ELEC-E8412 Power Electronics

HW#2_ Solutions

Problem 1:

For a half-wave rectifier with resistive load with *vin*=100sin(400t) V and *R*=10 Ω determine

(a) the average load current,

(b) the rms load current,

(c) the apparent power supplied by the source, and

(d) the power factor of the rectifier.

Solution:

Part (a)

The voltage across the resistor in a half-wave rectified sine wave with peak value $V_m = 100\ V$. The average voltage in a half-wave rectifier is Vm/π, and average current is:

$$\langle \mathrm{i} \rangle = \frac{V_m}{\pi R} = \frac{100}{\pi * 10} = 3.18\ A$$

Part (b) the rms load current

$$I_{rms} = \frac{V_m}{2R} = \frac{100}{2 * 10} = 5\ A$$

Part (c) the apparent power supplied by the source

Apparent power = $V_{in,rms} * I_{in,rms} = \frac{100}{\sqrt{2}} * 5 = 353.553\ \ VA$

Part (d) power factor

$$Pf = \frac{\mathrm{R} * \mathrm{I}_{\mathrm{rms}}^2}{V_{in-rms} * I_{in-rms}} = \frac{10 * 5^2}{\frac{100}{\sqrt{2}} * 5} = 0.707$$

Problem 2:

A half-wave rectifier has a source of 120 V rms at 60 Hz and an R-L load with R=10 Ω and L= 10 mH. Determine

(a) an expression for the load current,

(b) the average current,

(c) the power absorbed by the resistor, and

(d) the power factor.

Hint: Beta=3.5 Rad.

Solution:

Part (a)

For a half-wave rectifier with R-L load:

$$i(t) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t - \theta) + Ae^{-t/\frac{L}{R}}$$

Where, $\theta = tan^{-1}(\frac{L\omega}{R})$ and $A = \frac{V_m sin\theta}{\sqrt{R^2+(L\omega)^2}}$

For the given problem:

$$Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{(10)^2 + (0.01 * 377)^2} = 10.687$$

$$\theta = tan^{-1}\left(\frac{L\omega}{R}\right) = tan^{-1}\left(\frac{3.77}{10}\right) = tan^{-1}(0.377) = 0.36\, rad = 20.64^{\circ}$$

$$\mathrm{i}(\omega t) = \frac{120\sqrt{2}}{10.687} \sin(\omega t - 0.36) + \frac{120\sqrt{2} * \sin(0.36)}{10.687} \mathrm{e}^{-\omega t/0.377}$$

$\beta = 3.5$ rad is given by the problem, so

$$\mathrm{i}(\omega t) = 15.88 \sin(\omega t - 0.36) + 5.59\mathrm{e}^{-\omega t/0.377} \qquad for \quad 0 \le \omega t \le 3.5$$

$$\mathrm{i}(\omega t) = 0 \qquad for \quad 3.5 \le \omega t \le 2\pi$$

Part (b)

$$\langle i(\omega t)\rangle = \frac{1}{2\pi}\int_0^{3.5}\left[15.88\sin(\omega t - 0.36) + 5.59e^{-\omega t/0.377}\right]d(\omega t) = 5.23\ A$$

Part (c) the power absorbed by the resistor is $P = R * I_{rms}^2$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{3.5}\left[15.88\sin(\omega t - 0.36) + 5.59e^{-\omega t/0.377}\right]^2 d(\omega t)} = 8.037\ A$$

$$P = R * I_{rms}^2 = 10 * (8.037)^2 = 645.93\ W$$

Part (d)

$$Pf = \frac{P}{V_{in-rms} * I_{in-rms}} = \frac{645.93}{120 * 8.037} = 0.67$$

Problem 3:

A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 500 Ω. Determine the value of the filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V. Determine the average and peak values of the diode current.

Solution:

$$\Delta V_0 \approx \frac{V_m}{fRC} \qquad C > \frac{120\sqrt{2}}{60 * 500 * 2} = 2828\ \mu F$$

$$\Delta V_0 = V_m(1 - sin\alpha)$$

$$\alpha = sin^{-1}\left(1 - \frac{\Delta V_0}{V_m}\right) = sin^{-1}\left(1 - \frac{2}{120\sqrt{2}}\right) = 1.417\ rad = 81.20°$$

$$i_{d-peak} = V_m\left(c\omega cos\alpha + \frac{sin\alpha}{R}\right) = 120\sqrt{2}\left(2828 * 10^{-6} * 2\pi * 60 * \cos(81.2°) + \frac{\sin(81.2°)}{500}\right)$$

$$i_{d-peak} = 28.03\ A$$

$$\langle i_d \rangle = \frac{\langle V_o \rangle}{R}$$

$$\langle V_o \rangle \approx V_{o,peak} - \frac{\Delta V_0}{2} = V_m - \frac{V_m}{2fRC} \approx V_m$$

$$\langle i_d \rangle = \frac{V_m}{R} = \frac{120\sqrt{2}}{500} = 0.339\ A$$

Problem 4:

A controlled half-wave rectifier has an ac source of 240 V rms at 60 Hz. The load is a 30-Ω resistor.

(a) Determine the delay angle such that the average load current is 3.0 A.

(b) Determine the power absorbed by the load.

(c) Determine the power factor.

Solution:

Part (a):

$$\langle\, i\,\rangle = \frac{V_m}{2\pi R}(1 + cos\alpha) = \frac{240\sqrt{2}}{2\pi * 30}(1 + cos\alpha) = 3\ A$$

$$\rightarrow\ \alpha = cos^{-1}\left(\frac{6\pi R}{V_m} - 1\right) = 48.235° = 0.843\, rad$$

Part (b):

$$P = \frac{{V_{rms}}^2}{R} = \frac{V_m^2}{4R}\left(\frac{1}{2} - \frac{\alpha}{\pi} + \frac{sin2\alpha}{2\pi}\right) = 854$$

Part (c):

$$pf = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{sin2\alpha}{4\pi}} = \sqrt{\frac{1}{2} - \frac{0.843}{2\pi} + \frac{\sin{(2 * 0.843)}}{4\pi}} = 0.67$$