Lec 4

Wednesday 30. September 2020 11.46

$u \rightarrow [G(s)] \rightarrow y$ $\quad s+1,\ s=-1$

$u = e^{\lambda t}$ $\quad \ddot{y}+3\dot{y}+y=\dot{u}+u$

$= Ae^{\lambda t}$

$A\lambda^2 e^{\lambda t} + 3A\lambda e^{\lambda t} + Ae^{\lambda t} = \lambda e^{\lambda t} + e^{\lambda t}$

$A(\lambda^2+3\lambda+1) = \lambda + 1 = 0$

$\lambda = -1$

$A x = 0$ ($n\times n$, $n\times 1$) $\quad \det(A)=0$, rank of A is not full

$px = Ax + Bu$

$y = Cx + Du$

$(pI - A)x - Bu = 0$

$-Cx - Du = -y$

$$\begin{bmatrix} pI-A & B \\ -C & D \end{bmatrix} \begin{bmatrix} x \\ -u \end{bmatrix} = \begin{bmatrix} 0 \\ -y \end{bmatrix}$$

$2\times 2 \quad 2\times 1 \quad 2\times 1$

$A = A^*$

$(x^* A x)^* = x^* A^* x$ ($n\times 1$, $n\times n$, $n\times 1$)

$= x^* A x$ real

$\mathrm{eig}(A) = $ real

$Ae = \lambda e$

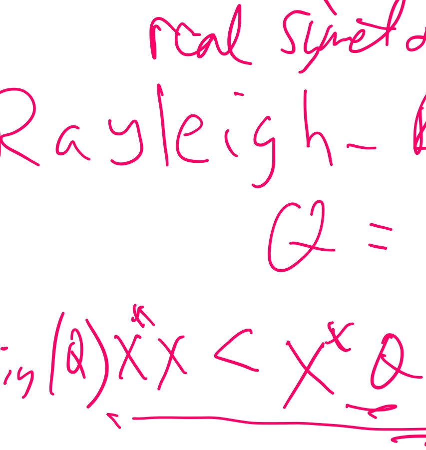
$\det(\lambda I - A) = 0$

$x^* A x$

$e^* A e = e^* \lambda e \quad > 0$ real

$= \lambda e^* e = \lambda |e|^2 > 0$ real

$\lambda > 0$



$x^* A x < 0$

$\Leftrightarrow x^*(-A)x > 0$

Sylvester

real symetric

Rayleigh–Ritz inequality

$Q = Q^*$

$\lambda_{min}(Q) x^* x < x^* Q x < \lambda_{max}(Q) x^* x$

$x = Tz$ $\quad TT^* = I$, $T^* = T^{-1}$ unitary

$x^* x = z^* T^* T z = z^* z$

$x^* Q x = z^* T^* Q T z = z^* T^{-1} Q T z$

$$\lambda_{min}(z^* z) < z^* \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{bmatrix} z < \lambda_{max} z^* z$$

$\lambda_{min}(z_1^* z_1 + z_2^* z_2 \dots) < \lambda_1 z_1^* z_1 + \lambda_2 z_2^* z_2 + \dots$ $\quad \lambda_{max}(z_1^* z_1 + z_2^* z_2)$

$A = U \Sigma V^*$

$AA^* = U \Sigma V^* V \Sigma^T U^*$ ($V^*V = I$)

$= U \Sigma \Sigma^T U^*$

$AA^* U = U \Sigma \Sigma^T$

$$AA^* [u_1 \vdots u_2 \vdots \dots \vdots u_n] = [u_1 \vdots u_2 \vdots \dots \vdots u_n] \begin{bmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \\ 0 & & \sigma_n^2 \end{bmatrix}$$

$n\times n \quad n\times n$

$\{AA^* u_1,\ AA^* u_2 \dots\} = \{\sigma_1^2 u_1 \vdots \sigma_2^2 u_2 \dots\}$

$AA^* u_j = \sigma_j^2 u_j$

$A = U \Sigma V^*$

$AV = U \Sigma V^* V = U\Sigma$ ($V^*V = I$)

$$A [v_1 \vdots v_2 \vdots \dots v_n] = [u_1\ u_2 \dots u_n] \begin{bmatrix} \sigma_1 & & 0 \\ & \sigma_2 & \\ 0 & & \sigma_n \end{bmatrix}$$

$A v_i = \sigma_i u_i$

Push-through rule

A ($n\times m$) $\quad B$ ($m\times n$) $\quad AB$ ($n\times n$) $\quad BA$ ($m\times m$)

$A(I+BA)^{-1} = (I+AB)^{-1}A$

$A(I+BA)^{-1} = (I+AB)^{-1}(I+AB)A(I+BA)^{-1}$

$= (I+AB)^{-1}(A+ABA)(I+BA)^{-1}$

$= (I+AB)^{-1}A(I+BA)(I+BA)^{-1}$

$= I$

$z + GF_y z = A$

$(I + GF_y)z = A$

$z = (I+GF_y)^{-1}A$

$L(j\omega) = G(j\omega)F_y(j\omega)$

$S(j\omega) = [I + L(j\omega)]^{-1}$

$T(j\omega) = [I + L(j\omega)]^{-1} L(j\omega)$

$= L(j\omega)[I + L(j\omega)]^{-1}$

$S + T = (I+L)^{-1} + (I+L)^{-1}L$

$(j\omega) = (I+L)^{-1}\{I + L\} = I$

$$\boxed{S(j\omega) + T(j\omega) = I}$$

$|S| + |T| \neq I$

Loop shaping