31E11100 - Microeconomics: Pricing

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Lectures 4-6: Monopoly pricing strategies
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Objectives for the second part of the course

- So far we have discussed linear pricing for a homogenous product in a competitive market
- Typically sellers have some market power
- More instruments are then available for the seller:
  - Different price for different individuals, or different market segments
  - Different versions with different prices
  - Different unit price for different quantities
  - Different prices for individuals with different purchase histories
  - Bundling of different products together
- We will analyze such strategies for a monopoly seller

Plan for the next three lectures

- Lecture 4 (today): personalized pricing and group pricing
- Lecture 5: menu pricing (September 21)
- Lecture 6: Bundling, price signalling (September 23)

Taxonomy of price discrimination

- Traditionally, price discrimination practices are classified as follows
  - First degree price discrimination, or *personalized pricing*: each buyer gets an individual offer
  - Second degree price discrimination, or *menu pricing*: consumers choose freely from a menu of offers
  - Third degree price discrimination, or *group pricing*: seller can identify different market segments and price them separately
- How are new technologies changing viability of various forms of price discrimination?
- In this lecture we will consider first and third degree price discrimination (since they are conceptually very similar)
- Second-degree price discrimination is conceptually different, since it relies on self-selection by consumers (next lecture)

Framework: Pricing in Monopoly

- The setup is a single firm setting its price in a given market
  - Interpretations: true monopoly
    - Natural monopoly
    - Legal monopoly (patent, copyright, etc.)
    - A unique product
  - One large firm with a competitive fringe of small firms
    - Small firms' reactions can be interpreted as part of the demand curve
    - No game theory needed this time
- We start by analyzing linear prices, then consider non-linear prices and price discrimination

Optimal Linear Price

- Large number of buyers represented by demand curve

$$q = d(p),$$

where $d'(p) < 0$.

- A single seller produces the good with cost function $c(q)$ for producing q units of the good.
- Monopolist chooses the price, and quantity is given by the demand curve.
- Prices are linear so that revenue is pq.
- The monopolist chooses p to maximize revenue net of cost.

Optimal Linear Price

- Monopolist's problem is

$$\max_{p,q\geq 0} pq - c(q)$$
$$\text{subject to } q = d(p).$$

- Substituting the constraint into the objective function gives:

$$\max_{p} pd(p) - c(d(p)).$$

- Notice that this objective function is not always concave. Hence you should check all points at which the first-order condition holds and also the point where p is high enough to make $q = 0$ and pick the point that results in the highest profit.

- First-order condition:

$$pd'(p) + d(p) - c'(d(p))\, d'(p) = 0.$$

- Dividing through by $d'(p)$, and rearranging yields:

$$\frac{p - c'(d(p))}{p} = -\frac{d(p)}{pd'(p)}.$$

- Writing $\varepsilon_p = -\frac{pd'(p)}{d(p)}$ for the price elasticity of demand and $q = d(p)$ for the amount demanded, we have:

$$\frac{p - c'(q)}{p} = \frac{1}{\varepsilon_p}.$$

- In words, the percentage markup of the optimal monopoly price over marginal cost is the inverse of the elasticity of demand in the market.
- Less elastic demand leads to higher markup
- What are examples of markets with inelastic demand? Implications for multi-product firms?

Example of a not so good principle: cost-plus pricing

- For concreteness, assume a cost function with a fixed cost $F = 2000$ and variable cost $60q$. Hence $c(q) = 2000 + 60q$ if $q > 0$ and $c(0) = 0$.
- The demand function is given by

$$q = 200 - p.$$

- Optimal monopoly price is $p^* = 130$, corresponding sales $q^* = 70$, and optimal profit is 2900. (check that you can derive these)

- Suppose next that the firm considers cost-plus pricing.
- This form of pricing is widely used across many industries.
- Its supposed virtue is that pricing decision are based on hard engineering data from production side rather than softer demand estimates.
- Suppose the capacity at the plant is at 100.
- Cost plus pricing takes the form of setting a margin requirement for the sales price.
- Suppose that the goal is to have a 100% margin on production costs.
- How to set the price? How to allocate the fixed cost?

- Variable cost is 60, at full capacity, average fixed cost is 20 per unit.
- Hence cost-plus pricing demands a price of

$$(1+\alpha)(60+20) = 160$$

  since at the required 100% margin, $\alpha = 1$.
- Realized demand is then 40 and total profit is 2000.
- But now the production department reports that the true fixed cost per unit sold is 50 since production is well below capacity.
- If the price is raised in response to this report to 220, the demand disappears altogether and the firm makes a loss of 2000.

- As this example illustrates: knowing your demand is the key to profits
- But firms do not often have accurate information about the elasticity of demand for their product.
- Nevertheless, they can perform thought experiments along the lines:
  - Suppose we increase price by x per cent.
  - What is the maximal drop in sales volume that makes this increase still profitable?
  - Do we consider a price response of that magnitude is likely?
- Room for consumer surveys.
- Room for experimenting with different prices to find out more about the demand function.
- Models on how to price when also learning about the demand curve are beyond the scope of this course.

Personalized Prices or First-Degree Price Discrimination

- Recall that the market demand is obtained by summing together all individual demand functions:

$$d(p) = \sum_{i=1}^{I} d_i(p),$$

where $d_i(p)$ is the individual demand function of buyer i.

- Suppose now that the seller knows all the $d_i(p)$ and can set individual prices p_i for each buyer.
- Let $\varepsilon_{P,i}$ be the price elasticity of the individual demand of buyer i.
- Optimal pricing is given by:

$$\frac{p_i - c'(q)}{p_i} = \frac{1}{\varepsilon_{P,i}}.$$

- Notice that the marginal cost depends on the aggregate demand.

- Special case: Unit Demands
  - Good is sold in discrete units.
  - Each buyer gets a utility v_i from the first unit, no additional utility from further units.
  - Without loss of generality, rename the buyers so that $v_1 \geq v_2 \geq ... \geq v_I$.
  - If each unit costs c to produce, sell to all buyers with $v_i \geq c$.
  - If n units cost $c(n)$ to produce, then sell to the first n^* buyers, where

$$n^* = \max\{n : v_n \geq c(n) - c(n-1)\}.$$

  - Interpretation?
  - For $i \leq n^*$, set $p_i = v_i$.
- With unit demands, monopolist extracts all consumer surplus in the market.
- This can be easily modeled also by assuming a continuum of consumers, with reservation value distributed over an interval on real line, e.g.: $v_i \sim U(0, 1)$

- With more general individual demands, the consumers do get some consumer surplus with linear individual prices.
- But what if the monopolist can use a two-part tariff for each consumer:
  - Let:

$$\begin{aligned} p_i(q_i) &= f_i + p_i q_i \text{ if } q_i > 0, \\ p_i(0) &= 0 \text{ if } q_i = 0. \end{aligned}$$

  - f_i is the fixed purchase fee of i.
  - p_i is the linear individual price for i.
  - Why is this helpful for the monopolist? How should the f_i and p_i be set?
  - The principle: choose p_i to maximize total surplus, and use f_i to exctract the consmer's surplus

- Do such two-part tariffs exist in reality?
- With two-part tariffs, the Pareto-efficient market outcome is obtained.
- Extreme distributional asymmetry. Sellers get all, buyers nothing.
  - This is Pareto-efficient, but is this a good societal outcome?
  - Relies on the seller's perfect knowledge of the preferences of the buyers.
  - Is this realistic?

- What about arbitrage, e.g. resale between buyers?
  - Always a question for models of price discrimination.
- Technological progress might make the model more relevant.
  - Collect information on individual buyers through loyalty cards, social media etc.
  - Tailor price offers available only on their loyalty card.
  - You can even experiment relatively cheaply by issuing coupons (price discounts) and observing the demand reactions.
  - Combined with statistical analysis of all data in the database of the selling firm, this is a potentially successful pricing tool.

Third-Degree Price Discrimination or Group Pricing

- What if the monopolist can identify different group and use separate price for each group?
  - Student/Pensioner/Disabled/Unemployed/Military Service discounts.
  - Geographically separate markets (e.g. countries)
  - What about differential insurance premiums based on sex/age etc.?
- Key assumption: membership in a market segment cannot be manipulated
- This is called third-degree price discrimination
- Can be thought of as a less extreme form of personalized pricing

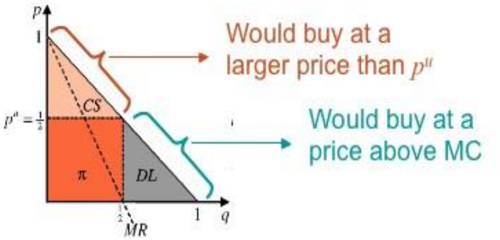
- N market segments.
- Each with a demand curve $d_n(p)$.
- Since the markets are separate, optimal pricing formula is as before:

$$\frac{p_n - c'(q)}{p_n} = \frac{1}{\varepsilon_{P,n}}.$$

- Implications are then clear: set higher prices for the segments with less elastic demand,
  - What does this mean in terms of the examples listed above?

- What is the value for the seller of this form of price discrimination?
  - What happens to the profit if there is more precise information available (i.e. finer grouping is possible)?
- What is the effect on consumer surplus?
- Let us next examine the effect of group pricing on welfare through an example

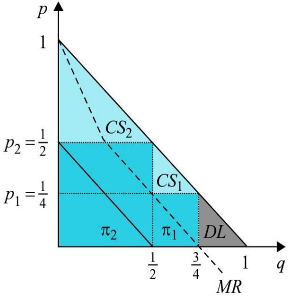
- Welfare effects of getting more refined information of consumers:
  - Example
    - Unit mass of consumers with unit demand
    - Valuation θ uniformly distributed over $[0,1]$
    - Buy if $\theta \geq p \rightarrow$ demand: $q = 1 - p$
    - Zero marginal cost; profits: $p(1 - p)$
    - If uniform price: $p^u = 1/2, \pi^u = 1/4, CS^u = 1/8, DL^u = 1/8$



- Refined information
  - Partition $[0,1]$ into N subintervals of equal length
  - Monopolist knows from which group each consumer comes & can charge a different price for each group
  - Take N = 2

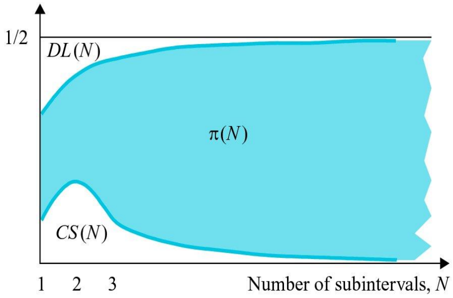
$$[0,1/2] \rightarrow q_1 = 1/2 - p_1$$
$$[1/2,1] \rightarrow q_2 = \max\{1/2,\, 1 - p_2\}$$

$$\pi(2) = \tfrac{1}{4} + \tfrac{1}{16} > \pi^u$$
$$CS(2) = \tfrac{1}{8} + \tfrac{1}{32} > CS^u$$
$$DL(2) = \tfrac{1}{32} < DL^u$$



- Refined information (cont'd)
  - N subintervals

$$\pi(N) = \frac{1}{2} - \frac{2N-1}{4N^2}$$
$$CS(N) = \frac{4N-3}{8N^2}$$
$$DL(N) = \frac{1}{8N^2}$$



Discussion and direction for the next lecture

- Both personal pricing (first-degree) and group pricing (third-degree) rely on the seller's ability to identify different buyers
  - In the first-degree case, individual identification
  - In the third-degree case, identification at the level of the segment
- When is grouping of consumers feasible?
- What if the buyer can manipulate the classification?
  - Second-degree price discrimination or menu pricing
  - Buyers self-select
  - For the next lecture

Further readings on the topics discussed so far

- A review of the economics of price discrimination: Armstrong (2006): "Recent developments in the economics of price discrimination", Advances in Economics and Econometrics: Theory and Applications. Ninth World Congress of the Econometric Society (contains also a lot of analysis of oligopoly that we do not cover in this course).
- For an example of empirical work on international price discrimination, see e.g. Goldberg and Verboven (2001): "The evolution of price dispersion in the European car market", Review of Economic Studies.
- Recent advances in the theory of group pricing include Aguirre, Cowan, and Vickers (2010): "Monopoly Price Discrimination and Demand Curvature", American Economic Review and Bergemann, Brooks and Morris (2015): "The Limits of Price Discrimination", American Economic Review.

Menu pricing

- So far, we have discussed price discrimination based on seller's direct information about buyer types
- But buyers' characteristics are to a large extent their private information
  - Some buyers value higher quality more than others, for example
  - Difficult for the seller to know the tastes of individual consumers
- Is there a profitable way to induce consumers self-select between different price-quality offers?

- In this lecture, we analyse this question with a simple theoretical model
- As a model of pricing, this is a model of second-degree price discrimination or menu pricing
  - How to design a menu of alternative price-quality bundels that consumers may choose from?
  - Or, how to design a non-linear pricing scheme, i.e. a set of different price-quantity bundles?
- But more broadly, this model is a classical example in information economics, within contract theory/mechanism design literature
  - How to design an incentive scheme under asymmetric information?

Examples of Second-Degree Price Discrimination or Menu Pricing

- Quantity discounts: "3 for the price of 2" -offers at supermarkets
- Differential fixed fee, variable fee combinations:
  - Pricing of different plans for smart phones.
  - Gym membership fees vs single entry fee
- Quality versioning
  - First-class, Business and Economy airfare.
  - Book versions: hardcover and papeback
  - Different speeds on broadband.
  - Insurance with different deductibles.
- Damaged goods?

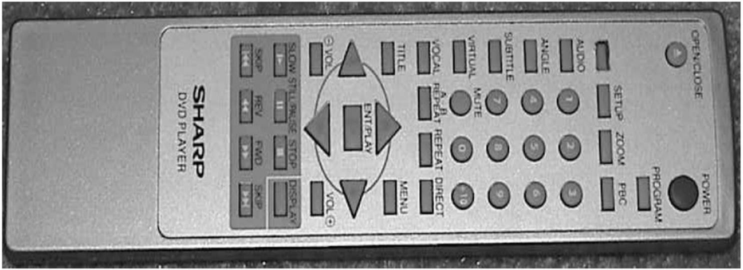


Figure 1: Hacked Remote Control of the DV740U (Courtesy of Area 450). Note the extra button in upper right portion of image.

Information Economics: Basic Model of Screening

- An uninformed party (principal) offers a menu of alternatives to an informed party (agent).
- The seller is the principal and the buyer is the agent.
- The menu consists of a list $\{(q^l, t^l)\}_{l=1}^{L}$.
  - q stands for a physical allocation to the agent: could be quality or quantity
  - t is the transfer that the agent makes to the principal: price
  - Hence choosing (q^l, t^l) means that the agent gets physical allocation q^l in exchange for paying t^l.
  - Notice that this is **not** a per unit price but a total price for q^l.

- Agent's utility from q depends on her private type $\theta \in \Theta$.
  - Assume here only two types: $\theta \in \{\theta^H, \theta^L\}$
- Quasilinear utility.
  - Agent:

$$u_A(\theta, q, t) = \theta v(q) - t.$$

  - Principal:

$$u_P(\theta, q, t) = t - c(q).$$

  - Here we interpret $v(q)$ as the utility from allocation q. Assume increasing utility with diminishing marginal utility: $v'(q) \geq 0$, $v''(q) \leq 0$
  - $c(q)$ is the cost of providing allocation (quantity or quality) q. Assume increasing convex cost: $c'(q) \geq 0, c''(q) \geq 0$.

- Seller makes an offer $\{q^l, t^l\}_{l=1}^{L}$.
  - She does not know the type of the buyer (but has a belief on the likelihoods of the different types).
  - With two types, set $\lambda = \Pr\{\theta = \theta^H\}, 1-\lambda = \Pr\{\theta = \theta^L\}$.
- Buyer of type θ picks the pair $\left(q^l, t^l\right)$ that gives her the maximal utility or picks nothing if that gives higher utility.
  - Since each type picks at most one pair, we can restrict the number of alternatives offered to be at most the number of different types of buyers.
  - With two types of buyers $\theta \in \{\theta^H, \theta^L\}$, enough to consider two pairs $\{(q^1, t^1), (q^2, t^2)\}$.
- Call the pair chosen by θ^i as $\left(q^i, t^i\right)$ for $i \in \{H, L\}$.
- Examples: Insurance company screening privately known risk types, Monopoly bank screening projects with privately known success rate, Regulator screening public utilities with privately known marginal cost, etc.

- Since θ^H chooses $\left(q^H, t^H\right)$ over $\left(q^L, t^L\right)$, we have

$$\theta^H v\left(q^H\right) - t^H \geq \theta^H v\left(q^L\right) - t^L.$$

- Similarly for θ^L

$$\theta^L v\left(q^L\right) - t^L \geq \theta^L v\left(q^H\right) - t^H.$$

- These constraints are called *incentive compatibility* constraints.

- If the agent can secure a payoff of $\underline{u}$ by not trading with the principal at all, then we also must have:

$$\begin{aligned} \theta^H v\left(q^H\right) - t^H &\geq \underline{u}, \\ \theta^L v\left(q^L\right) - t^L &\geq \underline{u}. \end{aligned}$$

  - These constraints are known as *individual rationality or participation constraints*.

Summary of the problem

The principal's problem is:

$$\max_{\{(q^H,t^H),(q^L,t^L)\}} \lambda\left(t^H - c\left(q^H\right)\right) + (1-\lambda)\left(t^L - c\left(q^L\right)\right)$$

subject to

$$\theta^H v\left(q^H\right) - t^H \geq \theta^H v\left(q^L\right) - t^L,$$

$$\theta^L v\left(q^L\right) - t^L \geq \theta^L v\left(q^H\right) - t^H,$$

$$\begin{aligned} \theta^H v\left(q^H\right) - t^H &\geq \underline{u}, \\ \theta^L v\left(q^L\right) - t^L &\geq \underline{u}. \end{aligned}$$

- This is a simple model of adverse selection:
  - The agent has private information at the time when the principal proposes the contract.
  - This private information gives (at least some type of) the agent some surplus even if the principal make a take-it-or-leave-it offer.
  - Model generates a genuine sharing of surplus.
  - Will the outcome be socially efficient as in the case where the principal knows θ?
- The more general theory framework encompassing this model is called Mechanism Design.
  - Treated in research track Microeconomics 4 in detail.

First- vs. Second-degree price discrimination

- Recall from last lecture that under first-degree price discrimination the monopolist could use a two-paritt tariff to extract all surplus from a buyer, i.e. choose $\left(\widehat{q}^i, \widehat{t}^i\right)$ for $i \in \{H, L\}$ such that:

$$\begin{aligned} \widehat{q}^i \text{ is efficient:} \quad & c'\left(\widehat{q}^i\right) = \theta^i v'\left(\widehat{q}^i\right), \\ \widehat{t}^i \text{ captures suplus} \quad : \quad & \widehat{t}^i = \theta^i v\left(\widehat{q}^i\right). \end{aligned}$$

- What goes wrong if the monopolist attempts this in the case where the type is not observable?
- Check if the incentive constraints hold

Analyzing the model

- We start with two observations:
- First, IC for H must bind.
  - If not, then you can increase profit by increasing t^H a little
  - Note, IR cannot bind for H, since she could get a positive payoff by choosing $\left(q^L, t^L\right)$

- Second, IR for L must bind
  - If not, then you could increase profit by increasing both prices by the same amount
- Using these, we can solve the model

- Use IR of type L to solve

$$t^L = \theta^L v\left(q^L\right).$$

- Use IC of H to solve

$$t^H = t^L + \theta^H v\left(q^H\right) - \theta^H v\left(q^L\right) = \theta^H v\left(q^H\right) - \left(\theta^H - \theta^L\right) v\left(q^L\right).$$

- But then:

$$\theta^H v\left(q^H\right) - t^H = \left(\theta^H - \theta^L\right) v\left(q^L\right) > 0 \text{ if } q^L > 0.$$

- We call $\left(\theta^H - \theta^L\right) v\left(q^L\right)$ the information rent of the high type.

- Hence the maximization problem becomes:

$$\max_{q^H,q^L}\{\lambda\left(\theta^H v\left(q^H\right)-\left(\theta^H-\theta^L\right)v\left(q^L\right)-c\left(q^H\right)\right)$$
$$+(1-\lambda)\left(\theta^L v\left(q^L\right)-c\left(q^L\right)\right)\}.$$

- FOC with respect to q^H :

$$\theta^H v'\left(q^H\right)=c'\left(q^H\right).$$

- We see from this that q^H is chosen efficiently.

- FOC with respect to q^L :

$$-\lambda\left(\theta^H-\theta^L\right)v'\left(q^L\right)=(1-\lambda)\left(c'\left(q^L\right)-\theta^L v'\left(q^L\right)\right).$$

- From this we see that q^L is smaller than the efficient level. This helps monopolist extract more profit from the high type.

Conclusions from the abstract model

- This abstract framework allows us to make some observations, that turn out to hold very generally in this type of models:
  - Higher types buy larger quantities, or better qualities, and earn a positive information rent
  - Low type earns no rents and is indifferent between participating and not
  - The allocation for the low type is distorted
- Profit maximizing solution hence trades off efficiency and rent extraction.

Applications

- We next consider the two main manifestations of screening by a monopolist seller:
- Quantity discounts
- Versioning:
  - Vertical vs Horizontal Differentiation
  - Quality Premia
  - Damaged Goods

Numerical Example on Quantity Discounts

- To illustrate quantity discounts, let us specify the model of the previous lecture as follows:
- $\theta^H = 2, \theta^L = 1.$
- $v(q) = \sqrt{q}.$
- $c(q) = cq.$
- $\Pr\{\theta = \theta^H\} = \frac{2}{5}.$

- Under full information, the monopolist sets:

$$\theta^i v'\left(\widehat{q}^i\right) = c'\left(q^i\right) \text{ for } i \in \{1, 2\}.$$

Hence

$$2 \times \frac{1}{2} \frac{1}{\sqrt{\widehat{q}^H}} = c,$$

or

$$\widehat{q}^H = \frac{1}{c^2},$$

and

$$\widehat{q}^L = \frac{1}{4c^2}.$$

- The corresponding transfers under full information are:

$$\widehat{t}^H = \frac{2}{c}, \widehat{t}^L = \frac{1}{2c}.$$

- Consider now the case where θ is private information to the buyer. If the monopolist chose $\left\{\left(\widehat{q}^H, \widehat{t}^H\right), \left(\widehat{q}^L, \widehat{t}^L\right)\right\}$, type θ^H would choose $\left(\widehat{q}^L, \widehat{t}^L\right)$. the resulting information rent to θ^H is

$$\left(\theta^H - \theta^L\right) v\left(\widehat{q}^L\right) = \frac{1}{2c}.$$

- Hence if $\left(\widehat{q}^L, \widehat{t}^L\right)$ is available to the buyers, the maximal t^H that will induce θ^H to choose $\left(\widehat{q}^H, t^H\right)$ over $\left(\widehat{q}^L, \widehat{t}^L\right)$ is

$$t^H = \widehat{t}^H - \left(\theta^H - \theta^L\right) v\left(\widehat{q}^L\right) = \frac{3}{2c}.$$

- The profit to the firm at $\left\{\left(\widehat{q}^H, t^H\right), \left(\widehat{q}^L, \widehat{t}^L\right)\right\}$ is given by:

$$\frac{2}{5}\left(\frac{3}{2c} - \frac{1}{c}\right) + \frac{3}{5}\left(\frac{1}{2c} - \frac{1}{4c}\right) = \frac{7}{20c}.$$

- How can the monopolist improve profit?
  - The only problem is the information rent going to θ^H.
  - The rent $\left(\theta^H - \theta^L\right) v\left(q^L\right)$ can be reduced by decreasing q^L.
  - For example, if $q^L = 0$, then θ^H gets no information rent.
  - Hence $\left\{\left(\widehat{q}^H, t^H\right), (0, 0)\right\}$ is an incentive compatible offer.
  - You can calculate the profit from this to be $\frac{2}{5c} > \frac{7}{20c}$.
- Even better: Choose q^L from the formula in the previous lecture:

$$-\lambda\left(\theta^H - \theta^L\right) v'\left(q^L\right) = (1-\lambda)\left(c'\left(q^L\right) - \theta^L v'\left(q^L\right)\right).$$

- Plugging in the functional forms, the values for θ^i and $\lambda = \frac{2}{5}$, we get:

$$-\frac{2}{5}\frac{1}{2\sqrt{q^L}} = \frac{3}{5}\left(c - \frac{1}{2\sqrt{q^L}}\right),$$

or

$$q^L = \frac{1}{36c^2}.$$

- Hence we can compute the optimal menu to be $\{(\widehat{q}^H, \widehat{t}^H), (\widehat{q}^L, \widehat{t}^L)\} = \{(\frac{1}{c^2}, \frac{11}{6c}), (\frac{1}{36c^2}, \frac{1}{6c})\}$.
- Total profit is then

$$\frac{22}{30c} - \frac{2}{5c} + \frac{3}{30c} - \frac{3}{5 \cdot 36c} = \frac{25}{60c} > \frac{2}{5c}.$$

- Notice that if $\lambda \geq \frac{1}{2}$, it is optimal to set $q^L = 0$ and to sell only to θ^H at the monopoly price.
  - You can see this from the fact that the derivative of the monopolist's profit is negative in q^L for all $q^L \geq 0$.
- Finally, we can compute the implied per unit price in the two options:

$$\frac{t^L}{q^L} = 6c,$$

$$\frac{t^H}{q^H} = \frac{11}{6}c.$$

Hence first q^L units are sold at a higher per unit price than the next $\left(q^H - q^L\right)$ units. We say, that the model shows quantity discounts in this case.

Endogenous Quality Choice

- Let us modify the model slightly.
- Here, it is more natural to interpret q as quality.
- $\theta^H = 2, \theta^L = 1$.
- $v(q) = q$.
- $c(q) = \frac{1}{2}q^2$.
- $\Pr\{\theta = \theta^H\} = \frac{2}{5}$.
- The full information quantities and transfers are $\left\{\left(\widehat{q}^H, \widehat{t}^H\right), \left(\widehat{q}^L, \widehat{t}^L\right)\right\} = \{(2,4), (1,1)\}$. The information rent to θ^H is $\left(\theta^H - \theta^L\right) q^L = 1$.

- $\{(2,3),(1,1)\}$ is incentive compatible and yields expected profit of $\frac{2}{5}(3-2)+\frac{3}{5}\left(1-\frac{1}{2}\right)=\frac{7}{10}$.
- By offering $\{(2,4),(0,0)\}$, the profit is increased to $\frac{8}{10}$.
- Again, the optimal offer to θ^L can be calculated from

$$-\lambda\left(\theta^H-\theta^L\right)v'\left(q^L\right)=(1-\lambda)\left(c'\left(q^L\right)-\theta^L v'\left(q^L\right)\right).$$

or

$$-\frac{2}{5}=\frac{3}{5}\left(q^L-1\right)\ \Leftrightarrow q^L=\frac{1}{3}.$$

- The profit at $\left\{\left(2,\frac{11}{3}\right),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$ is $\frac{2}{5}\left(\frac{11}{3}-2\right)+\frac{3}{5}\left(\frac{1}{3}-\frac{1}{18}\right)=\frac{2}{3}+\frac{1}{5}-\frac{3}{90}=\frac{75}{90}=\frac{5}{6}$.
- Notice that now the "per unit price" of the first $\frac{1}{3}$ quality units is 1 whereas for the higher quality level $q^H=2$, the per unit price is $\frac{11}{6}$. We say that this model of vertical quality differentiation displays quality premia.

- An extreme form of quality differentiation happens when the seller damages her goods intentionally and perhaps at a cost
- Various examples of such strategies are discussed in Deneckere and McAfee (1996): "Damaged goods", Journal of Economics and Management Strategy.

To sum up:

- We demonstrated in the simple two-type model two features of non-linear pricing:
  - Quantity discounts
  - Quality premia.
- Do these properties hold more generally?
  - For quantity discounts: Maskin and Riley (1984), "Monopoly with Incomplete Information", *Rand Journal of Economics*.
  - For quality premia: Mussa and Rosen (1978), "Monopoly and Product Quality", *Journal of Economic Theory*.

Further readings

- For a text-book treatment of menu pricing, see e.g. Belleflamme and Peitz: “Industrial Organization”, chapter 9.
- Screening models are also analyzed in advanced microeconomics text books, such as Jehle and Reny: "Advanced Microeconomic Theory" Chapter 8, or Mas-Colell, Whinston and Green: "Microeconomic Theory", Chapter 13.
- For a much deeper discussion about the type of models treated in this lecture, see Salanie: "The Economics of Contracts", MIT Press, or Bolton and Dewatripont: "Contract Theory", MIT Press.
- Seminal articles on monopoly pricing under asymmetric information are Mussa and Rosen (1978): "Monopoly and Product Quality", Journal of Economic Theory, and Maskin and Rilery (1984): "Monopoly with Incomplete Information", Rand Journal of Economics.

Bundling

- So far, we have considered menus with one good
- When the firm is producing multiple goods, another alternative is to bundle them together
- Why would a firm want to do that?

- Potential reasons to bundle separate goods:
  - Complementary products
    - A very natural reason for bundling. Extreme example: right and left shoes
  - Anti-competitive behavior
    - Extending market power across markets, entry deterrence (Microsoft: OS and other software products)
    - Competitive authorities take a grim view of this.
  - Price discrimination strategy that increases rent extraction opportunities for the seller.
    - Exploit different buyers differential willingness to pay
    - We will consider this next.

Bundling: Examples

- Subscriptions for cable TV channels.
  - Do you want to sell larger packages of channels at a discount relative to sum of individual channel prices?
  - Do you offer individual channels at all?
  - If only a large package is available, we talk about pure bundling.
  - If buyers can select packages or individual channels, we talk about mixed bundling.
- Mobile handsets and operator contracts.
  - Different regulations apply in different countries.

Bundling: Examples

- Bundling of computer operating system with other software (Windows with IE, Office etc.)
- Online and paper newspaper (HS, NYTimes,...).
- Hotel room with or without breakfast, with or without free wifi etc.
- Selling packages of academic journals to university libraries.
- Copy machines and maintenance contracts (Kodak), elevator sales and maintenance contracts (Kone), computer mainframes and consulting contracts (IBM).

Simple Example of Bundling

- Suppose a monopolist sells two different goods in a single market consisting of buyers with different valuations for the goods.
- The valuations are private information to the buyers.
- For simplicity, assume that the buyers have either a high or a low willingness to pay for each of the products.
- Let $v^i \in \{v^H, v^L\}$ with $v^H > v^L$ denote a buyer's willingness to pay for product i with $i \in \{1, 2\}$.

Simple Example Continued

- We can write a table for the probabilities of valuations as follows:

$$\begin{array}{ccc} v^1 \backslash v^2 & v^H & v^L \\ v^H & \pi^H & \frac{1}{2}\pi^M \\ v^L & \frac{1}{2}\pi^M & \pi^L \end{array}.$$

- Here π^H stands for the probability that a buyer has valuation v^H for both of the goods, π^L for the probability that valuation is v^L for both goods and π^M for the probability of mixed valuations.
- The case where $\pi^M = 0$ stands for perfectly correlated valuations across the goods. The case $\pi^H = \pi^L = 0$ stands for negatively (perfectly) correlated values.
- If $\pi^H \pi^L = \frac{1}{4}\left(\pi^M\right)^2$, we have independently distributed values across products. (For example if $\pi^H = \pi^L = \frac{1}{2}\pi^M = 1/4$).

Simple Example Continued

- Let's assume that the valuations of the buyers across the two goods are additive so that her willingness to pay for both goods is $v^1 + v^2$.
- The monopolist must decide whether to sell the two goods separately at prices p^1 and p^2, or whether to engage in pure bundling, i.e. sell them as a package at price $p^{1,2}$ or whether to give the buyers the option of either buying separately or as a package.
- Clearly in the last case, we must have $p^{1,2} < p^1 + p^2$ if buyers cannot be prevented from buying the two goods separately.

Simple Example Continued

- What is the optimal strategy under positively correlated values?
- What is the optimal strategy under negatively correlated values?
- What is the optimal strategy under independent values?

Simple Example Continued

- In the case of perfectly correlated valuations all buyers have high value for both products, or low value for both products.
  - It does not matter whether monopolist sells them separately or as a bundle - every buyer buys both or nothing in any case.
- In the case of pure negative correlation, $\pi^H = \pi^L = 0$, and so all consumers have valuation $v^H + v^L$ for the bundle consisting of both goods.
  - Seller extracts all surplus by selling as a bundle at price $v^H + v^L$!
  - This is clearly not possible by separate pricing.
- What about the independent case?
  - For example, let $\pi^H = \pi^L = \frac{1}{2}\pi^M = 1/4$ and $2v^L < v^H < 3v^H$.
  - Compute profit with bundle price $v^H + v^L$ and compare to separate pricing.
  - Bundling increases profits, but buyers retain some rents.

Independent valuations and Bundling*

- We saw that bundling can increase profits even with independent valuations
  - Intuition: Bundling reduces consumer heterogeneity and thereby allows better rent extraction
- For more detailed analysis, we move to a slightly richer setting
- An additional insight: mixed bundling can be even more profitable than pure bundling (sell separately + as a "discount price"-bundle)
- *The presentation here is dense; consider this as extra material. For a more detailed presentation of the next 10 slides, please consult pages 271-281 in the Belleflamme and Peitz book (see course syllabus for full reference)

- A monopolist sells two products $i \in \{1, 2\}$.
- There is a continuum of buyers that have independent valuations for the two products.v^i.
    - Each v^i is distributed on $[0, 1]$
    - Each v^i has a distribution function $F^i\left(v^i\right)$ with a density $f^i\left(v^i\right)$.
- Suppose the monopolist sets prices separately for the two products: p^1, p^2.
- Assume that production cost is zero (so that valuation is really the net valuation over production cost).

- At price p^i, the monopolist's profit in market i is:

$$p^i(1 - F^i(p^i)),$$

where $(1 - F^i(p^i))$ is the fraction of buyers with valuation above p^i.

- First order condition for optimal price:

$$p^{*i} \text{ solves } (1 - F^i(p^{*i})) - p^{*i} f^i(p^{*i}) = 0.$$

- Is it optimal for the monopolist to offer prices (p^{*1}, p^{*2}) with $p^{*1,2} = p^{*1} + p^{*2}$?
- Consider a change to prices $(p^{*1} + \varepsilon, p^{*2}, p^{*1,2})$.
  - In words, keep all other prices unchanged, just increase the price of good 1 by ε.

- What happens to total profit?
    - No change to buyers with $v^1 < p^{*1}$.
    - No change for buyers with $v^1 > p^{*1}$ and $v^2 > p^{*2}$.
    - Loss of sales to buyers with $p^{*1} < v^1 < p^{*1} + \varepsilon$ if $v^2 < p^{*2}$.
    - Gain in revenue of ε on those with $v^1 > p^{*1} + \varepsilon, v^2 < p^{*2} - \varepsilon$.
    - Gain in revenue of p^{*2} on those with $v^1 > p^{*1}$, $p^{*2} - \varepsilon < v^2 < p^{*2}$.

- Counting together the changes:

$$-\varepsilon p^{*1} f^1\left(p^{*1}\right) F^2\left(p^{*2}\right)+\varepsilon\left(1-F^1\left(p^{*1}+\varepsilon\right)\right) F^2\left(p^{*2}-\varepsilon\right)$$
$$+p^{*2}\left(1-F^1\left(p^{*1}\right)\right) \varepsilon f^2\left(p^{*2}\right).$$

- Since $F^2\left(p^{*2}-\varepsilon\right)=F^2\left(p^{*2}\right)-\varepsilon f^2\left(p^{*2}\right)$, $F^1\left(p^{*1}+\varepsilon\right)=F^1\left(p^{*1}\right)+\varepsilon f^1\left(p^{*1}\right)$, and $\left(1-F^1\left(p^{*1}\right)\right)-p^{*1} f^1\left(p^{*1}\right)=0$ (by monopolist's first order condition in the choice of p^{*1}), we have after ignoring terms of order ε^2 the net change as:

$$p^{*2}\left(1-F^1\left(p^{*1}\right)\right) \varepsilon f^2\left(p^{*2}\right)>0.$$

- Hence increasing one of the original separate monopoly prices results in an increase in profit.

Uniform distribution

- Assuming that v^i:s are drawn from the uniform distribution on $[0, 1]$, the model can be solved explicitly
- Start by deriving optimal monopoly prices for individual products, and compute associated profit
- Then consider optimal price if only pure bundling possible:
  - What is the demand function for the bundle?
  - What is the optimal price and associated profits?
- Finally, consider the mixed bundle.
  - Derive the demands for products 1 and 2 and for the bundle with some prices p^1, p^2, $p^{1,2}$
  - Argue that it is optimal to choose $p^1 = p^2 := p$
  - Find optimal p and $p^{1,2}$
- What kind of welfare effects can you identify?

Many Items for Sale

- What if the seller has more than two different products?
- Continue with the basic setting above.
- n items for sale.
- Valuation of each buyer for a collection $\{1, ..., k\}$ of the items is $v^1 + ... + v^k$.
- Assume that each v^i is an independent draw from the uniform distribution on $[0, 1]$.
- In other words, $F^I\left(v^i\right) = v^i$ for all i and all $0 \leq v^i \leq 1$.
- Easy to calculate the optimal monopoly price for single items to be $\frac{1}{2}$.

- We saw already that with $n = 2$, a local improvement in profits possible through bundling.
- One can compute the optimal mixed bundling solution explicitly (turns out, $p^1 = p^2 = \frac{2}{3},\ p^{1,2} = \frac{4-\sqrt{2}}{3} \approx 0.86$)
- What about $n = 3$? Can be done but gets harder
- $n = 4$? Can be done numerically.

- Is $n \to \infty$ even harder?
- To get full optimum, yes, but to get qualitative features of optimum, not so
- What can we say about the random variable $v = v^1 + ... + v^n$?
- If the v^i are independent , all with variance σ^2 and mean μ, then v has variance $n\sigma^2$.
- With uniform, $\mu = \frac{1}{2}$, $\sigma^2 = \frac{1}{12}$.
- On the other hand, the expected value of v is $n\mu$.
- Hence the willingness to pay per item $\frac{v}{n}$ has mean μ and variance $\frac{\sigma^2}{n}$.
- How does the aggregate demand function for a bundle of n goods change as n grows?
- Go back to your reading assignment 3...

Other modifications of the model

- Interrelated products
  - Bundle of products will become more attractive to buyers
  - At the same time, advantage of bundling strategy to the seller as compared to separate selling may diminish
- Correlated values
  - As our simple example above suggested, negative correlation makes bundling strategy more profitable
- Bundling and competition
  - Bundling can soften or increase competition.
  - See e.g. Belleflamme and Peitz: "Industrial Organization", Chapter 11.3.
- Marginal costs of production
  - Our example above has zero marginal cost (good approximation for information goods such as software)
  - A higher marginal cost of production makes bundling less attractive relative to separate selling (why?)

Conclusion on Price Discrimination

- Price discrimination can take many different forms as we have seen
- We have not covered all possibilities (e.g. behavior based pricing, exercise set 1 problem 4)
- Basic motive for monopolist seller: transform consumer surplus into profit.
  - Sometimes at the expense of efficiency.
- How successful this can be depends on:
  - Buyers' possibilities for undoing differentiation: breaking bundles and resale etc.
  - Legislative concerns.
- Not covered here, but also important: strategic product design.
  - Compatibility with competitors.
  - Differentiation to relax price competition.

Informed seller - uninformed buyers

- So far we have analyzed situations where buyers are better informed than the seller: they have private information on their own taste
- We now consider the opposite situation
- Seller has private information about the quality of the product
  - Does this lead to efficient trade?
  - Is seller's private information beneficial to her?
    - Problem is that buyers are suspicious about quality
  - Can the seller signal credibly the true quality level?

Setup

- A single seller offers a product of two potential qualities $q \in \left\{q^L, q^H\right\}$
- Assume the quality is given, and privately known by the seller (seller's type).
- Buyers do not know the quality, and assign probability λ for high quality so that expected quality is:

$$\lambda q^H + (1 - \lambda)\, q^L.$$

- (Opportunity) cost of selling is c^i, $i = L, H$. Assume $c^H > c^L$.
- A mass of identical buyers with unit demand and reservation utility equal to the quality of the product $r^i = q^i$, $i = L, H$.
- The consumers prefer a higher quality: $q^H > q^L$.
- Assume: $q^i > c^i$ for $i = L, H$. In other words, trading is always efficient.

Setup

- Formally, we can models this as a three stage game:
  - 1. stage: Nature draws the true value q from the known distribution (i.e. with probability λ we have $q = q^H$ and with $1 - \lambda$ we have $q = q^L$). Only the seller observes the true q.
  - 2. stage: Seller decides whether and which price to post
  - 3. stage: Buyer forms beliefs about q and makes purchase decision (buy / do not buy)
- Contrast: in the screening model, the uninformed player moves first (seller postes a menu of contracts)
- Here: the informed party moves first. This opens the possibility for signalling.
- How do the buyers form their beliefs? Let us illustrate...

Expectations and belief formation

- Suppose that the buyers expect that both types of seller set the same price p
  - Then the belief by the buyer upon observing p is that with probability λ we have $q = q^H$, and hence expected quality is

$$\lambda q^H + (1 - \lambda) q^L.$$

  - This is called pooling: both types use the same strategy
- Suppose that only low types offer a price p (and high types withdraw from the market)
  - The belief by the buyer upon observing price p is that quality is $q = q^L$ for sure
- Or, low type could offer p' and high type would offer $p'' \neq p'$
  - Then the buyer would know the quality upon observing price: separating case
- The point is: the strategy of the seller affects the belief of the buyer

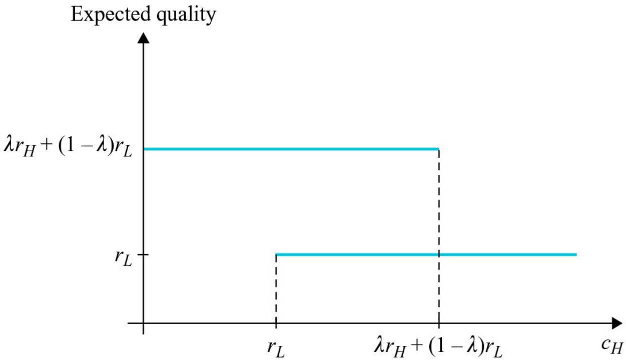
Possible equilibria

- Pooling equilibrium?
  - Then price must be $p = \lambda q^H + (1 - \lambda)\, q^L$ (why?)
  - Such an equilibrium is feasible if

$$c^H < \lambda q^H + (1 - \lambda)\, q^L,$$

    otherwise high type would withdraw.
- Equilibrium with adverse selection?
  - Low type sets price $p = q^L$ and high type withdraws
  - Such an equilibrium is feasible if $q^L < c^H$.
- For $q^L < c^H < \lambda q^H + (1 - \lambda)\, q^L$, both types of equilibria co-exist

Equilibrium prices for different opportunity cost of high type



Discussion

- When quality is not observed by the buyers, high-quality products may not be offered for sale at all
- What if there are more than two quality levels?
  - Full unraveling is possible, so that only the very lowest possible quality survives in the market
  - This is the logic in the famous "market for lemons" by Akerlof
- Why is fully separating equilibrium not possible here?
  - A low type would mimic.
  - Is it possible in some circumstances for the high type seller to signal high quality by choosing a high price?
  - Yes, but to make this work, mimicking must be more costly for the low type. We come back to this shortly...

Voluntary information disclosure

- A natural question to ask is: what if there is a credible way for the firms to publicly disclose their quality level?
  - Low type does not want to disclose
  - High type naturally wants to disclose
  - But then, if a buyer sees a seller who does not want to disclose, what should she conclude about quality?
- What if there are more than two types?
  - Unraveling result: all types disclose their quality, see Milgrom (1981): "Good news and bad news: Representation theorems and applications", Bell Journal of Economics.
  - This follows from an induction argument
  - Asymmetric information problem is solved
- But is such credible and costless disclosure feasible in reality?

Endogenous quality and moral hazard

- What if quality choice is endogenous?
- Assume the model as before, but in the beginning the seller can choose quality level
- Benchmark case: quality choice is observable
  - Since seller can extract all surplus, quality choice is efficient
  - If $r^H - c^H > r^L - c^L$, then seller chooses high quality
- What if quality choice is unobservable?
  - Seller always chooses low quality (why?)
- This is a very simple model of moral hazard
  - Instead of hidden type (as in adverse selection), we have hidden action

Signalling by price

- Let us now return to the idea that seller can signal its quality by price
- For this to work, signal must be credible, in other words, buyers must believe that high price truly signals high quality
- For this to be the case, mimicking high quality must be too costly for the low quality producer
- Possible reasons for such costs are, for example:
    - Repeat purchasing (true quality will be revealed in time) and reputational effects
    - Existence of some better informed consumers (increasing price will mean low quality producer will lose all such consumers)
    - ...

A model with some informed consumers: price signalling

- We will next demonstrate how signaling can work in a simple setting
- Assume the model as above with a mass of identical consumers with unit demand
- For simplicity, let $c^H = c^L := c$, and let $c < r^L < r^H$
- But now we assume that fraction λ of consumers know the true quality q
- Signalling models have typically multiple equilibria. Here we want to construct one.

- We want to construct a separating equilibrium: price posted by seller will reveal the true quality
- First consider a potential equilibrium, where high type chooses price $p^H = r^H$ and low type chooses price $p^L = r^L$
- If this is an equilibrium, then the buyers expect correctly that they get quality q^H at price p^H and q^L at price p^L
- Is this an equilibrium? We have to check if any player wants to deviate

- A high type gets the best possible deal, so naturally she does not want to deviate
- But a low type might want to mimick the high type. She wants to do that if

$$
\begin{aligned}
(1-\lambda)\left(r^H - c\right) &> r^L - c \\
&\Longleftrightarrow \\
\lambda &< \frac{r^H - r^L}{r^H - c} := \overline{\lambda}.
\end{aligned}
$$

- So, if $\lambda \geq \overline{\lambda}$, such deviations are not profitable. In that case, a fully separating equilibrium exists, where both types of seller can extract all surplus from the buyers

- What if $\lambda < \overline{\lambda}$?
- We can still construct a fully separating equilibrium, but the high type must lower its price to make mimicking less attractive for the low type
- If high type sets price $\overline{p}^H$, low type is indifferent between choosing r^L and p^H if

$$\begin{aligned} (1-\lambda)\left(\overline{p}^H - c\right) &= r^L - c \\ &\Longleftrightarrow \\ \overline{p}^H &= c + \frac{r^L - c}{1-\lambda} \end{aligned}$$

- To make sure this is an equilibrium, we must now also consider what happens if high type (or low) type deviate by setting price above $\overline{p}^H$

- To make sure that pricing above $\overline{p}^H$ is not profitable, we can assume that any deviation to higher prices would be interpreted by the buyers as low quality: they will not buy
- This is not really "assumption" about model, this is part of equilibrium description
- Formally, to define a "Perfect Bayesian Equilibrium" in a game like this, we must define beliefs of the buyers for all possible prices (also "out-of-equilibrium" prices) in such a way that sellers set optimal prices and all beliefs are consistent with their behavior
- Signalling models have typically a large number of different equilibria. Our purpose here is to construct just one equilibrium.
- See additional material of game theory for this

- To summarize this model:
  - when λ is sufficiently high, there is an equilibrium where high type sets price $p^H = r^H$ and low type sets price $p^L = r^L$
  - When λ is smaller, there is still a separating equilibrium, where high type must lower price in order to prevent low type from mimicing

Summary of models with privately informed seller

- When seller has private information about quality of product, this may lead to market break-down (adverse selection)
- This may also lead to choice of too low quality by sellers (moral hazard)
- Voluntary disclosure of quality can be helpful, if technologically feasible
- Signalling by prices can also work, if mimicking is sufficiently costly for a low quality producer
  - This is the case, e.g., when some consumers are informed about the quality
  - This makes high price less attractive for the low type, since she would lose all informed consumers
- Signalling can also work through other channels than prices:
  - For example, high quality firm can signal through costly advertising, even when advertising is not directly informative (see literature starting with Nelson (1974): "Advertising as information", Journal of Political Economy)

Further readings

- For a more detailed analysis of bundling, see McAfee, McMillan, and Whinston (1989): “Multiproduct Monopoly, Commodity Bundling, and Correlation of Values”, Quarterly Journal of Economics.
- Classical information economics papers relating to the case, where seller knows quality better than buyers are Akerlof (1970): ”The Market for Lemons: Quality Uncertainty and the Market Mechanism”, Quarterly Journal of Economics, and Spence (1973): ”Job market signaling”, Quarterly Journal of Economics.