ELEC-E8412 Power Electronics HW#1

Problem #1

The voltage and current for a device (using the passive sign convention) are periodic functions with T=100 ms described by

$$v(t) = \begin{cases} 50 * t \quad (V) & 0 < t < 70\,ms \\ 0 & 70\,ms < t < 100\,ms \end{cases}$$

$$i(t) = \begin{cases} 0 & 0 < t < 50\,ms \\ 4 \quad (A) & 50\,ms < t < 100\,ms \end{cases}$$

Determine

(a) the instantaneous power,

(b) the average power, and

(c) the energy absorbed by the device in each period.

 # 1

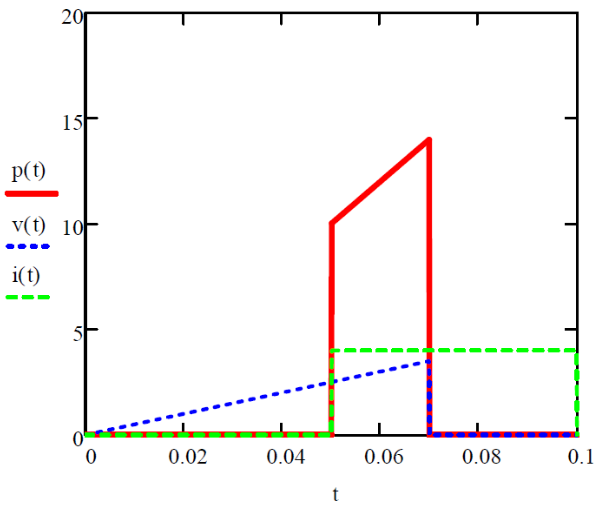
$$v(t) := \text{if}\left[\left(t < 70 \cdot 10^{-3}\right) \wedge (t > 0), 50 \cdot t, 0\right]$$

$$i(t) := \text{if}\left[\left(t < 100 \cdot 10^{-3}\right) \wedge \left(t > 50 \cdot 10^{-3}\right), 4, 0\right]$$

part (a)

$$p(t) := v(t) \cdot i(t)$$

$$p(t)= \begin{cases} 0 & 0<t<50 \text{ ms} \\ 200t & 50<t<70 \text{ ms} \\ 0 & 70<t<100 \text{ ms} \end{cases}$$



part (b)

T=100.10^{-3} sec

$$P = \frac{1}{T}.\int_0^T P(t)\,dt = \frac{1}{100.10^{-3}}.\int_{50.10^{-3}}^{70.10^{-3}} 200t\,dt = \frac{1}{100.10^{-3}}.100((70.10^{-3})^2 - (50.10^{-3})^2)$$

$$P = \frac{1}{100.10^{-3}}.100((70.10^{-3})^2 - (50.10^{-3})^2) = 2.4 \text{ W}$$

part(c)

$$W = \int_0^T P(t)\,dt = 0.24 \text{ J}$$

Problem # 2

A current of $10\sin(2\pi 60t)$ (A) enters an element. Sketch the instantaneous power and determine the average power absorbed by the load element when the element is

(a) a 5-V source (current into the positive terminal),

(b) a 6-Ω resistor, and

(c) a 7-mH inductor.

 # 2

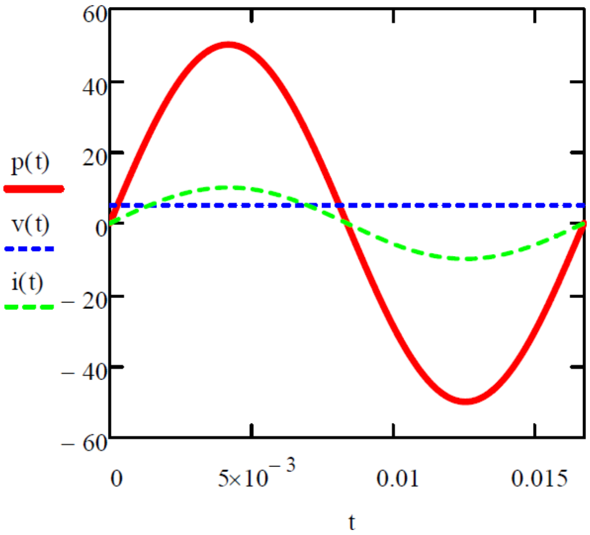
$$i(t) = 10\sin(2\pi 60t) \qquad T = \frac{1}{60}$$

part(a)

v(t)=5

$P(t) = V(t).i(t)$

$$P(t) = 50.\sin(120\pi t)$$



$$P = \frac{1}{T} \cdot \int_0^T P(t)\, dt = \frac{1}{1/60} \cdot \int_0^{1/60} 50.\sin(120.\pi.t)\, dt = 60 \cdot \frac{-50}{120.\pi} . (\cos(2.\pi) - \cos(0)) = 0$$

Part (b)

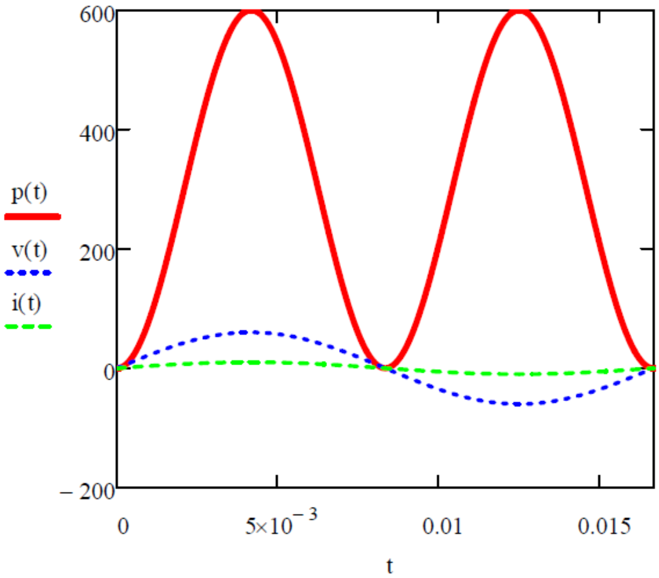
R = 6

$V(t) = R.i(t)$

$$V(t) = 60.\sin(120\pi t)$$

$P(t) = V(t).i(t)$

$$P(t) = 600.\sin(120\pi t)^2$$



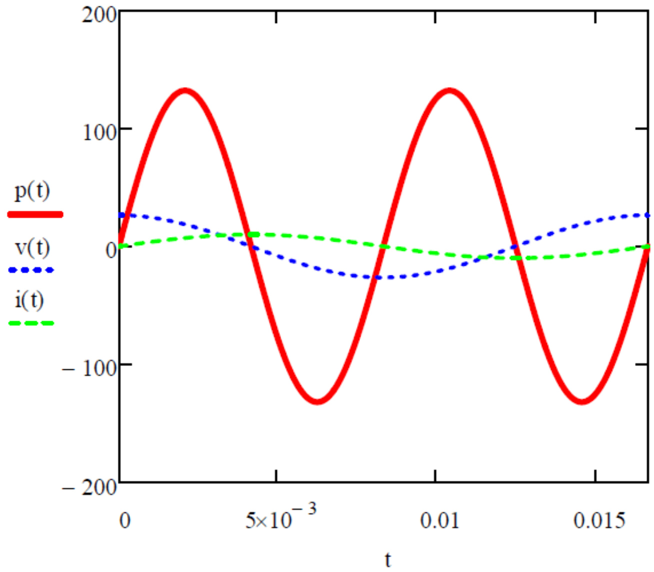
$$P = \frac{1}{T} \cdot \int_0^T P(t)\,dt \qquad or \qquad P = V_{rms} I_{rms} \cos(0) \; = \frac{60}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} = 300$$

part(c)

$L = 7.10^{-3}$

$V(t) = L.\frac{di}{dt} = 8.4\,\pi \cos\,(120\pi t)$

$p(t) = V(t).\,i(t) = 84.\pi.\cos(120\pi t)\,.\,sin(120\pi t)$



$$P = \frac{1}{T} \cdot \int_0^T P(t)\,dt = 60.\int_0^{1/60} 84.\pi.\cos(120\pi t)\,.\,sin(120\pi t)\,dt = \frac{10.5}{\pi}(\cos(4\pi) - \cos(0)) = 0$$

Note: $\cos(120\pi t)\,.\,sin(120\pi t) = \frac{1}{2}(\sin(240\pi t) - \sin(0)) = \frac{\sin(240\pi t)}{2}$ (general)

Problem # 3

The voltage and current for a circuit element are

$v(t) = 2 + 8\cos(2\pi 60t) + 6\cos(4\pi 60t + 55^\circ)$ V and

$i(t) = 1 + 3\cos(2\pi 60t + 25^\circ) + 6\cos(4\pi 60t - 10^\circ)$ A.

(a) Determine the rms values of voltage and current.

(b) Determine the power absorbed by the element.

3

$$v(t) := 2 + 8\cos(2\pi 60\cdot t) + 6\cos\left(4\pi 60t + \frac{55}{180}\pi\right)$$

$$i(t) := 1 + 3\cos\left(2\pi 60\cdot t + \frac{25}{180}\pi\right) + 6\cos\left(4\pi 60t - \frac{10}{180}\pi\right)$$

part (a)

$$Vrms := \sqrt{2^2 + \left(\frac{8}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2}$$

$$Vrms \rightarrow 3\cdot\sqrt{6}$$

$$Vrms = 7.348$$

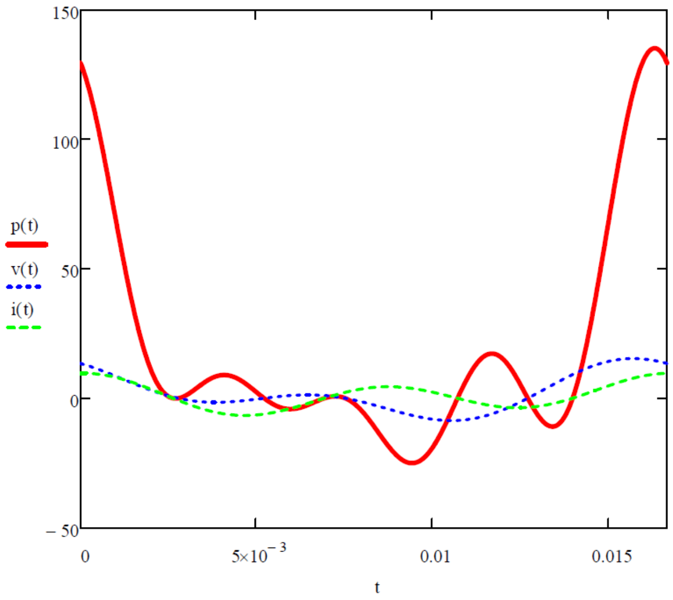
$$Irms := \sqrt{1^2 + \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2}$$

$$Irms \rightarrow \frac{\sqrt{2}\cdot\sqrt{47}}{2}$$

$$Irms = 4.848$$

part (b)

$P(t) = V(t).i(t)$



$$P = 2 + \frac{8}{\sqrt{2}}.\frac{3}{\sqrt{2}}\cos\left(\frac{25}{180}\pi\right) + \frac{6}{\sqrt{2}}.\frac{6}{\sqrt{2}}\cos\left(\frac{65}{180}\pi\right) = 20.483$$

Problem # 4

A sinusoidal voltage source of $v(t) = 100\cos(2\pi 60t)$ V is applied to a nonlinear load, resulting in a non-sinusoidal current which is expressed in Fourier series as $i(t) = 15\cos(2\pi 60t + 35^\circ) + 6\cos(4\pi 60t + 15^\circ) + 1\cos(8\pi 60t - 10^\circ)$ A. Determine

(a) the power absorbed by the load,

(b) the power factor of the load, and

(d) the total harmonic distortion of the load current.

 # 4

$$v(t) := 100\cos(2\pi 60 \cdot t)$$

$$i(t) := 15\cos\left(2\pi 60 \cdot t + \frac{35}{180}\pi\right) + 6\cos\left(4\pi 60t + \frac{15}{180}\pi\right) + 1\cos\left(8\pi 60t - \frac{10}{180}\pi\right)$$

Part (a)

$P(t) = V(t).i(t)$

Based on page 19 of chapter 1:

$$P = \frac{100}{\sqrt{2}} . \frac{15}{\sqrt{2}} \cos\left(\frac{35}{180}\pi\right) = 614.364$$

Note: average of second term and third term are zero

Part (b)

$$V_{rms} = \frac{100}{\sqrt{2}}$$

$$V_{rms} = 70.711$$

$$I_{rms} = \sqrt{\left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$I_{rms} = 11.446$$

$$pf = \frac{P}{V_{rms} I_{rms}}$$

$$pf = 0.759$$

Part (c)

$$\mathrm{THD} = \frac{\sqrt{\left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}}{\frac{15}{\sqrt{2}}}$$

THD= 0.406