ELEC-E8412 Power Electronics

/ Exercise # 3 _ Solutions 22.10.2020

Problem 1:

A half-wave rectifier with a 1-kΩ load has a parallel capacitor. The source is 120 V rms, 60 Hz. Determine the peak-to-peak ripple of the output voltage when the capacitor is

a) 4000 μF and

b) 20 μF.

Solution:

Part (a):

the approximation $\Delta V_0 \approx \frac{V_m}{fRC}$ is valid when $\omega RC \gg 1$

C=4000 μF $\omega RC = 377 * 1000 * 4 * 10^{-3} = 1508 \gg 1$

$$\Delta V_0 \approx \frac{V_m}{fRC} = \frac{120\sqrt{2}}{60 * 1000 * 4 * 10^{-3}} = 0.707\ V$$

Part (b):

C=20 μF $\omega RC = 377 * 1000 * 20 * 10^{-6} = 7.54 > 1$ (approximation is not valid here)

$\theta = \pi - tan^{-1}(RC\omega) = \pi - tan^{-1}(7.54) = 1.7027 rad$

$sin\alpha - sin\theta . e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$

$\alpha = 30.2° = 0.5275 rad$

$\Delta V_0 = V_m(1 - sin\alpha) = 120\sqrt{2}(1 - 0.5034) = 83.26V$ (correct answer)

$$\Delta V_0 \approx \frac{V_m}{fRC} = \frac{120\sqrt{2}}{60 * 1000 * 20 * 10^{-6}} = 141.42\ V$$

Matlab code for calculating α (you do not need it in your exam, because α will be given to you in your exam)

```
>> syms a
>> sola=solve(sin(a)-0.991*exp(-(2*pi+a-1.7027)/7.54)==0,a)

sola =

0.527461058625877600152913378307 98
```

Problem 2:

A half-wave rectifier with a capacitor filter has V_m=200 V, R=10 kΩ, C=1000 μF, and ω=377.

a) Determine the peak-to peak ripple voltage using the exact equations.

b) Determine the ripple (using the approximate formula).

Solution:

Part (a):

$$\theta = \pi - tan^{-1}(RC\omega) = \pi - tan^{-1}(10000 * 1000 * 10^{-6} * 377) = 1.5711 rad$$

$$sin\alpha - sin\theta . e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$$

$$\alpha = 86.7° = 1.513 rad$$

$$\Delta V_0 = V_m(1 - sin\alpha) = 200(1 - sin\alpha) = 200(1 - 0.9983) = 0.34\, V$$

Part (b):

$$\Delta V_0 \approx \frac{V_m}{fRC} = \frac{200}{60 * 10000 * 1000 * 10^{-6}} = 0.333\, V$$

Problem 3:

For the controlled half-wave rectifier with resistive load, the source is 120 V rms at 60 Hz. The resistance is 100 Ω, and the delay angle α is 45^0.

a) Determine the average voltage across the resistor.

b) Determine the power absorbed by the resistor.

c) Determine the power factor as seen by the source.

Solution:

Part (a): Average voltage across the resistor

$$V_o = \frac{V_m}{2\pi}(1 + cos\alpha) = \frac{120\sqrt{2}}{2\pi}(1 + cos45) = 46.11\ V$$

Part (b): the power absorbed by the resistor

$$P = \frac{{V_{rms}}^2}{R}$$

$$V_{rms} = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{sin2\alpha}{2\pi}} = \frac{120\sqrt{2}}{2}\sqrt{1 - \frac{0.785}{\pi} + \frac{sin2\alpha}{2\pi}} = \frac{120\sqrt{2}}{2} * 0.953 = 80.91V$$

$$\alpha = 45° = \frac{\pi}{4}rad = 0.785rad$$

$$P = \frac{{V_{rms}}^2}{R} = \frac{(80.91)^2}{100} = 65.46W$$

Part (c): Power factor

$$pf = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{sin2\alpha}{4\pi}} = 0.67$$

Problem 4:

A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 750 Ω.

a) Determine the value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V.

b) Determine the average and peak values of diode current.

Solution:

Part (a): the value of C

$$\Delta V_0 \approx \frac{V_m}{fRC} \qquad C > \frac{120\sqrt{2}}{60 * 750 * 2} = 1885\ \mu F$$

Part (b): average and peak values of diode current

$\Delta V_0 = V_m(1 - sin\alpha)$

$\alpha = sin^{-1}\left(1 - \frac{\Delta V_0}{V_m}\right) = sin^{-1}\left(1 - \frac{2}{120\sqrt{2}}\right) = 1.417\ rad = 81.20°$

$$i_{d-peak} = V_m\left(c\omega cos\alpha + \frac{sin\alpha}{R}\right) = 120\sqrt{2}\left(1885 * 10^{-6} * 2\pi * 60 * \cos(81.2°) + \frac{\sin(81.2°)}{750}\right)$$

$$i_{d-peak} = 18.7\ A$$

$\langle i_d \rangle = \frac{\langle V_o \rangle}{R}$

$\langle V_o \rangle \approx V_{o,peak} - \frac{\Delta V_0}{2} = V_m - \frac{V_m}{2fRC} \approx V_m$

$\langle i_d \rangle = \frac{V_m}{R} = \frac{120\sqrt{2}}{750} = 0.2262\ A$