Aalto University
School of Business

Intermediate Microeconomics

Tools for Decision Analysis

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Tools for Economic Decisions

- Expected value (*odotusarvo*)
- Present value (*nykyarvo*)
- Decision trees (*päätöspuut*)
- Sensitivity analysis (*herkkyysanalyysi*)
- Value of information
- Certainty equivalent (*varmuusekvivalentti*)
- Real options (*reaalioptiot*)

Decision Analysis

- What is the objective, how are results measured?
- What are the relevant facts?
- What are the relevant choices and their consequences?

So you've found the best alternative course of action.
Additional questions:

- How sensitive is the "best decision" to uncertainty in information and assumptions?
- Is it worth getting more information?
- What could be the unintended consequences?

Decision Trees

Elements

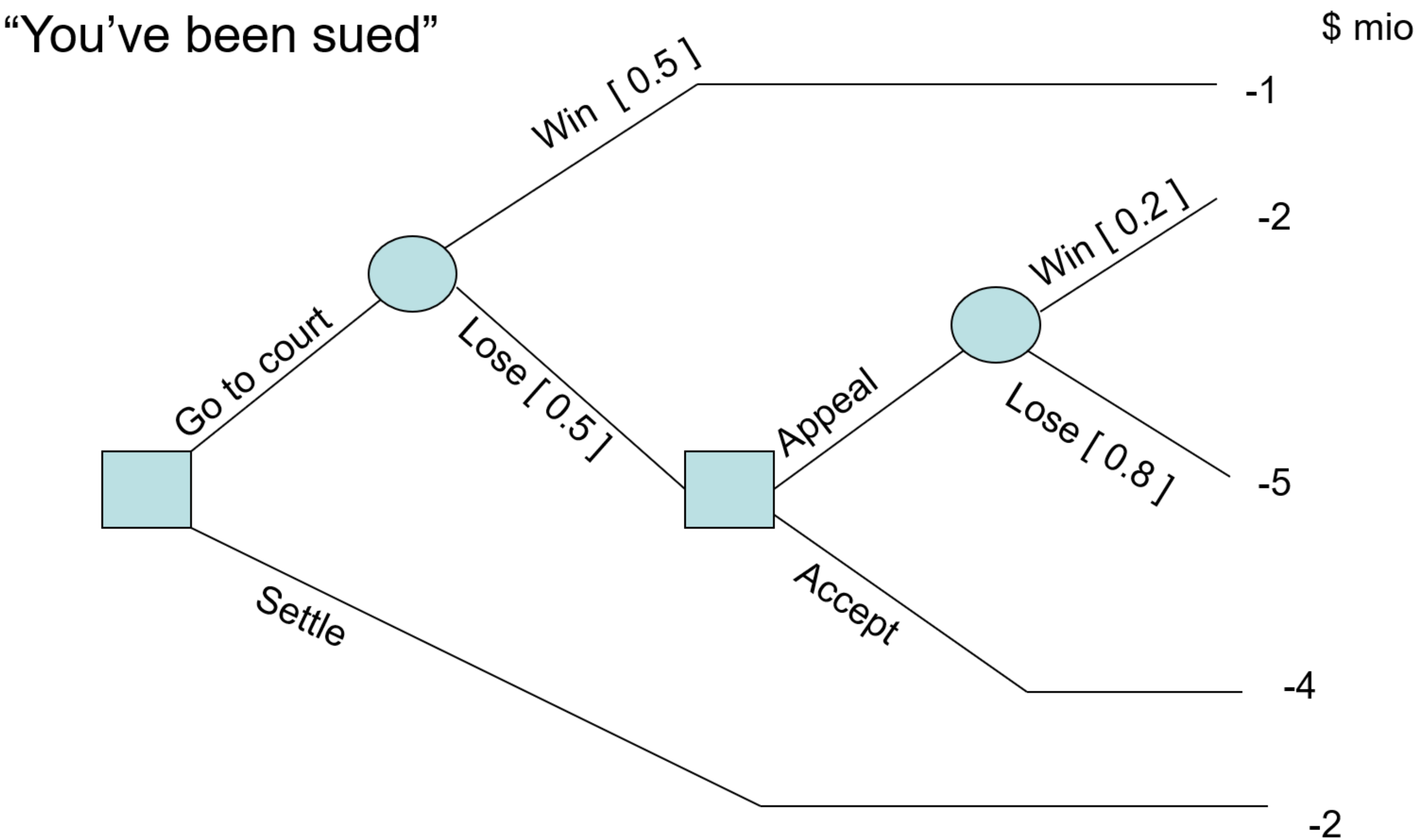
1. Decision nodes (*päätösnoodi*)
2. Outcomes (*tulema*) as branches
3. Payoffs (*tulos*) are final values of the objective
4. Chance nodes (*satunnaisnoodi*), probability for each branch

Decision tree is a method for analyzing decisions (strategic or not)

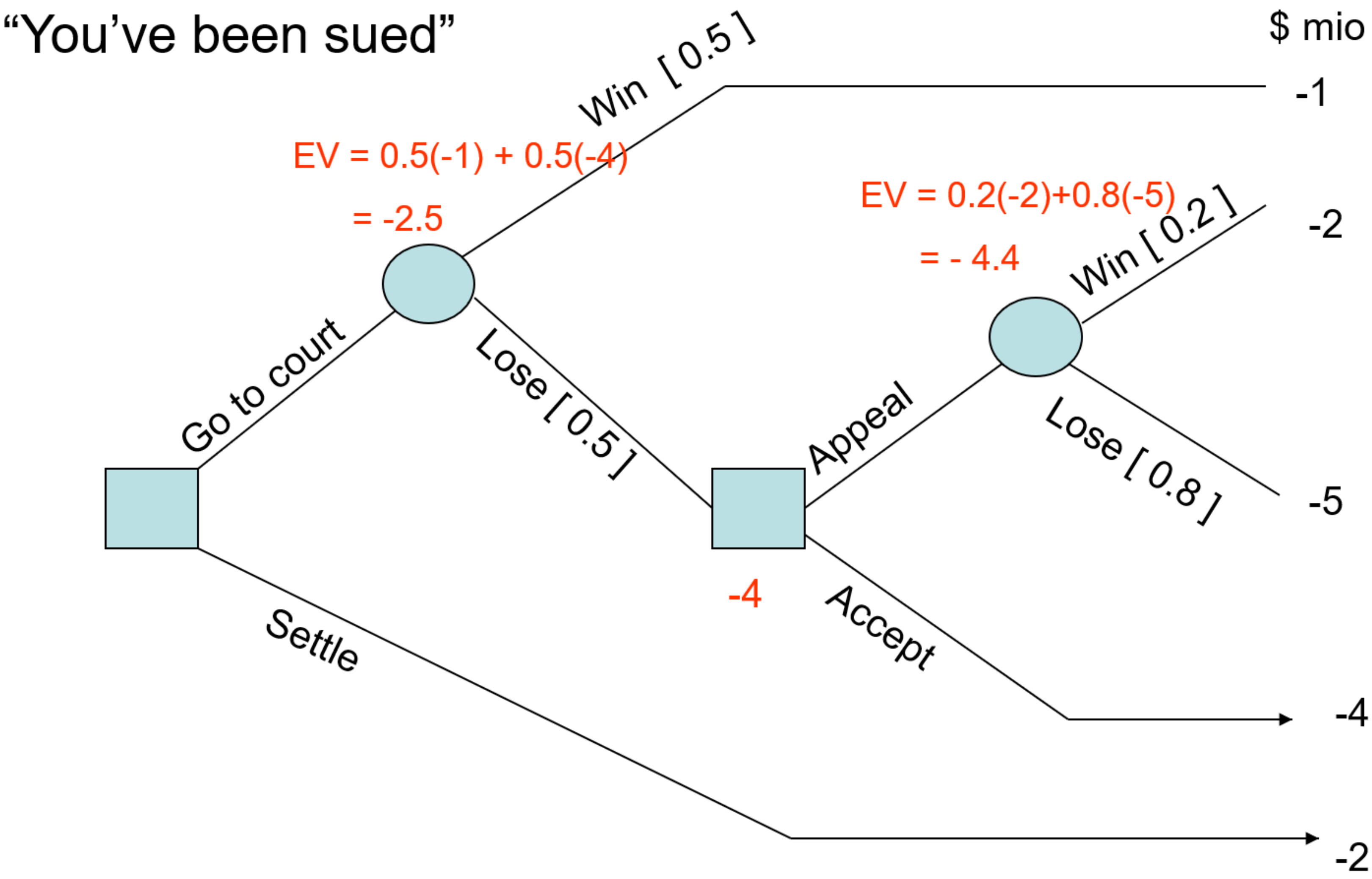
- State assumptions, clarify existing alternatives
- Force quantification of uncertainty
- Facilitate communication between decision-makers
- Encourage sensitivity analysis

Solved with backward induction

Decision Trees: Example



Decision Trees: Example



Calculate EV at chance nodes, choose best EV at decision nodes

Decision Trees: Example assumptions

"You've been sued"

- Plaintiff offers to settle for \$2m
- Legal costs \$1m per court
- Probability of winning at lower court 50%
- Win and pay no damages, Lose and pay \$3m to plaintiff
- Loser can appeal to higher court (but plaintiff wouldn't)
- Appeal has 20% probability of winning

Decision Trees: Example payoffs

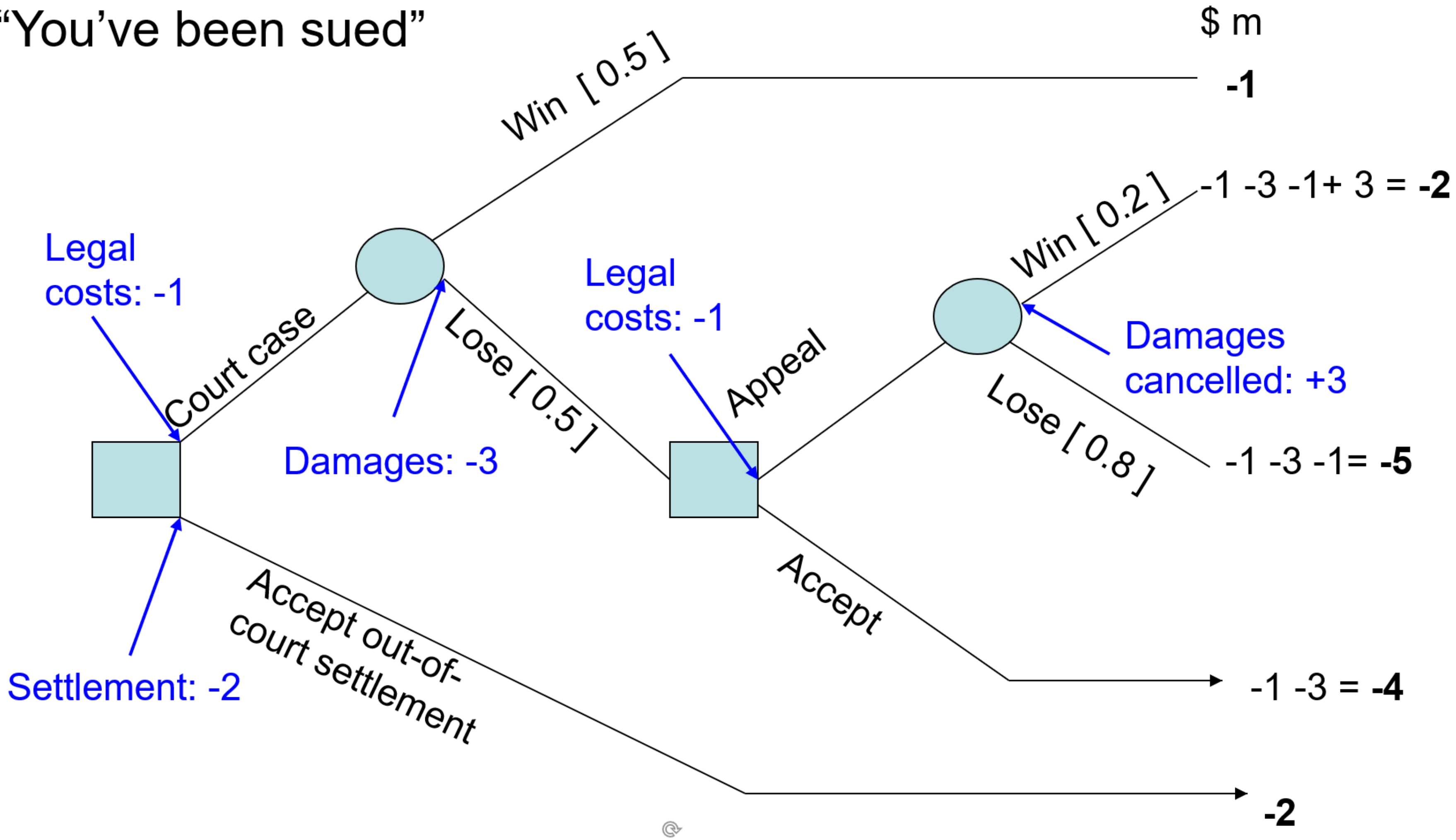
"You've been sued"

Payoffs for each possible outcome ($m)

- Go to court, Win: -1
- Go to court, Lose, Appeal, Win: $-1 - 1 = -2$
- Go to court, Lose, Appeal, Lose: $-1 - 1 - 3 = -5$
- Go to court, Lose, Accept: $-1 - 3 = -4$
- Settle: -2

Decision Trees: Example with payoff details

"You've been sued"



Final payoff is the sum of all costs and benefits along the branch

Decision Trees: Another example

3G Auctions:

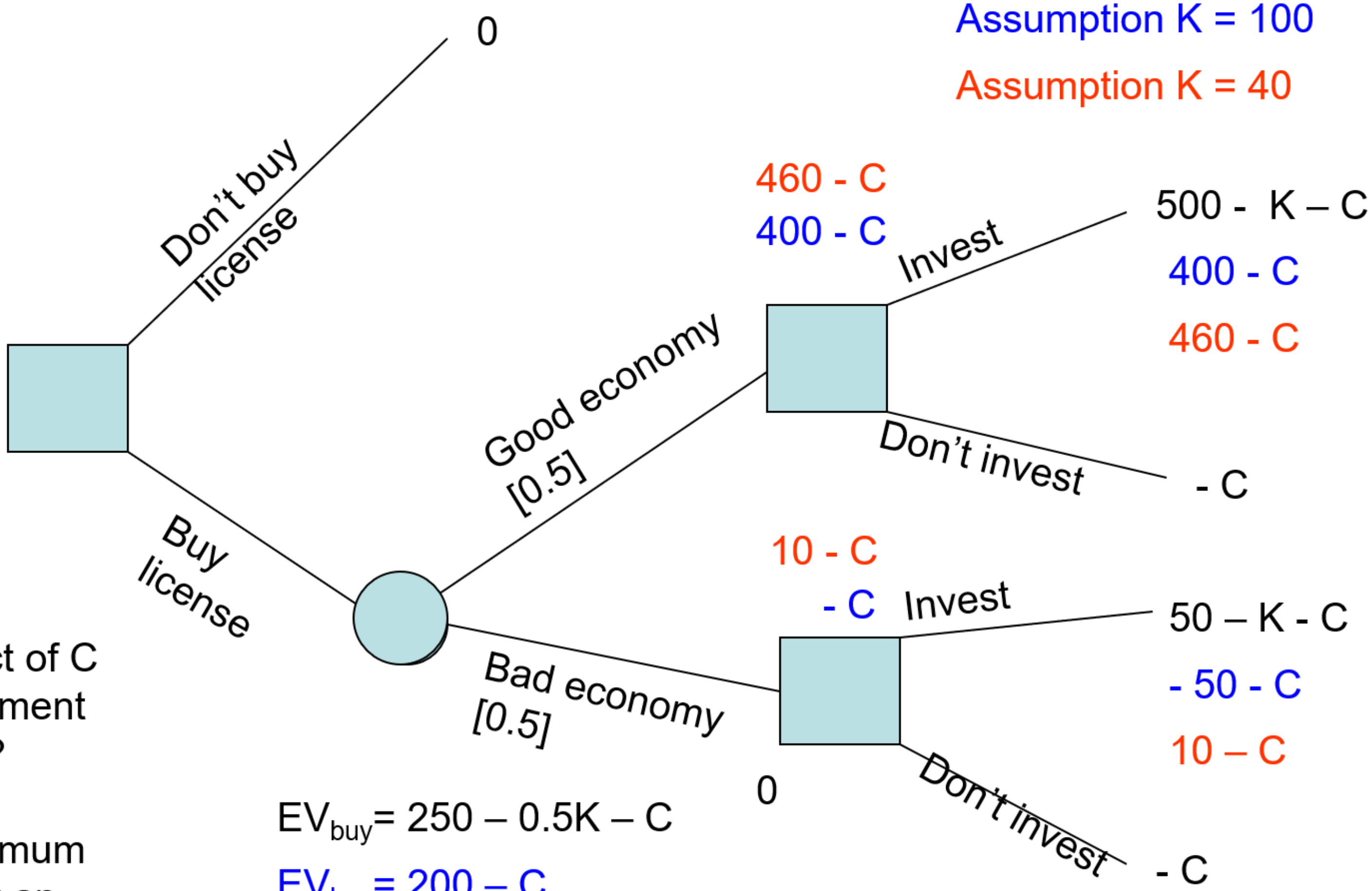
The sunk cost fallacy

K = 3G investment cost

C = Price for the 3G license

Assumption K = 100

Assumption K = 40



Q1. Effect of C on investment decision?

Q2. Maximum price that an operator should pay?

EV_{buy}= 250 – 0.5K – C

EV_{buy}= 200 – C

EV_{buy}= 235 – C

Sensitivity analysis

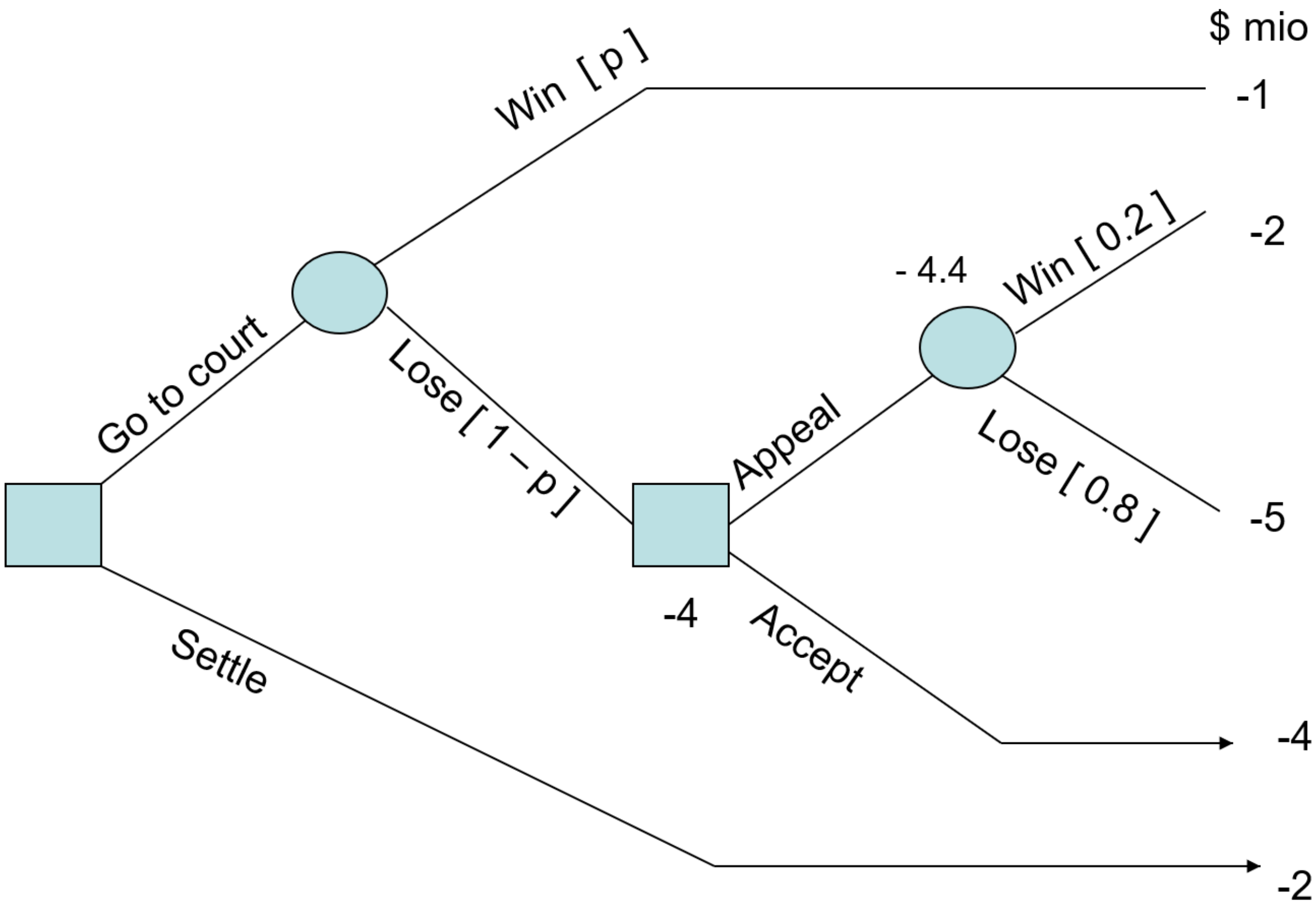
How much does our objective value (e.g. EV) change if an assumption is wrong?

How much can an assumption be wrong before the optimal decision would change?

Baseline assumptions: parameter values in the main case

- Replace the baseline value with a parameter, such as “p” or “x”
- Solve the optimal decision as a function of the parameter
- In what range can parameter vary for baseline decision to be optimal
  (many parameters: region of parameter space instead of range)

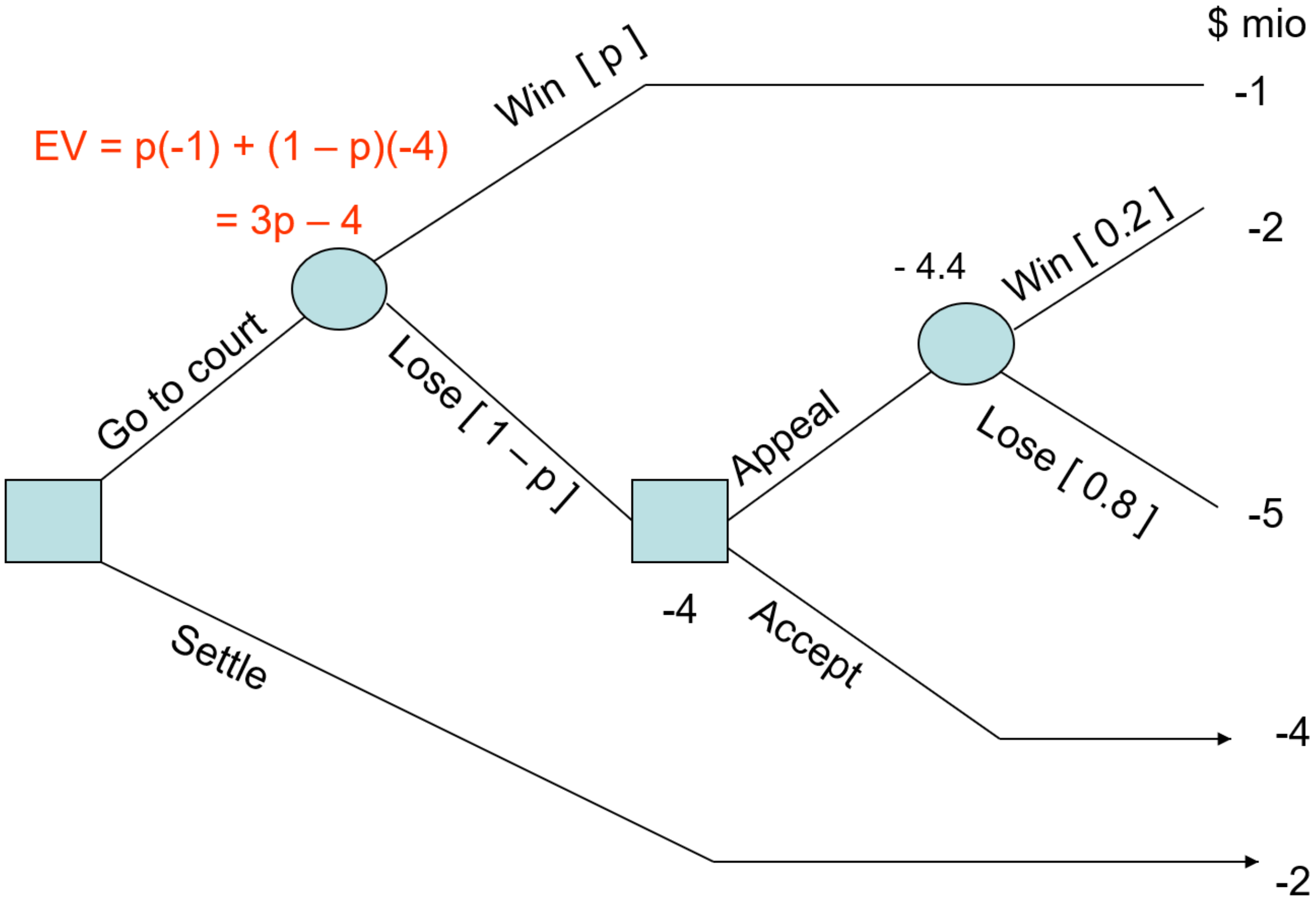
Sensitivity analysis: Example



What if our assumptions are wrong?

Suppose the true win probability is *p*

Sensitivity analysis: Example



What if our assumptions are wrong?

Suppose the true win probability is *p*

Better to settle if $3p - 4 < -2 \Leftrightarrow p < 2/3$

Value of Information

What is the impact the information is expected to have on the value of optimal decision?
Value of information = difference between expected value with and without the information.

Information has no economic value if it cannot affect any decision (unless “nice-to-know” is posited as an objective)

Example “you’ve been sued” continued:
- How valuable would it be to know the winner of the lower court case in advance?
- What about the appeal court case?

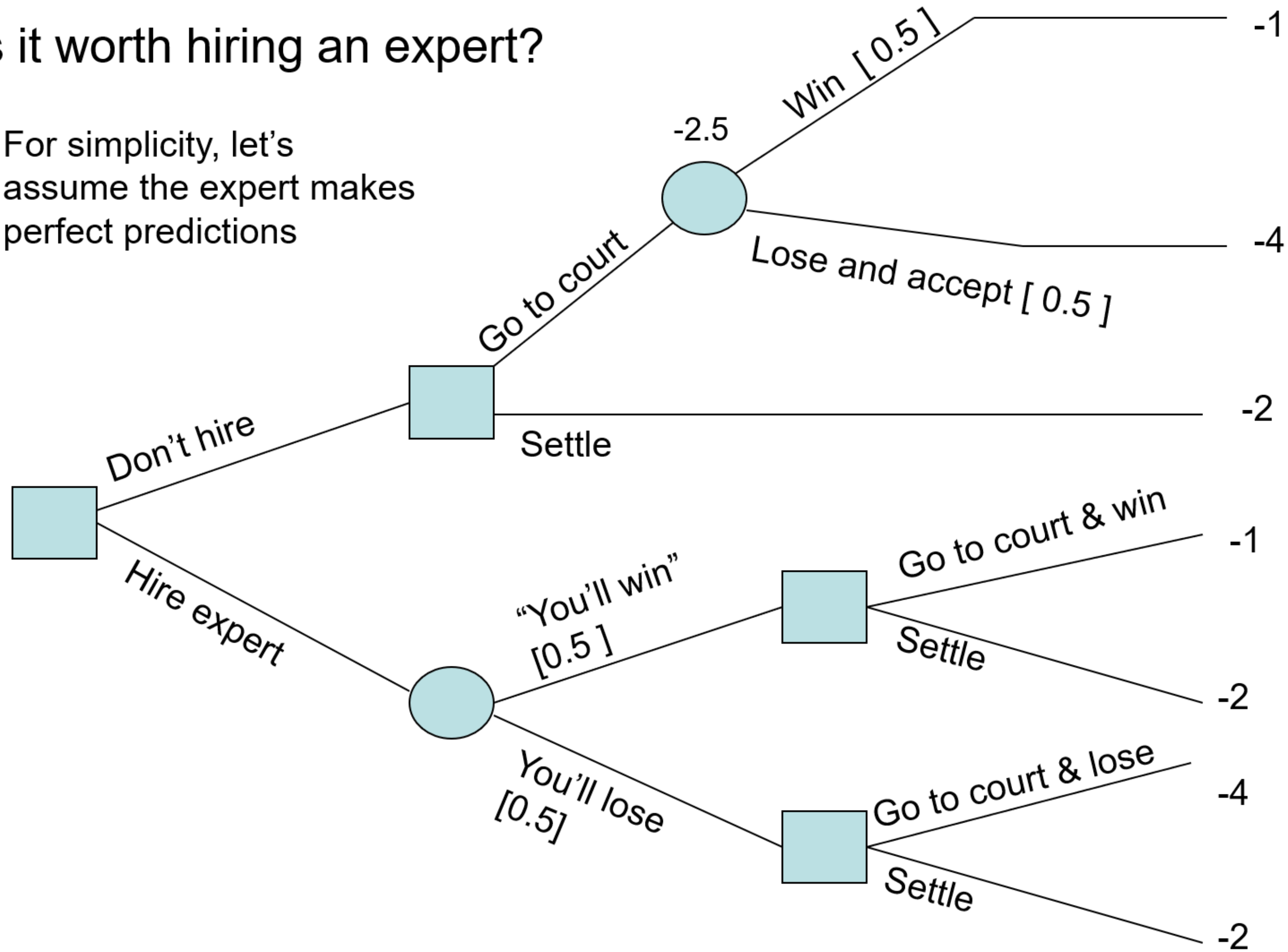
Value of Information: Example

"You've been sued"

Is it worth hiring an expert?

For simplicity, let's assume the expert makes perfect predictions

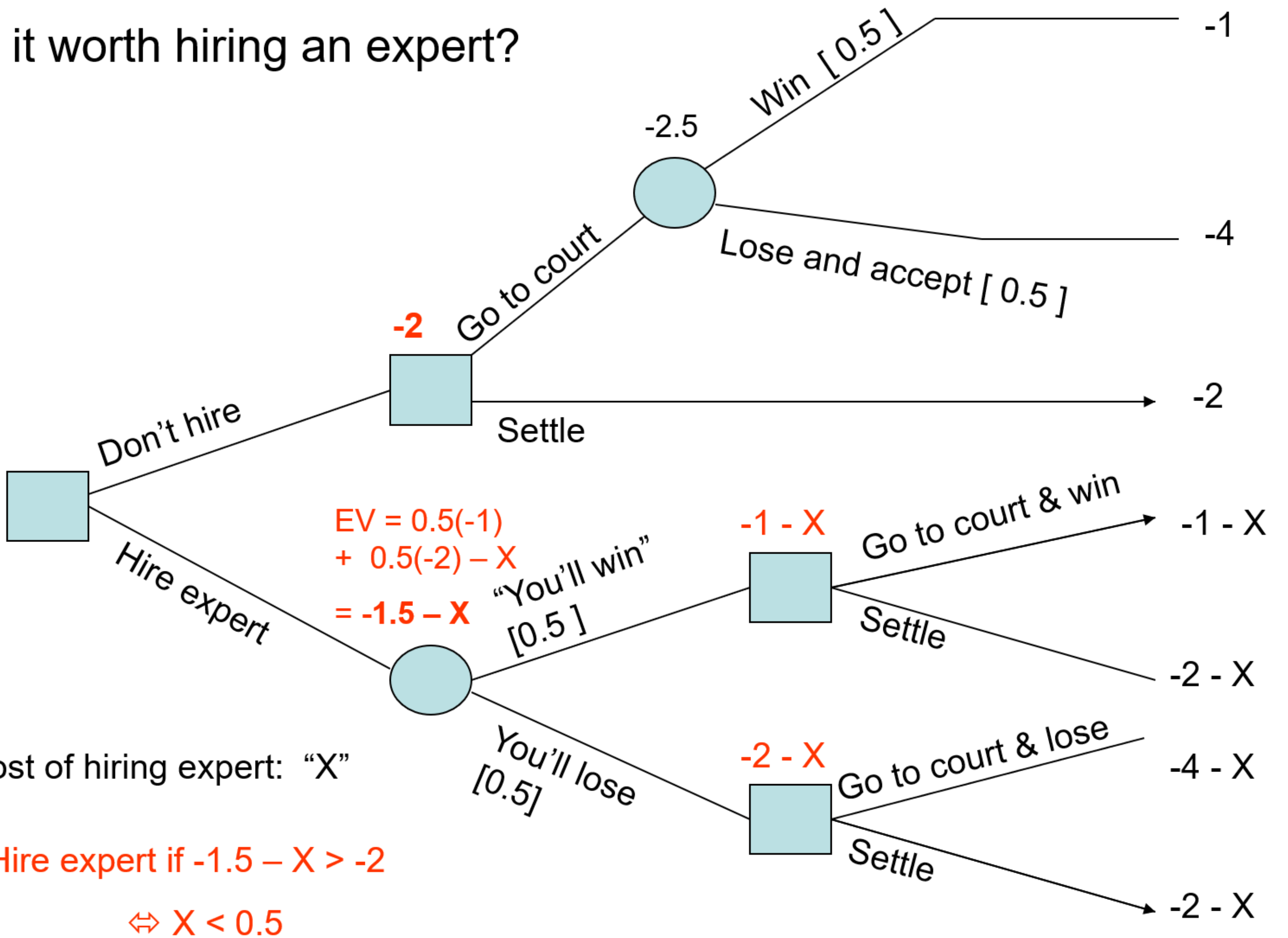
Payoff before cost of expert $ mio



Value of Information: Example

"You've been sued"

Is it worth hiring an expert?

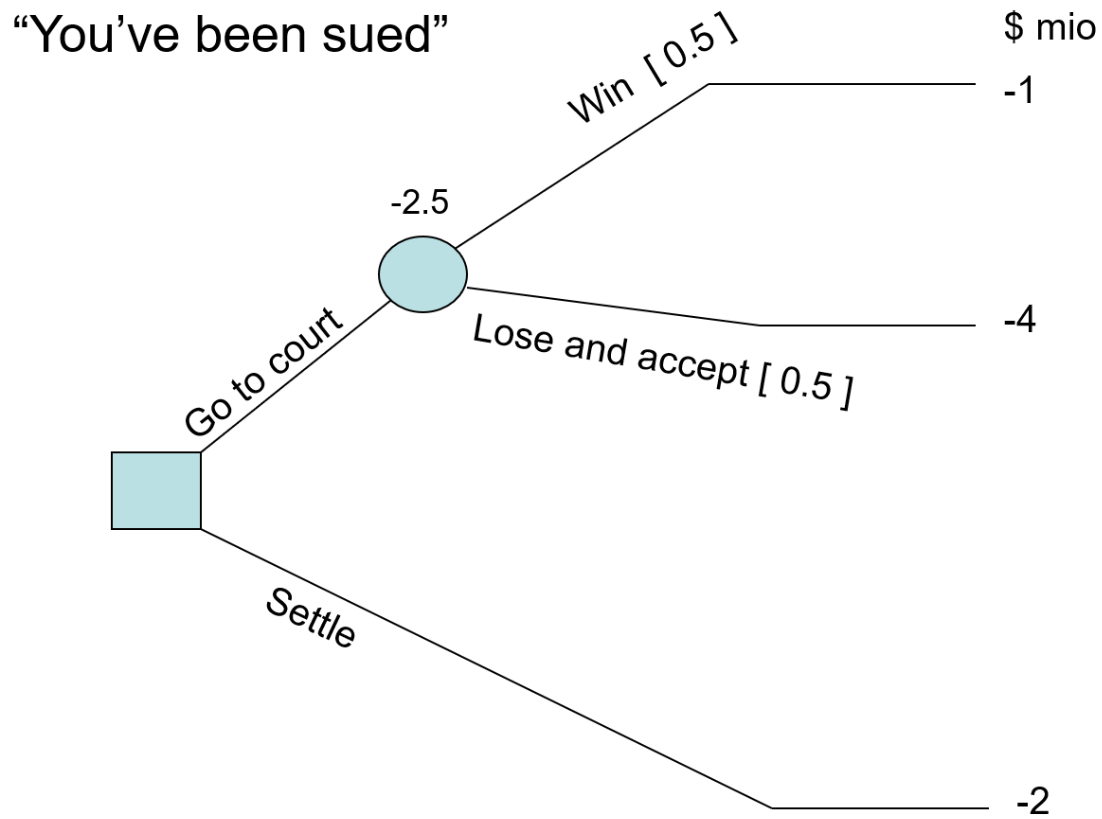


Cost of hiring expert: "X"

Hire expert if -1.5 – X > -2

⇔ X < 0.5

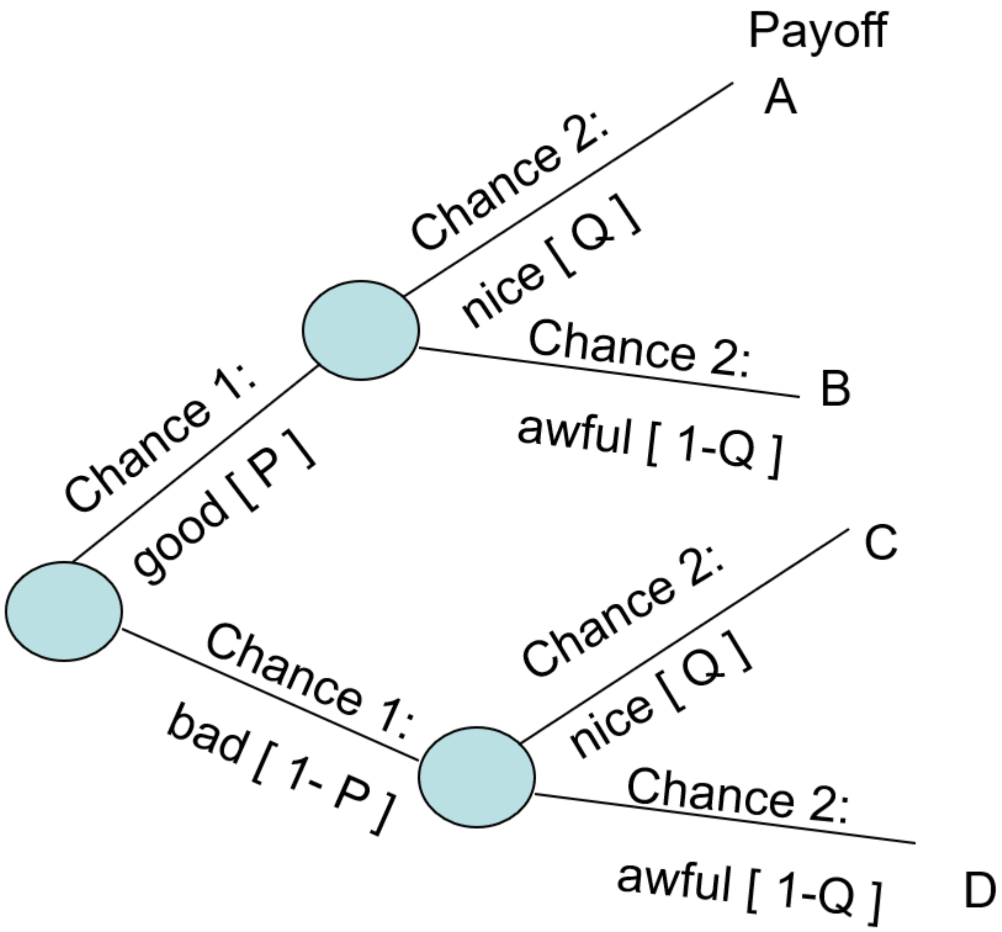
Decision Tree: Simplify by pruning

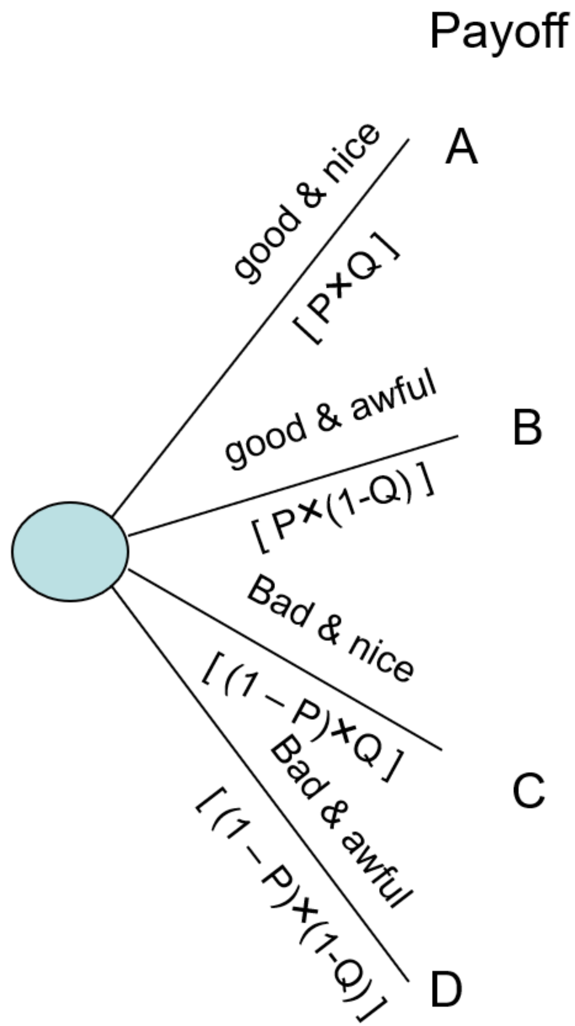


Trees can be pruned by removing branches we'll never end up in

Irrelevance based on sensitivity analysis or a judgment call

Decision Tree: Simplify by merging chance nodes





Successive chance nodes can be merged for a simpler tree

Reminder: Expected Value (EV)

Expected value can reflect randomness or our lack of information

Discrete outcomes $i = 1, \ldots, N$ with probabilities p_i and values V_i

$$\mathrm{EV} = p_1 V_1 + p_2 V_2 + \cdots + p_N V_N$$

Example: two outcomes

Success: $V_1 = 100$, $p_1 = 0.8$

Failure: $V_2 = -20$, $p_2 = 0.2$

$$\mathrm{EV} = 0.8 \times 100 + 0.2 \times (-20) = 76$$

Reminder: Expected Value (EV)

Continuous outcome V distributed such that $\Pr(V \leq v) = F(v)$

$$\mathrm{EV} = \int v f(v) \mathrm{d} v$$

where $f(v) = \partial F(v)/\partial v$ is the PDF and $F(v)$ the CDF of V

Example: uniform distribution $V \in [-10, 30]$

$$F(v) = \begin{cases} 0 & v \leq -10 \\ \frac{v-(-10)}{30-(-10)} = \frac{v+10}{40} & v \in (-10, 30] \\ 1 & v > 30 \end{cases}$$

$f(v) = 1/40$ if $v \in [-10, 30]$, else $f(v) = 0$

$$\mathrm{EV} = \int_{-10}^{30} v \frac{1}{40} \mathrm{d} v = \Big|_{-10}^{30} \frac{v^2}{80}[v] = \frac{30^2 - (-10)^2}{80} = 10$$

Reminder: Discounting, NPV

Discounting is the inverse of compound interest

Deposit €X at rate of r per compounding period $\longrightarrow$ after t periods you have €$X(1+r)^t$

Net Present Value (NPV) criterion: A project is worth doing if it has NPV positive, i.e., if it yields a higher return than is the cost of capital

$$\text{NPV} = \sum_{t=0}^{\infty} \frac{X_t}{(1+r)^t}$$

where X_t is net income in period t

Opportunity cost of capital could be forgone return to funds in alternative use, or cost of borrowing

Reminder: Discounting, NPV

Example: Current cost of a project €1000, gives income €1200 5 years from now. Alternative is to deposit €1000 in risk-free account at 3%. Which is better?

$$\text{NPV} = \frac{-1000}{(1.03)^0} + \frac{1200}{(1.03)^5} = -1000 + 1035.13 = 35.13 > 0$$

If you deposit 1000€ on the risk-free account, in 5 years you have $1000(1.03)^5 = 1159.27 < 1200$

Units of NPV are in current year Euros. This is more useful than "5 years from now Euros", but calculations in either units will always support the same decision

Discounting: Perpetuity

Discount rate r

Discount factor $B := \frac{1}{1+r}$

$$\text{NPV} = X_0 + BX_1 + B^2X_2 + B^3X_3 + \cdots$$

If $X_t = X$ in every year, starting in one year ($t = 1$), "to perpetuity"

$$\text{NPV} = BX + B^2X + B^3X + \cdots \quad || \text{ multiply both sides by } (1-B)$$

$$(1-B)\text{NPV} = BX + B^2X + B^3X + \cdots - B(BX + B^2X + B^3X + \cdots) \Rightarrow$$

$$\text{NPV} = \frac{B}{1-B}X$$

$$\text{NPV} = \frac{\frac{1}{1+r}}{1-\frac{1}{1+r}}X = \frac{X}{r}$$

Reminder: Present value

Example: At $r = 4\%$, the PV of receiving 100€ every year starting today, is $100 + 100/0.04 = 2600€$

Example: Borrow at $r = 8\%$ to invest in project with these flows:

Year	Earnings	Costs	Net Income
0	0	400	-400
1	300	100	200
2	300	0	300
3	200	300	-100
4	200	50	150

$$\text{NPV} = -400 + \frac{200}{1.08} + \frac{300}{(1.02)^2} - \frac{100}{(1.02)^3} + \frac{150}{(1.02)^4} = 73.26$$

User cost of capital

What is the opportunity cost of a durable good during one year?

Required data:

- Discount rate r
- Value of good now V_0
- Resale value in one year V_1

User cost during the next year is $V_0 - \frac{V_1}{1+r}$

Example. Own a laptop with current value $V_0 = 1200€$, expected resale value next year $V_1 = 500€$, $r = 6\%$.

User cost i.e. opportunity cost of capital “tied into” the laptop is $1200 - 500/(1.08) \approx 728€$.

Consider the buy-or-rent decision in a competitive market

Risk preferences

In economics a “gamble” or a “lottery” is a combination of mutually exclusive outcomes, each with a payoff (value) and a probability. With n outcomes the gamble is L=($\{v_1, v_2, \ldots, v_n\}$,$\{p_1, p_2, \ldots, p_n\}$) where probabilities p_i sum to one.

Example: A genie offers you the following coin flip gamble.
({€1m,0}, {0.5,0.5})
So $\mathrm{EV} =$ €500k. Would you prefer a sure thing €499k to the gamble? If so, you are risk averse.

The lowest certain value that you find at least as good as the uncertain gamble is your certainty equivalent (CE) for the gamble.

CE depends both on the gamble and individual preferences

Certainty Equivalent

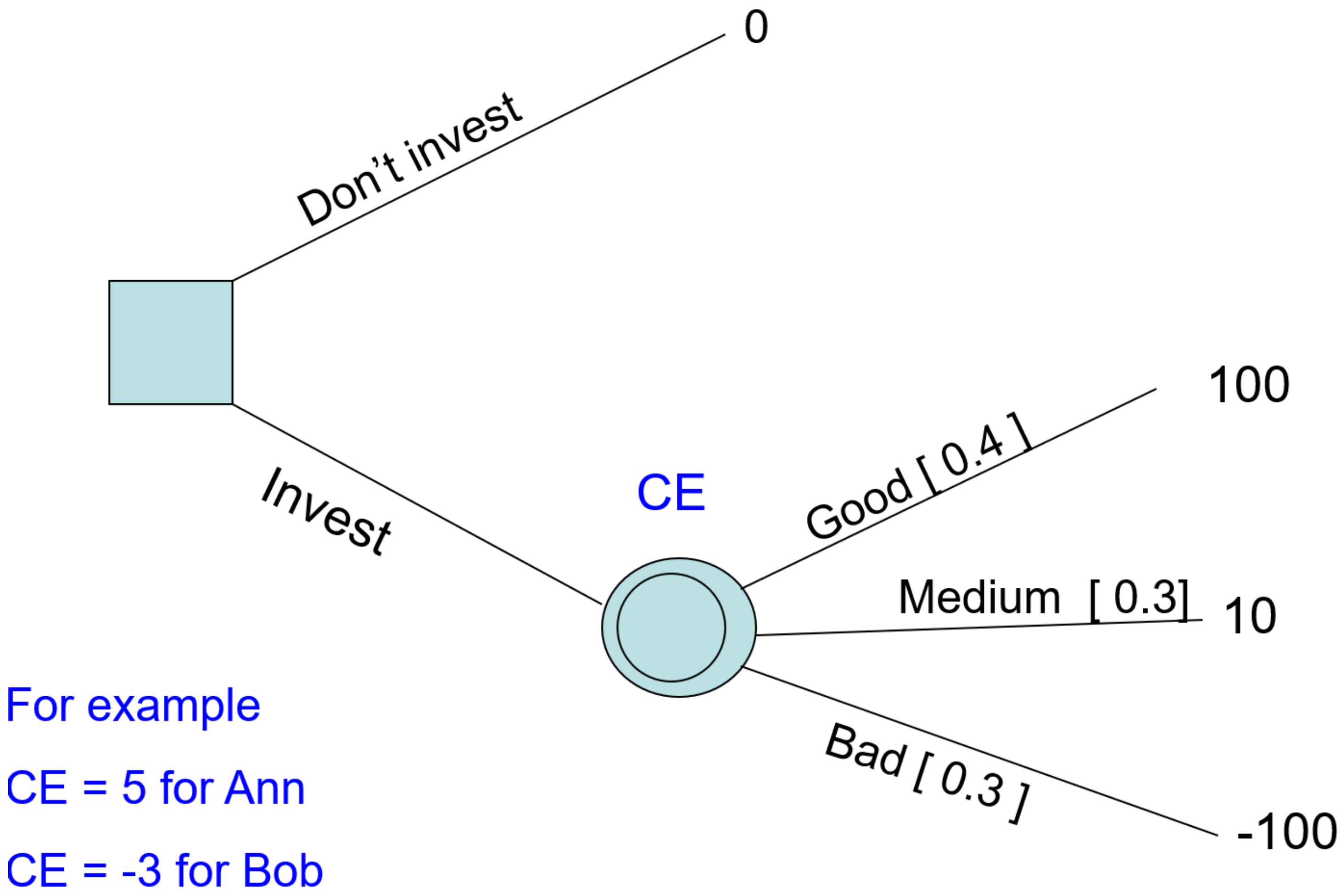
Consider a gamble with some EV. Different people will typically have different CE

- Risk averse preferences $\mathrm{CE} < \mathrm{EV}$
- Risk neutral preferences $\mathrm{CE} = \mathrm{EV}$
- Risk loving (risk seeking) preferences $\mathrm{CE} > \mathrm{EV}$
- A person with a smaller CE for the same gamble is more risk averse

Risk premium $= \mathrm{EV} - \mathrm{CE}$ is the reservation value for a perfect insurance against the risk in the gamble

How to find out the CE? What is your CE for the genie gamble?

Risk preferences in a decision tree



$\mathrm{EV} = 0.4 \times 100 + 0.3 \times 10 + 0.4 \times (-100) = 10$
CE varies by person. For the most risk-averse $\mathrm{CE} < 0$ and this project is not worth it

Utility functions, risk preferences

Bernoulli's utility function (1738) U maps wealth to utility

Risk aversion = concavity of U

Expected utility EU = $p_1 U(v_1) + p_2 U(v_2) + \cdots p_n U(v_n)$

Of alternative gambles, one with highest EU is most preferable

Units of EU are meaningless, units of CE from values v

$$U(\mathrm{CE}) = p_1 U(v_1) + p_2 U(v_2) + \cdots p_n U(v_n)$$

Example: St. Petersburg paradox

$L = \{2^{s-1}, 2^{-s}\}_{s=1}^{\infty} = (\{1, 2, 4, 8, \ldots\}, \{1/2, 1/4, 1/8, 1/16, \ldots\})$

$\mathrm{EV} = \infty$

If $U(v) = \log(v)$ then $\mathrm{CE} = 2$

If $U(v) = \sqrt{v}$ then $\mathrm{CE} \approx 2.91$

Risk preferences, behavioral issues

- Wealth effect on risk-taking

  What happens to risk aversion in $u(W+v)$ as W grows?

  Relative risk aversion $\rho(x) = -\frac{xu''(x)}{u'(x)}$

- Gambler's fallacy
- Favorite-longshot bias (?)
- Loss aversion (Prospect theory)

  Utility function kinks around "a reference point"

- Framing
- Importance of learning and experience

Real options

If you face a now-or-never decision you have no real options

Real option is the ability to change a decision at a later date
e.g. option to invest or divest later

Option value of experimentation from the option to undo/ignore the result of the experiment

Sometimes a real option is obtained at a cost. Value of real option gives the right reservation price

Example 1. Option to wait (Acme Ltd)

Example 2. Option value of experimentation (Sähkö Oy)

Financial option: right to buy or sell an asset later at a pre-specified price

Option to wait: Example

Acme Ltd could launch a new gadget, if it invests €1m

The gadget would generate expected profit of €400k, starting a year later, for three years, at which point this gadget type will becomes obsolete. Acme faces capital cost is 5%

After one year Acme would have a more accurate picture of revenue for this type of gadget
- Good scenario, $\Pr = 0.5$, €700k
- Bad scenario, $\Pr = 0.5$, €100k

Option to wait: Example

Invest now: $X_0 = -1000$, $X_1 = X_2 = X_3 = 400$

NPV = $-1000 + \frac{400}{1.05} + \frac{400}{1.05^2} + \frac{400}{1.05^3} = 89.3$

Wait: $X_0 = 0, X_1 = -1000, X_2 = X_3 > 400$

Wait + good news:

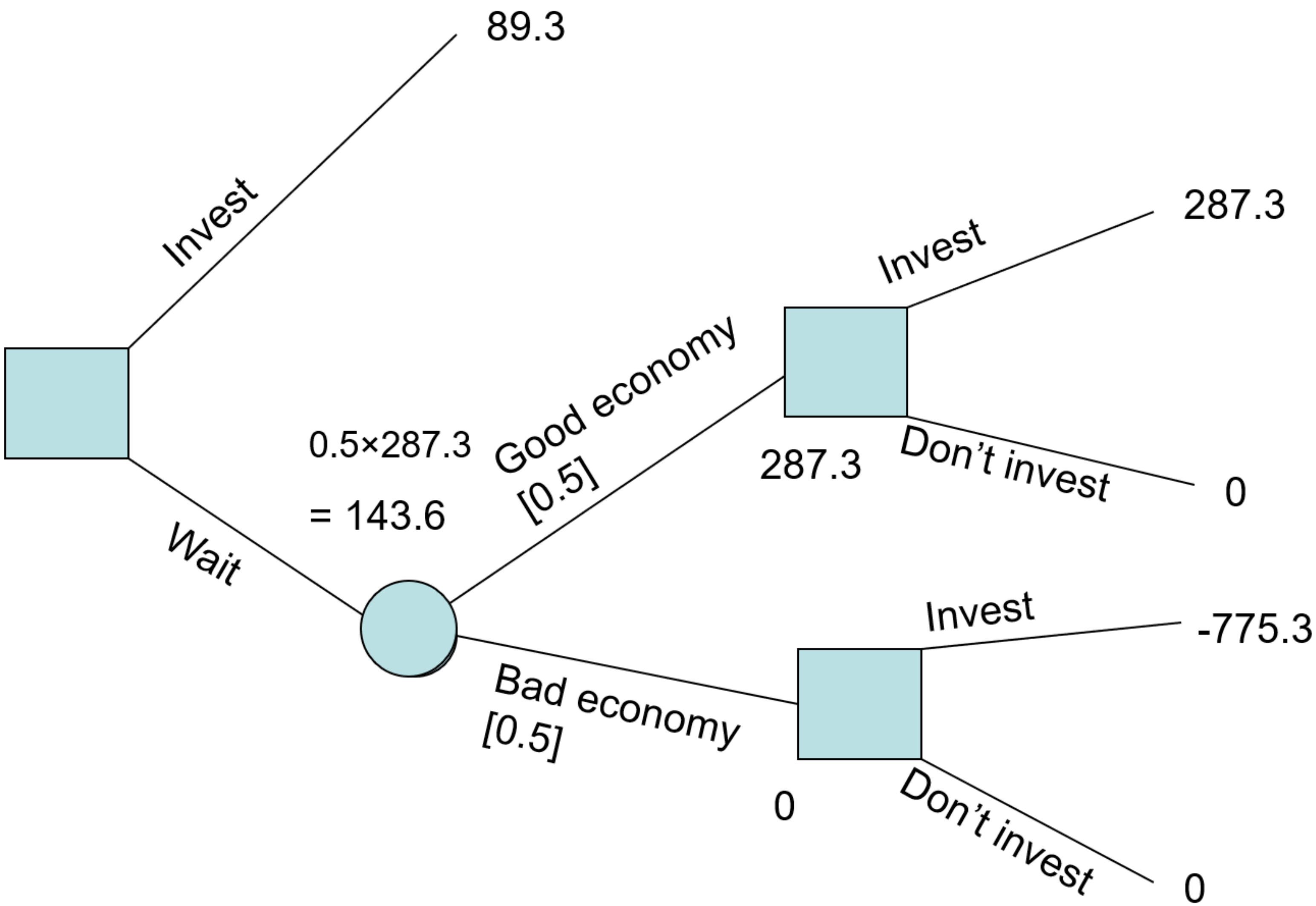
NPV = $\frac{-1000}{1.05} + \frac{700}{1.05^2} + \frac{700}{1.05^3} = 287.3$

Wait + bad news:

NPV = $\frac{-1000}{1.05} + \frac{100}{1.05^2} + \frac{100}{1.05^3} = -775.3$

$\Longrightarrow$ don't invest if bad news, so actually NPV = 0

Option value of waiting: Acme Ltd



Option to wait is valuable: $143.6 - 89.3 = 54.3$

Option value of experimentation: Example

Sähkö Oy has for long purchased a key component from Oldie Inc
Profits are €1m per year, $r = 10\%$.

Alternative supplier Newbie Inc could be better or worse than Oldie
Profits would be $(X + 1)$€m or $(X - 1)$€m per year, equally likely

Sähkö can choose the supplier for one year at a time. By trying Newbie as the supplier even once Sähkö would find out whether it is better or worse than Oldie.

How good should Newbie be in expectation (X) for Sähkö to try it?

Option value of experimentation: Example

How well will Sähkö Oy under alternative decisions?

Expected NPV (€m)

Stick with Oldie:

$PV_{\text{old}} = B \times 1 + B^2 \times 1 + B^3 \times 1 + \cdots] = \frac{1}{r} = 10$

Try Newbie:

Newbie better, profits $X + 1$ forever $[\Pr = 0.5]$

$PV_{\text{new+}} = B \times (X + 1) + B^2 \times (X + 1) + B^3 \times (X + 1) + \cdots] = \frac{X+1}{r} = 10(X + 1)$

Newbie worse, profits $X - 1$ once, then switch to Oldie $[\Pr = 0.5]$

$PV_{\text{new-}} = B \times (X - 1) + B^2 \times 1 + B^3 \times 1 + \cdots] =$
$B \times (X - 1 + PV_{\text{old}}) = (X + 9)/1.1 = 0.909X + 8.18$

Option value of experimentation: Example

In expectation the value of trying Newbie is

$$
\begin{aligned}
PV_{\text{new}} &= 0.5 PV_{\text{new+}} + 0.5 PV_{\text{new-}} \\
&= 0.5 \times 10(X+1) + 0.5 \times 0.909X + 8.18 \\
&= 9.1 + 5.45X
\end{aligned}
$$

Experimentation is valuable if

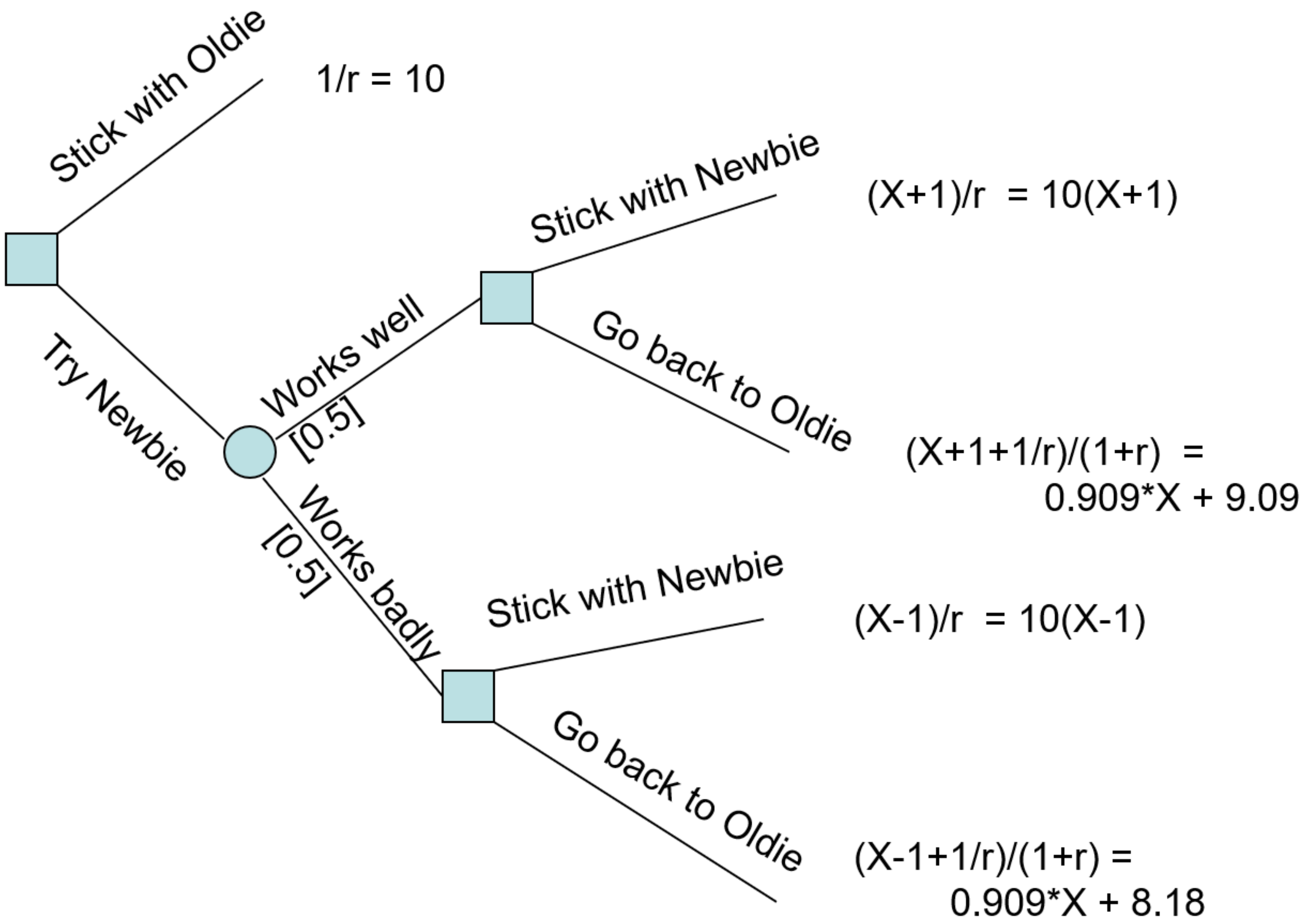
$$
\begin{aligned}
PV_{\text{new}} &> PV_{\text{old}} \\
9.1 + 5.45X &> 10 \\
X &> 0.17
\end{aligned}
$$

Newbie is worse than Oldie in expectation, but worth trying

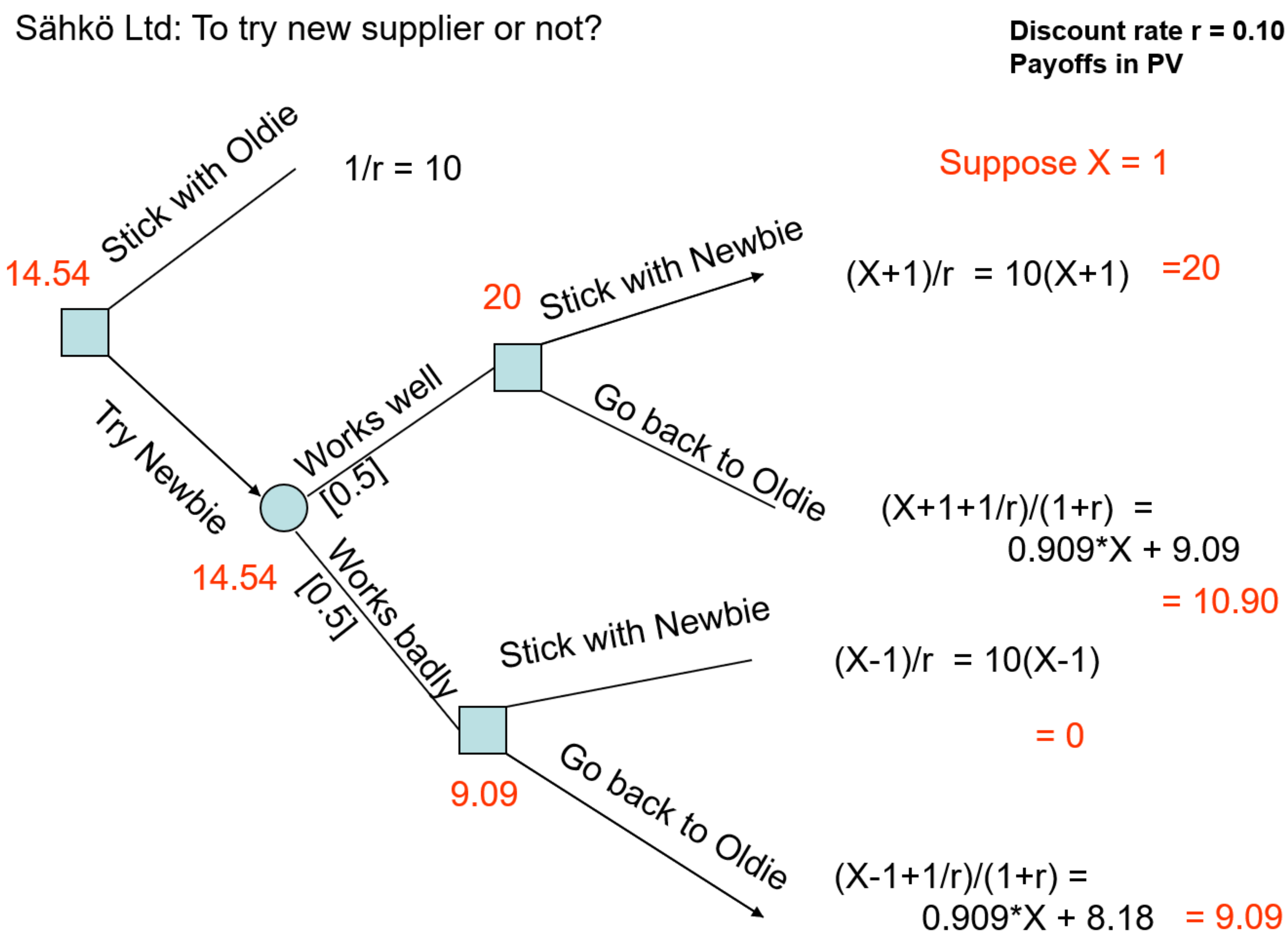
Option value of experimentation

Sähkö Ltd: To try new supplier or not?

Discount rate r = 0.10
Payoffs in PV



Option value of experimentation



With $X = 1$ the option to experiment is valuable: $14.5 - 10 = 4.5$

Option value of experimentation

Trying an alternative with lower expected value can be valuable

Experimentation–trying Newbie for a year–produced valuable information because Sähkö Oy had the option to return to use Oldie Inc

Option value is increasing in risk

Suppose that Newbie were even riskier, so that profits in good and bad case were $X + 2$ and $X - 2$. What would now be the minimum X required for experimentation to be valuable?

Answer: -0.67

Decision analysis: key skills

- Construct decision tree from a verbal description of a situation, assumptions, data
- Calculate comparable objective values across outcomes: expected value, net present value
- Conduct sensitivity analysis to uncertain assumptions
- Calculate value of additional information (upper bound from value of perfect forecast)
- Understand the value of options to delay or undo decisions