1. (**A little finger warm-up**) (2p)

   When deriving the lower hybrid frequency during the lectures, we left a little part for you (where it says "algebra"). Starting from the expression $M(1-\frac{\Omega_i^2}{\omega^2}) = -m(1-\frac{\Omega_e^2}{\omega^2})$, derive the expression $\omega = \sqrt{\Omega_i\Omega_e} = \omega_{LH}$.

2. (**A little physics warm-up**) (3p)

   Calculate the value for the sound speed for a one-dimensional sound wave in

   (a) a typical deuterium fusion plasma: $n = 10^{20}\,\mathrm{m}^{-3}$, $T = 20\,\mathrm{keV}$,

   (b) Earth's magnetosphere, mainly protons: $n = 1\,\mathrm{cm}^{-3}$, $T = 5000\,\mathrm{K}$,

   (c) air: mainly nitrogen: $n = 3\times10^{25}\,\mathrm{m}^{-3}$, $T = 20\,°\mathrm{C}$.

3. (**Straight-forward and useful**) (6p)

   In the lectures, we derived a bunch of dispersion relations for various electrostatic waves. For the ion acoustic wave not all the details were carried out. Derive the dispersion relation for the ion acoustic wave, but with the following simplification: use the plasma approximation one step earlier, i.e., before (and instead of) using Gauss' law for the electric field. You should obtain the ion acoustic wave in the limit $k^2\lambda_D^2 \to 0$. Assume that the plasma is stationary ($\boldsymbol{v}_0 = \mathbf{0}, \dot{n}_0 = 0$), homogeneous ($\nabla n_0 = \mathbf{0}$), isothermal ($T_e, T_i =$ constant), and initially unperturbed ($\boldsymbol{E}_0 = \mathbf{0}$).

4. (**Plasma as a dielectric medium – do it yourself!**) (6p)

   Revisit the high-frequency electrostatic perturbation for another useful result: Linearize the electron equation of motion and continuity equation, and use them together with the linearized Gauss' law to show that the Gauss' law can be expressed in terms of the *plasma dielectric coefficient* ϵ_p as $\nabla\cdot(\epsilon_p\mathbf{E}) = 0$, where $\epsilon_p = 1-\omega_p^2/\omega^2$. Use the cold plasma approximation for the electron temperature, and additionally assume that the plasma is stationary, homogeneous, isothermal, and initially unperturbed.

5. (**Food for thought: How to measure things...**)

   Let us assume you have the means to measure electrostatic emissions from a plasma, i.e., you can measure not only amplitude but also the frequency and wavelength, allowing identification of the wave. If you would like to know things like what is the density and temperature of the plasma and what kind of, if any, magnetic field there is, what waves would you use and why?