



# **Distributed Generation Technologies**

By: Edris Pouresmaeil

Department of Electrical Engineering and Automation (EEA)  
Aalto University, 02150 Espoo, Finland

E-Mail : [edris.pouresmaeil@aalto.fi](mailto:edris.pouresmaeil@aalto.fi)

Fall 2020

**Chapter4:**  
**Control and Operation of Grid-  
Connected DC/AC Converters**

## **The Main Objectives of this Session:**

At the end of this session you will be able to answer the following questions:

- 1. How we can obtain dynamic model of a grid-connected DC/AC converter?**
- 2. How we can control the operation of grid-connected converters under steady state operating condition?**
- 3. How we can control a grid-connected converter for power injection from DG sources to the power grid?**
- 4. How we can compensate the harmonic current components of main grid by integration of RESs into the grid?**
- 5. How we can fix a unit value for the power factor of main grid?**

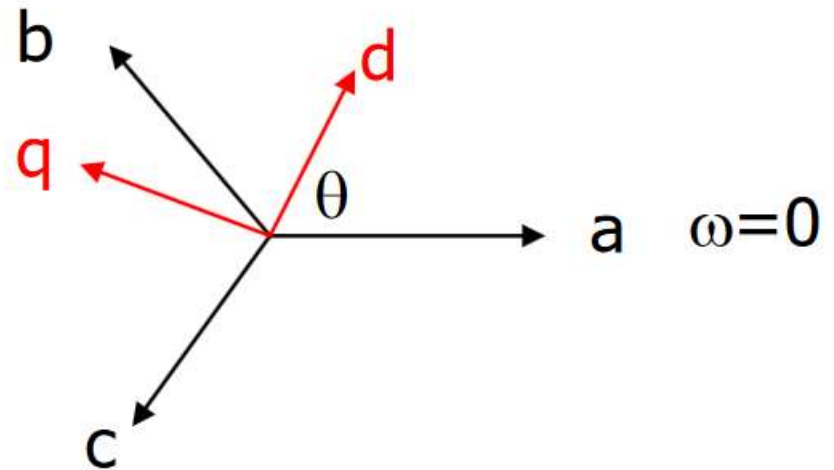
## Park Transformation

Park transformation is a tool to decouple three-phase quantities into two-phase variables.

$$\begin{bmatrix} f_{dq0} \end{bmatrix} = \begin{bmatrix} T_{dq0}(\theta) \end{bmatrix} \times \begin{bmatrix} f_{abc} \end{bmatrix}$$

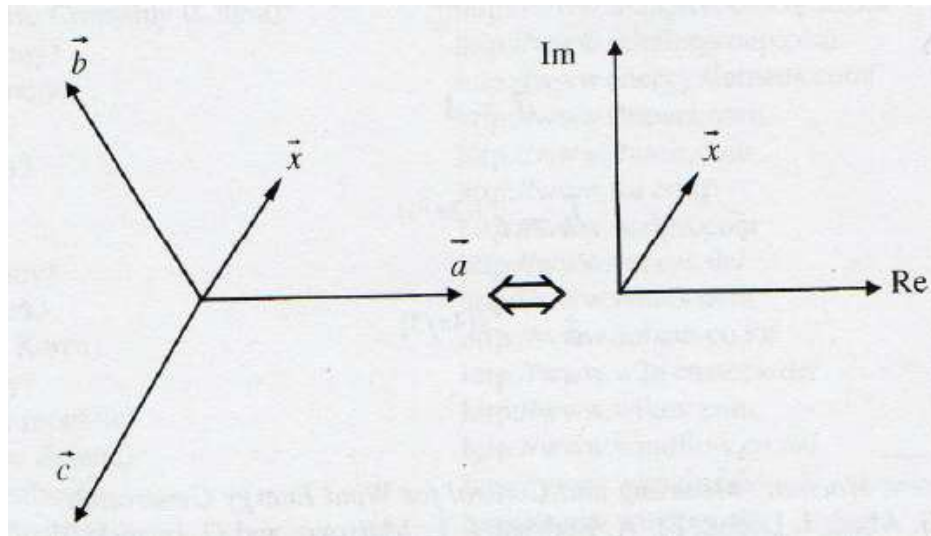
$$\begin{bmatrix} T_{dq0}(\theta) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} T_{dq0}(\theta) \end{bmatrix}^{-1} = \frac{2}{3} \begin{bmatrix} \cos \theta & -\sin \theta & 1 \\ \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix}$$



## $\alpha$ - $\beta$ Transformation

Three phase vectors can also be represented by two “phase magnitudes,” called  $x_\alpha$  and  $x_\beta$  in the real-imaginary complex plane, as illustrated below.



This transformation is also called the “**Clarke transformation**” for the person who developed it, Edith Clarke.

The relation between abc and  $\alpha\beta$  can be represented in matrix form as follows:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

## Three-Phase Converter

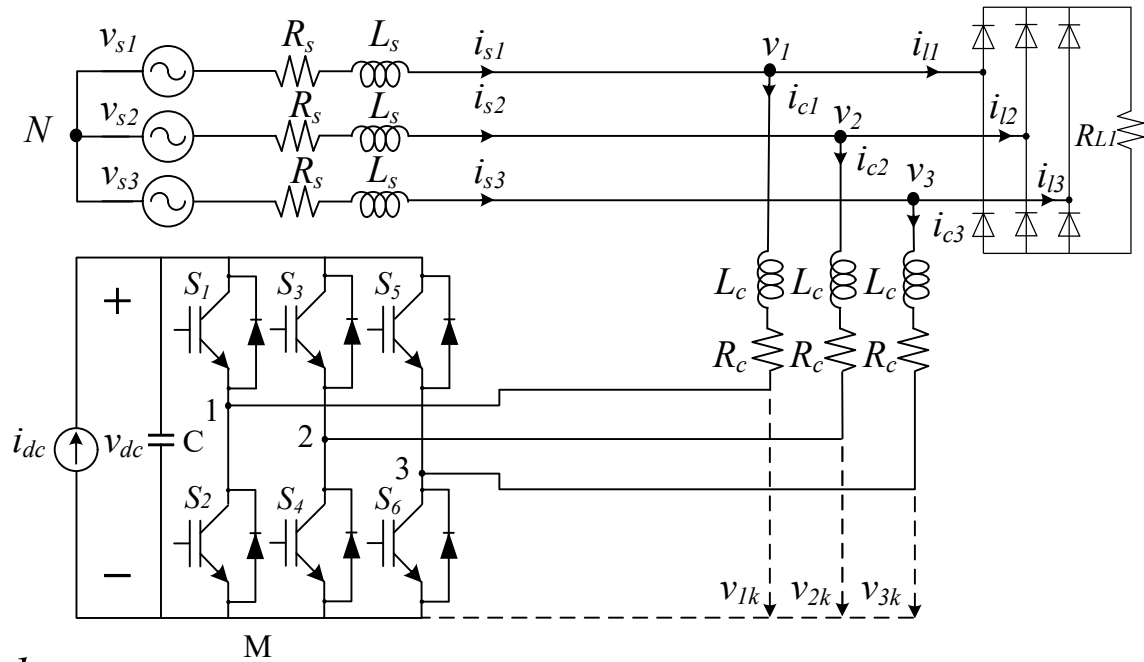
$$\begin{cases} v_{1M}(t) = v_{1k}(t) + v_{NM}(t) \\ v_{2M}(t) = v_{2k}(t) + v_{NM}(t) \\ v_{3M}(t) = v_{3k}(t) + v_{NM}(t) \end{cases}$$

$v_{1M}(t)$  : Leg Voltage,  $v_{1k}(t)$  : Phase Voltage,  
and  $v_{NM}(t)$  : Common Mode Voltage

$$v_{1M}(t) + v_{2M}(t) + v_{3M}(t) = (v_{1k}(t) + v_{2k}(t) + v_{3k}(t)) + 3 \times v_{NM}(t)$$

$$v_{1k}(t) + v_{2k}(t) + v_{3k}(t) = 0$$

$$v_{NM}(t) = \frac{v_{1M}(t) + v_{2M}(t) + v_{3M}(t)}{3}$$



**Fig. 4.1. General model of a grid-connected converter.**

## Dynamic model analysis

To draw an appropriate plan to control the interfaced converter for integration of energy sources into the power grid, a dynamic analytical model of the proposed model should be developed. According to Fig. 4.1, the KVL law for load voltage (PCC) can be express as:

$$v_{1M}(t) = v_{1k}(t) + v_{NM}(t) \rightarrow v_{1M}(t) - v_{1k}(t) - v_{NM}(t) = 0 \quad v_{1k}(t) = -Ri_1 - L \frac{di_1}{dt} + v_1$$

$$\left. \begin{array}{l} -v_1 + L \frac{di_1}{dt} + Ri_1 + v_{1M} + v_{MN} = 0 \\ -v_2 + L \frac{di_2}{dt} + Ri_2 + v_{2M} + v_{MN} = 0 \\ -v_3 + L \frac{di_3}{dt} + Ri_3 + v_{3M} + v_{MN} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} v_1 = L \frac{di_1}{dt} + Ri_1 + v_{1M} + v_{MN} \\ v_2 = L \frac{di_2}{dt} + Ri_2 + v_{2M} + v_{MN} \\ v_3 = L \frac{di_3}{dt} + Ri_3 + v_{3M} + v_{MN} \end{array} \right. \quad (1)$$

$$\underbrace{v_1 + v_2 + v_3}_0 = L \frac{d}{dt} \left( \underbrace{i_1 + i_2 + i_3}_0 \right) + R \left( \underbrace{i_1 + i_2 + i_3}_0 \right) + (v_{1M} + v_{2M} + v_{3M}) + 3v_{MN}$$

$$(v_{1M} + v_{2M} + v_{3M}) + 3v_{MN} = 0 \Rightarrow v_{MN} = -\frac{v_{1M} + v_{2M} + v_{3M}}{3} = -\frac{1}{3} \sum_{m=1}^3 v_{mM} \quad (2)$$

## Dynamic model analysis

$$\begin{cases} \frac{di_1}{dt} = -\frac{R}{L}i_1 - \frac{v_{1M}}{L} - \frac{v_{MN}}{L} + \frac{v_1}{L} \\ \frac{di_2}{dt} = -\frac{R}{L}i_2 - \frac{v_{2M}}{L} - \frac{v_{MN}}{L} + \frac{v_2}{L} \\ \frac{di_3}{dt} = -\frac{R}{L}i_3 - \frac{v_{3M}}{L} - \frac{v_{MN}}{L} + \frac{v_3}{L} \end{cases} \quad (3)$$

By substituting Eq. (2) and  $v_{mM}=S_m V_{dc}$  in Eq. (3), Eq. (4) can be obtained as: (4)

$$\left. \begin{cases} \frac{di_1}{dt} = -\frac{R}{L}i_1 - \frac{S_1}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 v_{mM}\right) + \frac{v_1}{L} \\ \frac{di_2}{dt} = -\frac{R}{L}i_2 - \frac{S_2}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 v_{mM}\right) + \frac{v_2}{L} \\ \frac{di_3}{dt} = -\frac{R}{L}i_3 - \frac{S_3}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 v_{mM}\right) + \frac{v_3}{L} \end{cases} \right\} \Rightarrow \begin{cases} \frac{di_1}{dt} = -\frac{R}{L}i_1 - \frac{S_1}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_1}{L} \\ \frac{di_2}{dt} = -\frac{R}{L}i_2 - \frac{S_2}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_2}{L} \\ \frac{di_3}{dt} = -\frac{R}{L}i_3 - \frac{S_3}{L}V_{dc} - \frac{1}{L}\left(\frac{-1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_3}{L} \end{cases}$$



## Dynamic model analysis

Eq. (4) leads to:

$$\left. \begin{aligned} \frac{di_1}{dt} &= -\frac{R}{L}i_1 - \frac{1}{L}\left(S_1 - \frac{1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_1}{L} \\ \frac{di_2}{dt} &= -\frac{R}{L}i_2 - \frac{1}{L}\left(S_2 - \frac{1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_2}{L} \\ \frac{di_3}{dt} &= -\frac{R}{L}i_3 - \frac{1}{L}\left(S_3 - \frac{1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_3}{L} \end{aligned} \right\} \Rightarrow \frac{di_k}{dt} = -\frac{R}{L}i_k - \frac{1}{L}\left(S_k - \frac{1}{3}\sum_{m=1}^3 S_m\right)V_{dc} + \frac{v_k}{L} \quad (5)$$

Equation (5) represents the phase  $k$  dynamic equation of the proposed model. Further, let us define a function  $d_{nk}$  that is called **switching state function** and is given by the following equation:

$$d_{nk} = \left(S_k - \frac{1}{3}\sum_{m=1}^3 S_m\right) \quad (6)$$

Equation (6) shows that the value of  $d_{nk}$  depends on the switching state  $\mathbf{n}$  and on the phase  $\mathbf{k}$ . In other words,  $d_{nk}$  depends simultaneously on the switching functions of the three legs of the interfaced converter. This shows the interaction between the three phases.

## Dynamic model analysis

Based on equation (6) and for the eight permissible switching states of the interfaced converter, the conversion of  $[S_k]$  to  $[d_{nk}]$  is given by the relation (7):

$$\begin{bmatrix} d_{n1} \\ d_{n2} \\ d_{n3} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \quad (7)$$

By substituting Eq. (6) in Eq. (5), Eq. (8) can be expressed as:

$$\frac{di_k}{dt} = -\frac{R}{L}i_k - \frac{1}{L}d_{nk}V_{dc} + \frac{v_k}{L} \quad \text{for } k = 1, 2, 3 \quad (8)$$

OR

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = -\frac{R}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} - \frac{1}{L} \begin{bmatrix} d_{n1} \\ d_{n2} \\ d_{n3} \end{bmatrix} V_{dc} + \frac{1}{L} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (9)$$

### Dynamic model analysis

For a balance system:  $i_1 + i_2 + i_3 = 0$  and  $v_1 + v_2 + v_3 = 0$ .

In addition, based on (7),  $d_{n1} + d_{n2} + d_{n3} = 0$ . Therefore, Eq(9) can be written as :

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = -\frac{R}{L} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} - \frac{1}{L} \begin{bmatrix} d_{n1} \\ d_{n2} \end{bmatrix} V_{dc} + \frac{1}{L} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (10)$$

Eq. (10) is the general dynamic equation of the proposed model.

## Model transformation into dq referential

In order to facilitate the control, the general dynamic model in Eq. (9) can be transformed to the synchronous orthogonal frame rotating at the supply frequency  $\omega$ . With this time varying transformation, given by (12), the positive-sequence components at fundamental frequency become constant.

$$L \frac{di_k}{dt} + Ri_k + d_{nk} V_{dc} = v_k \quad \text{for } k = 1, 2, 3 \quad (11)$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) & 1 \\ \cos(\omega t - \frac{2\pi}{3}) & -\sin(\omega t - \frac{2\pi}{3}) & 1 \\ \cos(\omega t + \frac{2\pi}{3}) & -\sin(\omega t + \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} \quad (12)$$

## Model transformation into dq referential

Eq. (12) can be expressed as:

$$\begin{aligned}
 u_1 &= u_d \cos(\omega t) - u_q \sin(\omega t) + u_0 \\
 u_2 &= u_d \cos\left(\omega t - \frac{2\pi}{3}\right) - u_q \sin\left(\omega t - \frac{2\pi}{3}\right) + u_0 \\
 u_3 &= u_d \cos\left(\omega t + \frac{2\pi}{3}\right) - u_q \sin\left(\omega t + \frac{2\pi}{3}\right) + u_0
 \end{aligned} \tag{13}$$

By substituting Eq. (11) in (13):

**(a). Phase (1):**

$$\begin{aligned}
 L \frac{di_1}{dt} + Ri_1 + d_{n1}V_{dc} &= v_1 \\
 L \left[ \frac{di_d}{dt} \cos(\omega t) - \omega i_d \sin(\omega t) - \frac{di_q}{dt} \sin(\omega t) - \omega i_q \cos(\omega t) + \frac{di_0}{dt} \right] + \\
 R \left[ i_d \cos(\omega t) - i_q \sin(\omega t) + i_0 \right] + \left[ d_{nd} \cos(\omega t) - d_{nq} \sin(\omega t) + d_{n0} \right] V_{dc} \\
 &= \left[ v_d \cos(\omega t) - v_q \sin(\omega t) + v_0 \right]
 \end{aligned} \tag{14}$$

## Model transformation into dq referential

(b). Phase (2):

(15)

$$L \frac{di_2}{dt} + Ri_2 + d_{n2}V_{dc} = v_2$$

$$\begin{aligned} & L \left[ \frac{di_d}{dt} \cos\left(\omega t - \frac{2\pi}{3}\right) - \omega i_d \sin\left(\omega t - \frac{2\pi}{3}\right) - \frac{di_q}{dt} \sin\left(\omega t - \frac{2\pi}{3}\right) - \omega i_q \cos\left(\omega t - \frac{2\pi}{3}\right) + \frac{di_0}{dt} \right] + \\ & R \left[ i_d \cos\left(\omega t - \frac{2\pi}{3}\right) - i_q \sin\left(\omega t - \frac{2\pi}{3}\right) + i_0 \right] + \left[ d_{nd} \cos\left(\omega t - \frac{2\pi}{3}\right) - d_{nq} \sin\left(\omega t - \frac{2\pi}{3}\right) + d_{n0} \right] V_{dc} \\ & = \left[ v_d \cos\left(\omega t - \frac{2\pi}{3}\right) - v_q \sin\left(\omega t - \frac{2\pi}{3}\right) + v_0 \right] \end{aligned}$$

## Model transformation into dq referential

(c). Phase (3):

(16)

$$L \frac{di_3}{dt} + Ri_3 + d_{n3}V_{dc} = v_3$$

$$L \left[ \frac{di_d}{dt} \cos(\omega t + \frac{2\pi}{3}) - \omega i_d \sin(\omega t + \frac{2\pi}{3}) - \frac{di_q}{dt} \sin(\omega t + \frac{2\pi}{3}) - \omega i_q \cos(\omega t + \frac{2\pi}{3}) + \frac{di_0}{dt} \right] +$$

$$R \left[ i_d \cos(\omega t + \frac{2\pi}{3}) - i_q \sin(\omega t + \frac{2\pi}{3}) + i_0 \right] + \left[ d_{nd} \cos(\omega t + \frac{2\pi}{3}) - d_{nq} \sin(\omega t + \frac{2\pi}{3}) + d_{n0} \right] V_{dc}$$

$$= \left[ v_d \cos(\omega t + \frac{2\pi}{3}) - v_q \sin(\omega t + \frac{2\pi}{3}) + v_0 \right]$$

## Model transformation into dq referential

By summing up (14), (15) and (16), the following relations are achieved as.

$$\begin{aligned}
 & \left[ L \frac{di_d}{dt} - L\omega i_q + Ri_d + d_{nd}V_{dc} - v_d \right] \left[ \cos(\omega t) + \cos\left(\omega t - \frac{2\pi}{3}\right) + \cos\left(\omega t + \frac{2\pi}{3}\right) \right] + \\
 & \left[ -L \frac{di_q}{dt} - L\omega i_d - Ri_q - d_{nq}V_{dc} + v_q \right] \left[ \sin(\omega t) + \sin\left(\omega t - \frac{2\pi}{3}\right) + \sin\left(\omega t + \frac{2\pi}{3}\right) \right] + \\
 & 3 \left[ L \frac{di_0}{dt} + Ri_0 + d_{n0}V_{dc} - v_0 \right] = 0
 \end{aligned} \tag{17}$$

By considering (17), the final equations in 0dq can be driven as,

$$\left\{ \begin{array}{l} L \frac{di_d}{dt} - L\omega i_q + Ri_d + d_{nd}V_{dc} - v_d = 0 \\ -L \frac{di_q}{dt} - L\omega i_d - Ri_q - d_{nq}V_{dc} + v_q = 0 \\ L \frac{di_0}{dt} + Ri_0 + d_{n0}V_{dc} - v_0 = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} L \frac{di_d}{dt} - L\omega i_q + Ri_d + d_{nd}V_{dc} = v_d \\ L \frac{di_q}{dt} + L\omega i_d + Ri_q + d_{nq}V_{dc} = v_q \\ L \frac{di_0}{dt} + Ri_0 + d_{n0}V_{dc} = v_0 \end{array} \right. \tag{18}$$



## Model transformation into dq referential

Eq.(18) can be written as :

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & 0 \\ \omega & -\frac{R}{L} & 0 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} - \frac{1}{L} \begin{bmatrix} d_{nd} \\ d_{nq} \\ d_{n0} \end{bmatrix} V_{dc} + \frac{1}{L} \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} \quad (19)$$

For a balance system:  $i_1 + i_2 + i_3 = 0$  and  $v_1 + v_2 + v_3 = 0$  therefore, homopolar values are zero.

In addition, based on (7),  $d_{n1} + d_{n2} + d_{n3} = 0$ . Therefore, Eq.(19) can be written as :

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ \omega & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \frac{1}{L} \begin{bmatrix} d_{nd} \\ d_{nq} \end{bmatrix} V_{dc} + \frac{1}{L} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \quad (20)$$

where the homopolar component has been omitted.

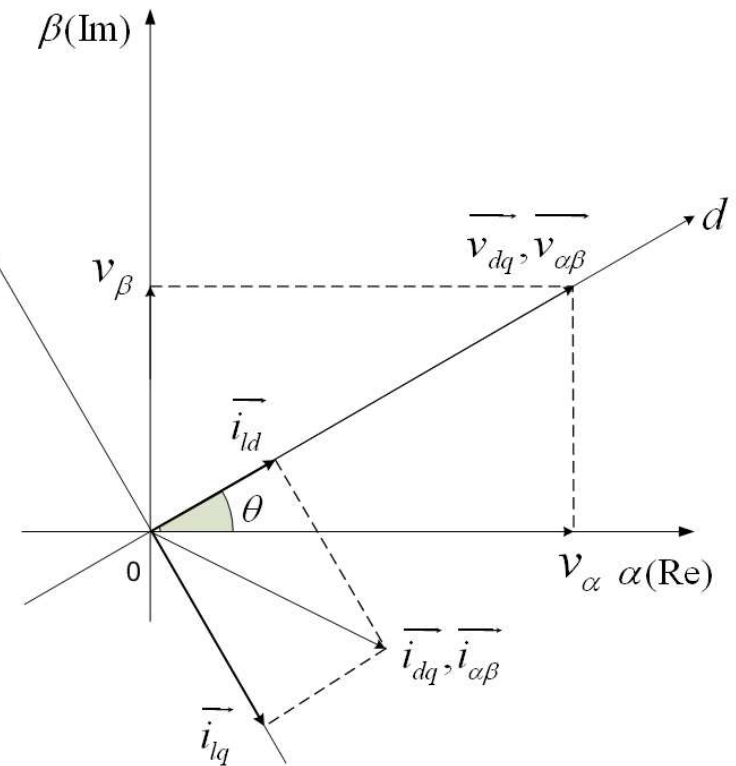
## Model transformation into dq referential

Considering the original position of the load voltage vector in  $d$ -axis, voltage vector of  $q$ -axis will be zero ( $v_q = 0$ ), and the other vector's value will be equal to the line-to-line *rms* voltage of grid voltage. Therefore, (20) can be expressed as:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \frac{1}{L} \begin{bmatrix} d_{nd} \\ d_{nq} \end{bmatrix} V_{dc} + \frac{1}{L} \begin{bmatrix} v_d \\ 0 \end{bmatrix} \quad (21)$$

$$\theta = \tan^{-1} \frac{v_\beta}{v_\alpha} \quad (21-1)$$

$$v_d = |\vec{v}_{dq}| = |\vec{v}_{\alpha\beta}| = \sqrt{v_\alpha^2 + v_\beta^2} \quad (21-2)$$



**Fig. 4. 2. Voltage and current components in special reference frames.**

### Current Control Technique for the Proposed Model

In order to obtain a **low overshoot**, **high accuracy**, and **fast dynamic response** to provide load active and reactive power and also harmonic current components of the grid-connected loads, two equations in the equivalent model of the proposed model (21) must be controlled in two different and independent loops.

As we mentioned before, all the Park-transformed variables of the proposed model will become a constant value in steady-state condition. By this technique, it is possible to design the current controllers using the well-known controller design tools meant for regulation problems instead of having to design more complex controllers to track general time-varying reference signals.

$$\begin{aligned} L \frac{di_d}{dt} - L\omega i_q + Ri_d + d_{nd}V_{dc} &= v_d \\ L \frac{di_q}{dt} + L\omega i_d + Ri_q + d_{nq}V_{dc} &= 0 \end{aligned} \tag{22}$$

### Current Control Technique for the Proposed Model

By referring to (22) and considering  $\lambda_{dq} = L(di_{dq}/dt) + Ri_{dq}$ , switching state functions can be calculated as:

$$d_{nd} = \frac{-\lambda_d + L\omega i_q + v_d}{V_{dc}} \quad (23-1)$$

$$d_{nq} = \frac{-\lambda_q - L\omega i_d}{V_{dc}} \quad (23-2)$$

To achieve a fast dynamic response and zero steady-state errors, particularly during connection of nonlinear loads to the grid, which main grid is polluted by these types of loads, a proportional–integral (PI)-type regulator is needed. The parameter of the proposed regulator can be obtained as:

$$(\lambda_{dq}) = k_p (\Delta i_{cdq}) + k_i \int (\Delta i_{cdq}) dt \quad (24)$$

where terms  $k_p$  and  $k_i$  are proportional and integral gains, and  $(\Delta i_{cdq}) = (i_{cdq}^*) - (i_{cdq})$  denotes a comparison of the calculated reference currents and the actual injection currents generated by the interfaced converter which create error signals and control the switches of the converter according to objectives of the connection of interfaced converter to the grid.

## Current Control Technique for the Proposed Model

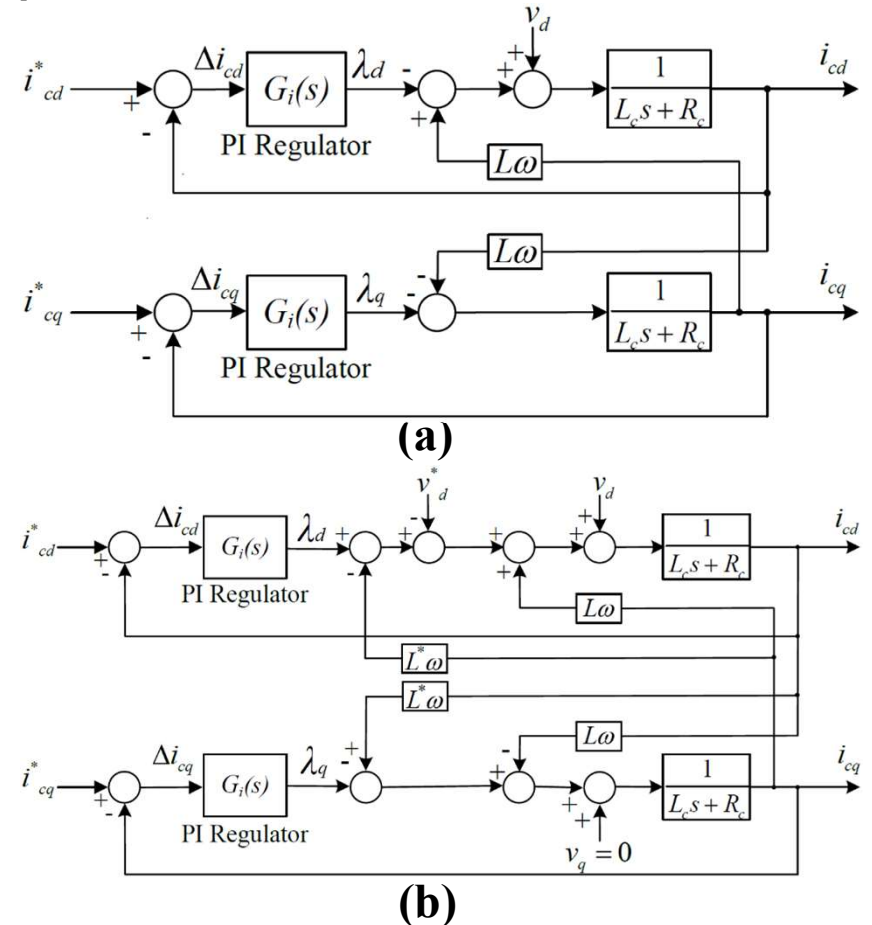
The transfer function of the PI regulator for current control loops of the proposed strategy is given as:

$$(\lambda_{dq}) = k_p (\Delta i_{cdq}) + k_i \int (\Delta i_{cdq}) dt \rightarrow \lambda_{dq}(s) = k_p \Delta i_{cdq}(s) + k_i \frac{1}{s} \Delta i_{cdq}(s)$$

$$G_i(s) = \frac{\lambda_d(s)}{\Delta I_d(s)} = \frac{\lambda_q(s)}{\Delta I_q(s)} = k_p + \frac{k_i}{s} \quad (25)$$

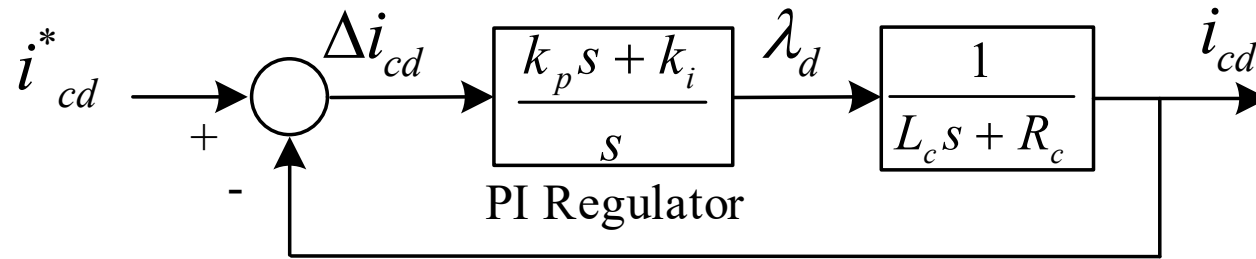
To design PI regulator in circuit of current controller, it is necessary to decouple the model of the system by adding the measured voltage of d-axis and crosscoupling terms as shown in Fig. 4.3 (b), where  $L^*$  and  $v^*$  are estimated values of coupling inductance and grid voltages.

**Fig. 4.3. Inner control loop of the  $i_{cd}$  and  $i_{cq}$ .**



## Current Control Technique for the Proposed Model

Thus, the inner control loops of the current  $i_{cd}$  can be simplified as shown in Fig. 4.4. As shown in Fig. 4.3, the current loops of  $i_{cd}$  and  $i_{cq}$  are the same. Thus, in dq reference frame, decoupled control for the reactive and active power can be conveniently achieved by independently controlling the d- and q-axis currents.



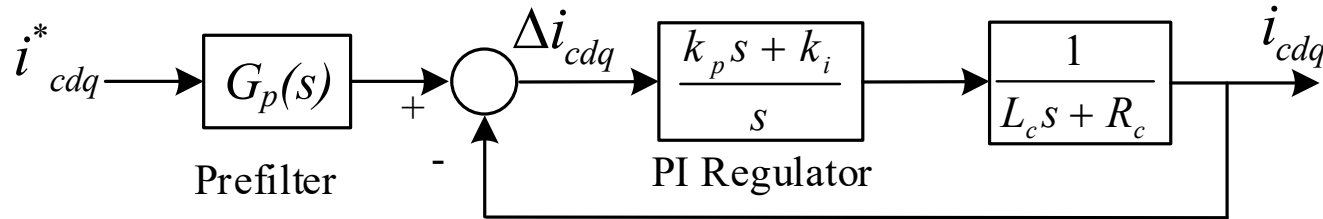
**Fig. 4. 4. Equivalent diagram of d-axis current control loop.**

The closed-loop transfer function of the current loop can be calculated as:

$$\frac{I_{cq}(s)}{I_{cq}^*(s)} = \frac{I_{cd}(s)}{I_{cd}^*(s)} = \frac{k_p}{L_c} \frac{s + \frac{k_i}{k_p}}{s^2 + \frac{(R_c + k_p)}{L_c} s + \frac{k_i}{L_c}} \quad (26)$$

## Current Control Technique for the Proposed Model

The transient response of the currents will be affected by the presence of the zero in (26). In particular, the actual percent overshoot will be much higher than expected.



**Fig. 4. 4. 2. Equivalent diagram of d-axis current control loop.**

For the optimal value of the damping factor  $\zeta = \frac{\sqrt{2}}{2}$ , the theoretical overshoot is 20.79%. To eliminate the effect of zero on transient response in (26), a pre-filter is added as shown in Fig. 4. 4. 2. The response of the current loops becomes that of a second-order transfer function with no zero. Comparison between general model of a second-order transfer function  $\omega_n^2 / (s^2 + 2\xi\omega_n s + \omega_n^2)$  and (26) leads to the following design relations:

$$k_p = 2L_c \xi \omega_n - R_c \quad \text{and} \quad k_i = L_c \omega_n^2 \quad (27)$$

### Reference Current Calculation

According to Fig. 4.2, and considering the instantaneous complex value of load power will be obtained by:

$$S = VI^* = P_l + jQ_l = \frac{3}{2}(v_d + jv_q)(i_{ld} - ji_{lq}) \quad (28)$$

$$S = \frac{3}{2}((v_d i_{ld} + v_q i_{lq}) + j(v_q i_{ld} - v_d i_{lq})) \quad (29)$$

$$P_l = \frac{3}{2}(v_d i_{ld} + v_q i_{lq}) \text{ and } Q_l = \frac{3}{2}(v_q i_{ld} - v_d i_{lq}) \quad (30)$$

#### *A. Calculation of Reference Current to Supply Load Active Power in Fundamental Frequency*

Considering the reference voltage vector in direction of d-axis vector in this transformation, the vertical component of the load voltage (q-component of voltage) in the rotating synchronous reference frame is always zero ( $v_q=0$ ); therefore, d-component of reference current to provide active current in fundamental frequency can be calculated by:

$$i_{cd}^* = \frac{2P_{ref}}{3v_d} \quad (31)$$



### Reference Current Calculation

#### *B. Calculation of harmonic components of reference current of d-axis*

Calculation of the load current in the dq reference frame makes it possible to separate fundamental and harmonic current components. d-component of the load current ( $i_{ld}$ ) can be separated into dc and alternative current components as:

$$i_{ld} = \tilde{i}_{ld} + I_{ld} \quad (32)$$

where  $\tilde{i}_{ld}$  is alternative d-component of load current in rotating synchronous reference frame which is related to harmonic components of load current and  $I_{ld}$  is the dc term of load current in this frame which is related to fundamental frequency of load current.

To compensate harmonic current it is necessary to separate alternative and constant current components. For this purpose, a high pass filter (HPF) must be used. To minimize the influence of the HPF's phase responses, by means of a low-pass filter (LPF), a minimal phase high pass filter (MPHPF) can be obtained, the transfer function of this LPF has order and cut-off frequency the same as HPF.

### Reference Current Calculation

Therefore, the proposed MHPF can be achieved simply by the difference between the input current signal and the filtered current, which is equivalent to performing:

$$H_{MPHPF}(s) = 1 - H_{LPF}(s) \quad (33)$$

To use the proposed model based on renewable energy resources as an active filter device, harmonic current components of load current must be supplied. For this purposes d-component of non-linear link reference current is achieved by doing the sum of currents in fundamental frequency and alternative terms of load current as:

$$i_{ld} = \tilde{i}_{ld} + I_{ld} \longrightarrow i_{cd}^* = \tilde{i}_{ld} + I_{cd}^* \quad (34)$$

### Reference Current Calculation

#### *C. Calculation of Reference Current to Supply Load Reactive Power*

In a rotating synchronous reference frame, q-component of load current is perpendicular on d-component of load voltage ( $v_d \perp i_{lq}$ ). Therefore, q-component of load current indicates required reactive power of the grid-connected loads. To compensate load reactive power, VSC must inject a current with q-component equal to  $i_{lq}$ . For this purpose, it is sufficient to set q-component of reference current in circuit loop of the proposed control strategy equal to q-component of the load current.

$$i_{lq} = \tilde{i}_{lq} + I_{lq} \longrightarrow i_{cq}^* = i_{lq} \quad (35)$$

By this consideration, reactive current of grid-connected loads is compensated in fundamental and harmonic frequencies.

### General schematic diagram

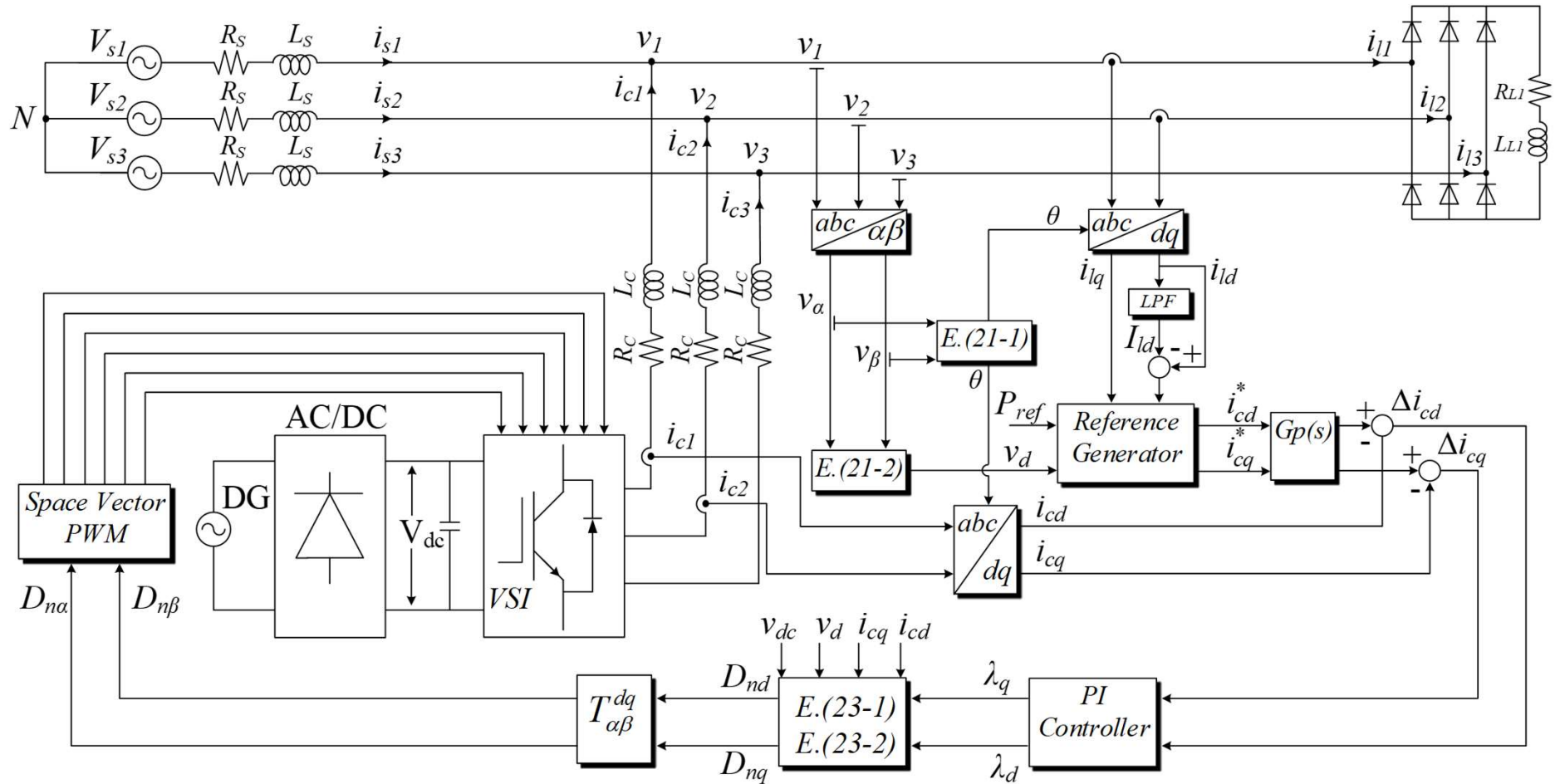
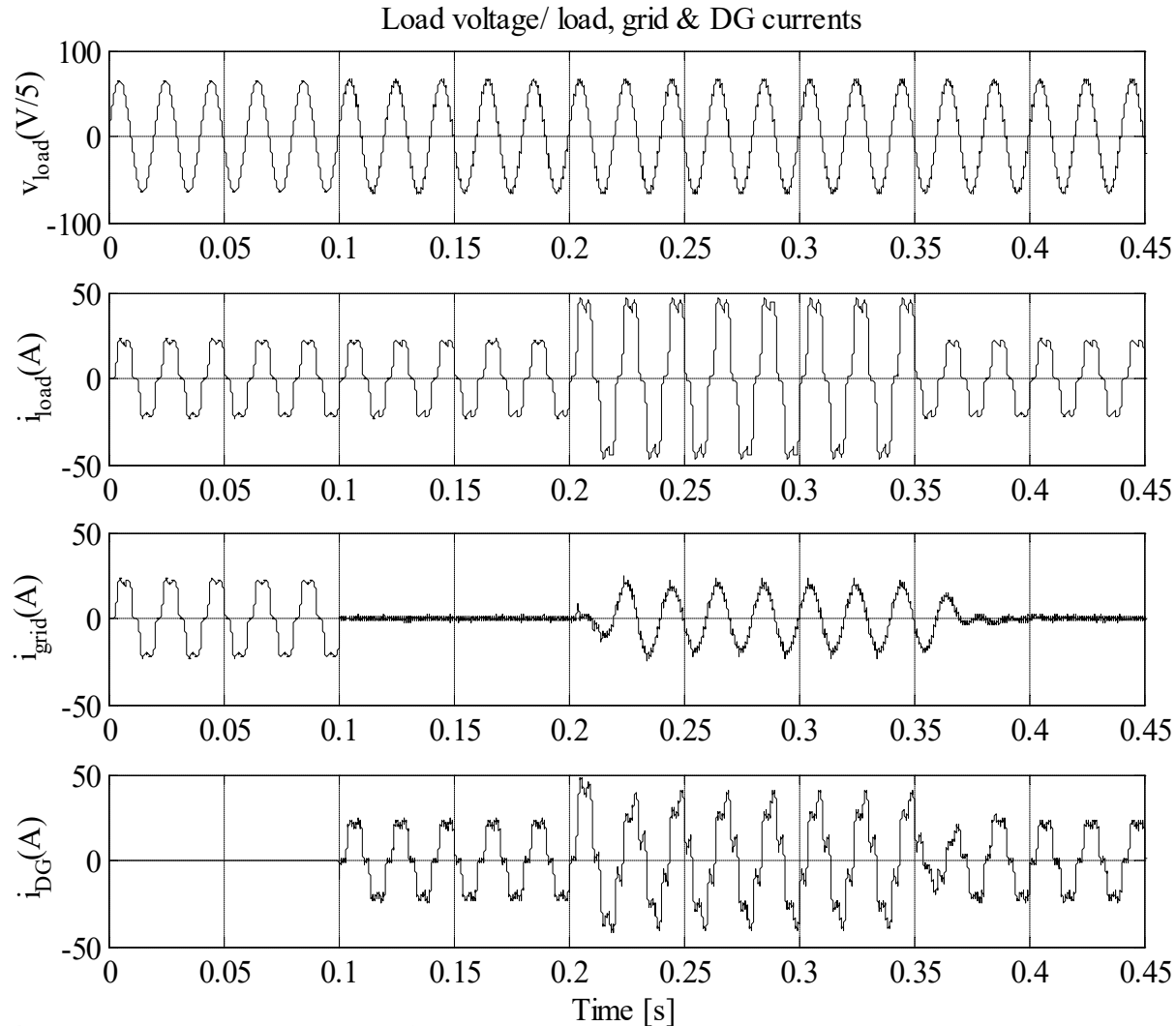


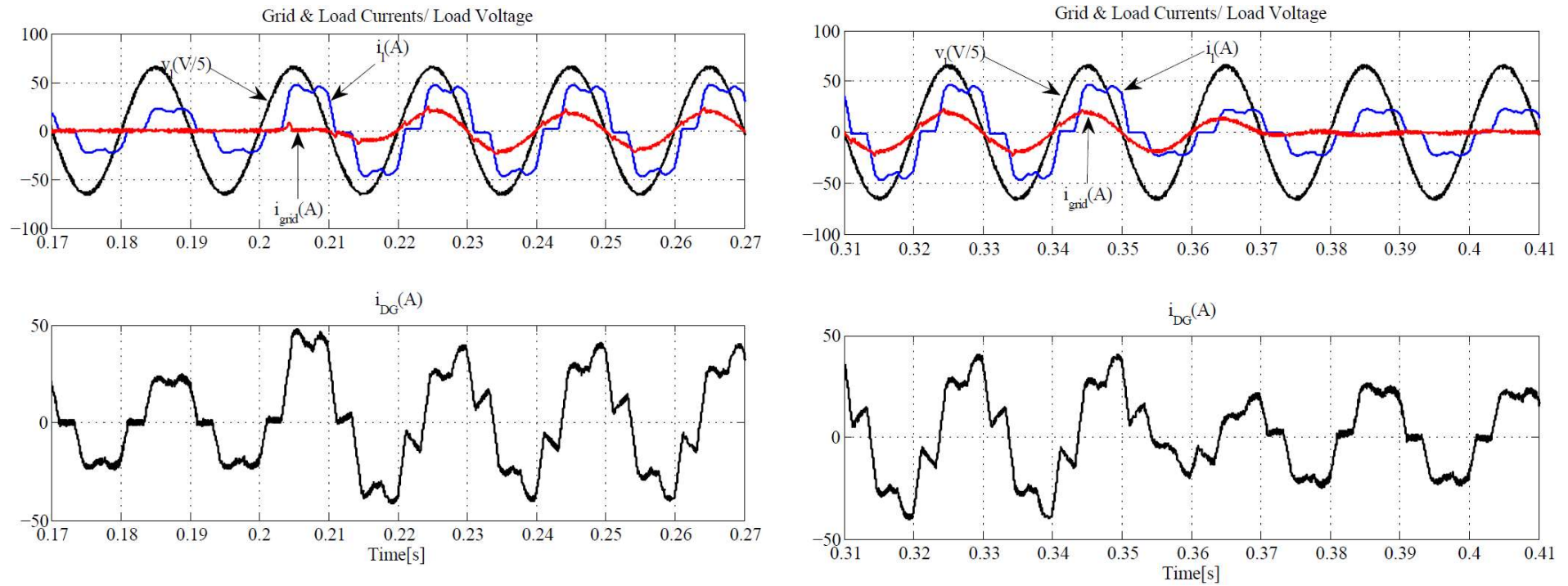
Fig. 4. 5. General schematic diagram of the proposed control strategy for DG system.

## Results and Discussions



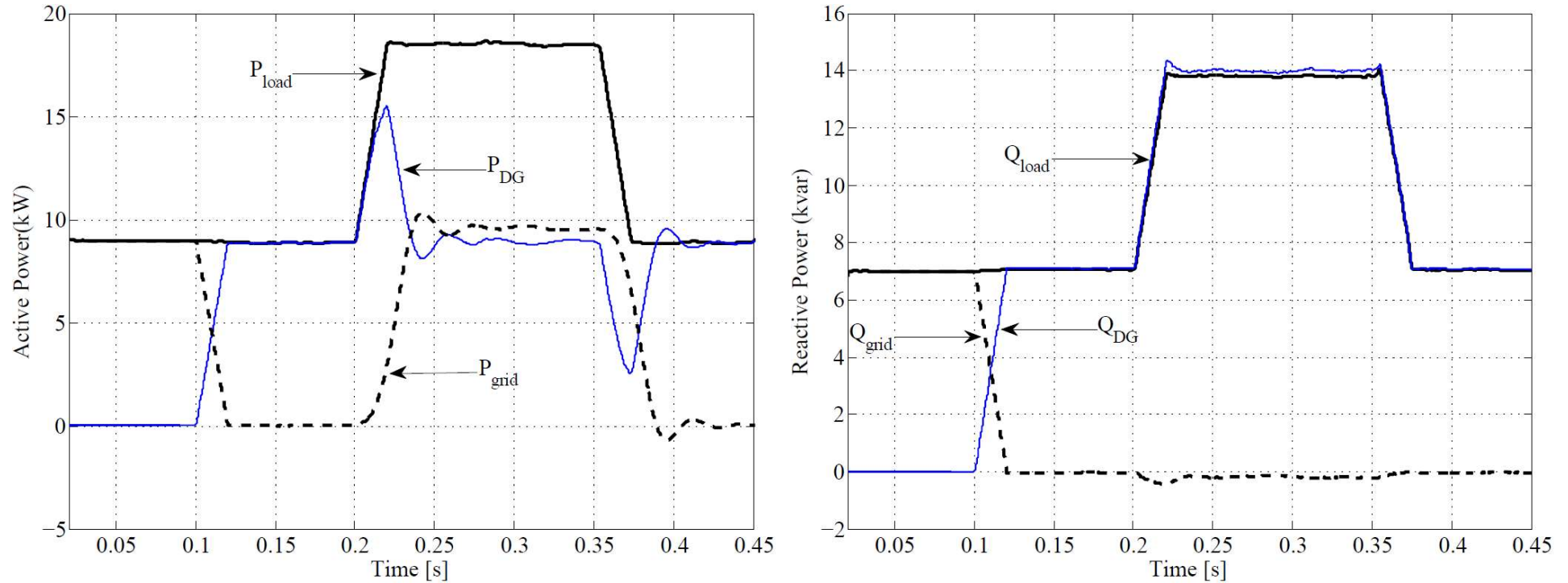
**Fig. 4. 6. Load current, Converter current and Source current before and after connection and before and after additional load connection to the grid.**

## Results and Discussions



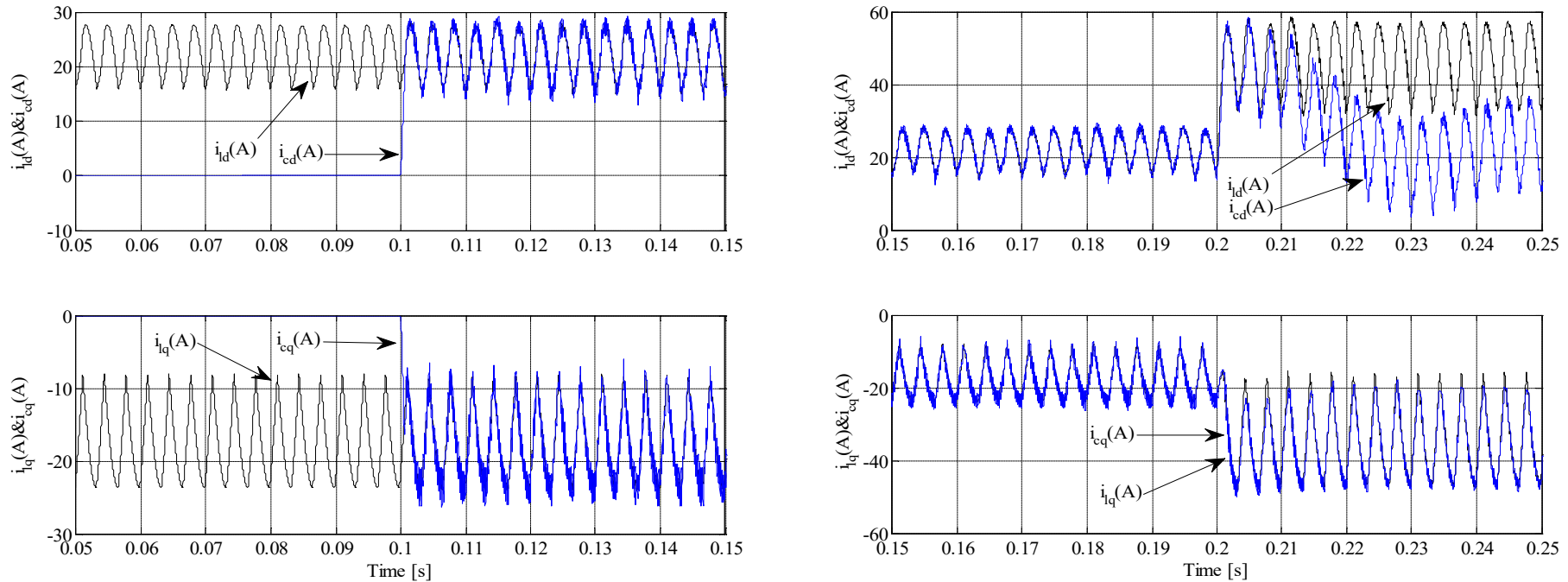
**Fig. 4. 7. Source current, load voltage and load current before and after connection of additional load.**

## Results and Discussions: Active and Reactive Power Injection



**Fig. 4. 8. Active and reactive power sharing between the main grid, load and DG.**

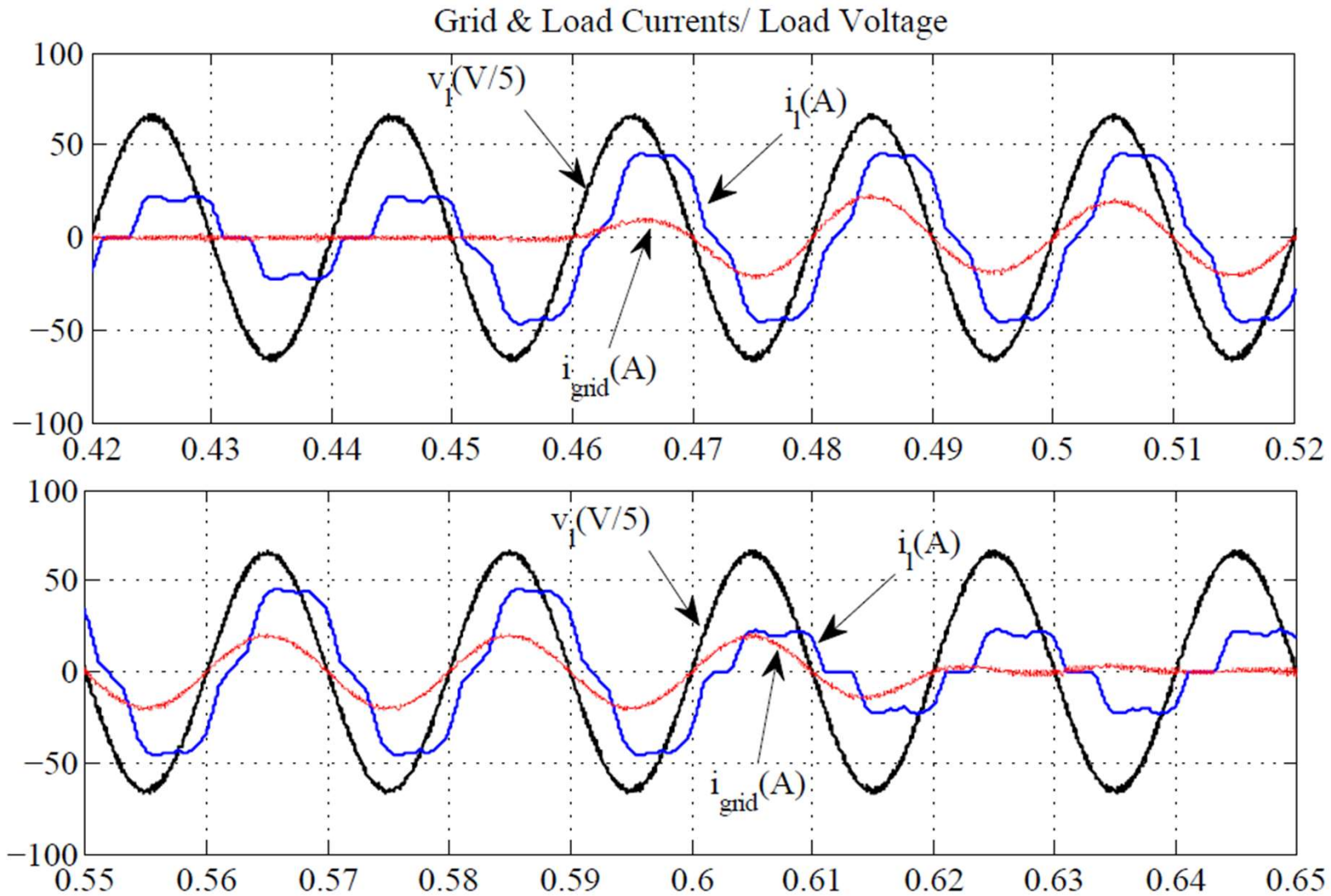
## Results and Discussions: Reference Currents Tracking



**Fig. 4.9. Reference currents track the load current (a) after interconnection of DG resources and (b) after additional load increment.**

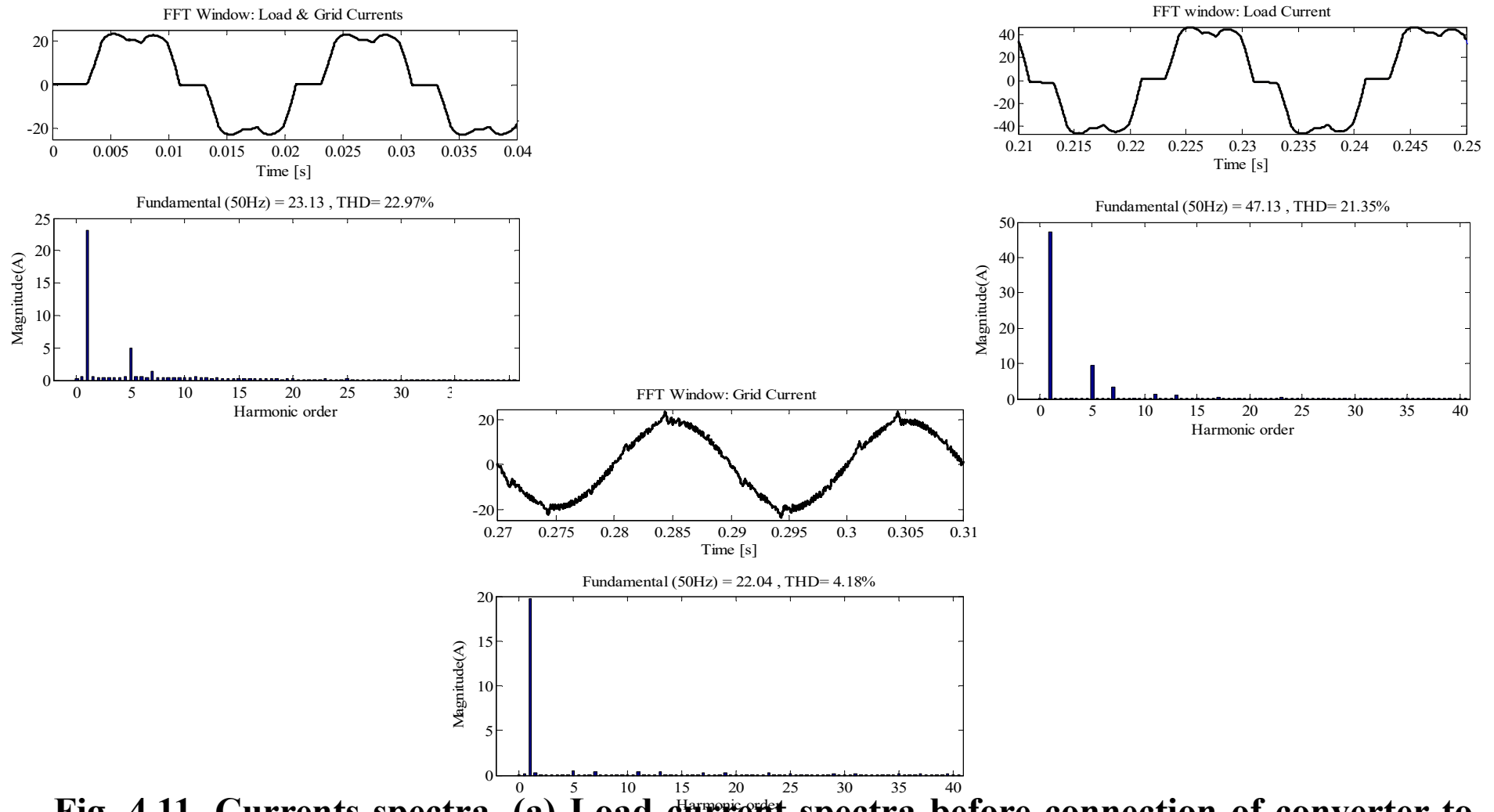


## Results and Discussions: Power Factor Correction (PFC)



**Fig. 4. 10. Source current, load voltage and load current before and after connection of additional load.**

## Results and Discussions: Active Power Filter (APF)



**Fig. 4.11. Currents spectra. (a) Load current spectra before connection of converter to the grid, (b) Load current spectra after connection of additional load, and (c) Source current spectra after connection of additional load.**

**Questions and comments are  
most welcome!**