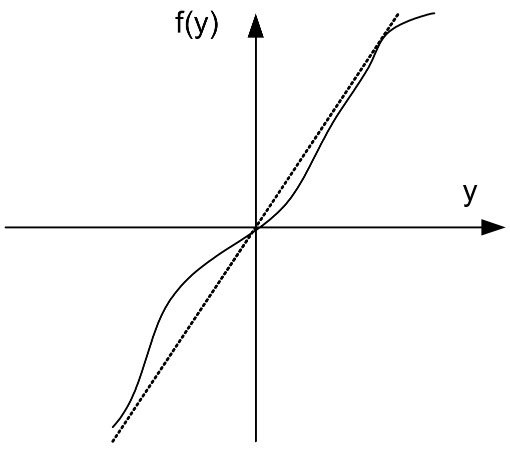


$$G(s) = \frac{0.4}{s+1}$$

$$e(t) = r(t) - f(y(t))$$

$$\|G\| = 0.4$$

If *K* < 2.5 the system is stable for sure



Slope is K

It was discussed....

- Introduction of the course
- Example of a multivariable system; analysis is difficult by classical SISO methods
- Example of a nonlinear system; analysis difficult
- General system models
- Signal and system norms
- Small Gain Theorem