ELEC-E8412 Power Electronics

/ Exercise # 2 _ Solutions 15.10.2020

Problem 1: (Chapter 1)

A sinusoidal voltage source of $v(t) = 40\cos(2\pi 50t)$ V is applied to a nonlinear load, resulting in a non-sinusoidal current which is expressed in Fourier series as $i(t) = 3 + 7\cos(2\pi 50t + 20^{\circ}) + 4\cos(6\pi 50t + 15^{\circ}) + 3\cos(8\pi 50t + 25^{\circ})$ A. Determine

(a) the power absorbed by the load,

(b) the power factor of the load, and

(d) the total harmonic distortion of the load current.

Solution:

$\mathrm{P(t) = V(t).i(t)}$

Part (a): The power absorbed by the load is

$$P = \frac{40}{\sqrt{2}}.\frac{7}{\sqrt{2}}\cos\left(\frac{20}{180}\pi\right) = 131.56 \ W$$

Part (b):

The power factor of the load can be calculated by

$$pf = \frac{P}{V_{rms}I_{rms}}$$

$$V_{rms} = \frac{40}{\sqrt{2}} = 28.28 \ V$$

$$I_{rms} = \sqrt{(3)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{46} = 6.78 \ \ A$$

$$pf = \frac{P}{V_{rms}I_{rms}} = \frac{131.56}{28.28 * 6.78} = 0.686$$

Part (c):

$$THD = \frac{\sqrt{(3)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}}{\frac{7}{\sqrt{2}}} = \frac{4.637}{4.95} = 0.936$$

Problem 2: (Chapter 1)

The voltage and current for a circuit element are

$v(t) = 3 + 5\cos(2\pi 60t + 15^{\circ}) + 2\cos(4\pi 60t)$ V and

$i(t) = 2 + 7\cos(2\pi 60t + 45^{\circ}) + 3\cos(6\pi 60t + 25^{\circ})$ A.

(a) Determine the rms values of voltage and current.

(b) Determine the power absorbed by the element.

Solution:

Part (a):

$$V_{rms} = \sqrt{(3)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 4.85\ V$$

$$I_{rms} = \sqrt{(2)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 5.74\ A$$

Part (b): The power absorbed by the load is

$$P = 6 + \frac{5}{\sqrt{2}} . \frac{7}{\sqrt{2}} \cos\left(\frac{30}{180}\pi\right) = 21.15\ W$$

Problem 3:

For the half-wave rectifier, the source is a sinusoid of 300 V rms at a frequency of 50 Hz. The load resistor is 25. Determine

(a) the average load current,

(b) the power absorbed by the load,

(c) the apparent power supplied by the source, and

(d) the power factor of the circuit.

Solution:

Part (a):

The voltage across the resistor in a half-wave rectified sine wave with peak value $V_m = V_{rms} * \sqrt{2} = 300\sqrt{2} = 424.4\,V$. The average voltage in a half-wave rectifier is Vm/π, and average current is:

$$\langle i_o \rangle = \frac{V_m}{\pi R} = \frac{300\left(\sqrt{2}\right)}{25\pi} = 5.4A$$

Part (b):

The rms voltage across the resistor for a half-wave rectified sinusoid is:

$$V_{orms} = \frac{V_m}{2} = \frac{300\left(\sqrt{2}\right)}{2} = 212.132V$$

$$I_{rms} = \frac{V_m}{2R} = \frac{300\sqrt{2}}{2*25} = 8.488\ A$$

The power absorbed by the resistor is:

$$P = R \times I_{orms}{}^2 = \frac{V_{orms}{}^2}{R} = \frac{(212.132)^2}{25} = 1800\ W$$

The rms current in the resistor is 8.488 A, and the power could also be calculated from $RI^2_{orms} = 25 \times (8.488)^2 = 1800$ w

Part (c):

Apparent Power= $V_{in-rms} * I_{in-rms}$=300*8.488= 2545

Part (d): The power factor is:

$$pf = \frac{\text{avreage power}}{\text{apparent power}} = \frac{P}{V_{in-rms} \times I_{in-rms}} = \frac{1800}{300 \times 8.488} = 0.707$$

Problem 4:

For the half-wave rectifier with R-L load, R=100Ω, L=0.1 H, ω=377 rad/s, and V_m=100V. Determine

(a) an expression for the current in this circuit,

(b) the average current,

(c) the rms current

(d) the power absorbed by the resistor, and

(e) the power factor.

Hint: $\boldsymbol{\beta = 3.5\, rad}$

Solution:

Part (a):

For a half-wave rectifier with R-L load:

$$i(t) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t - \theta) + Ae^{-t/\frac{L}{R}}$$

Where, $\theta = tan^{-1}(\frac{L\omega}{R})$ and $A = \frac{V_m sin\theta}{\sqrt{R^2+(L\omega)^2}}$

For the given problem:

$$Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{(100)^2 + (0.1 * 377)^2} = 106.87$$

$$\theta = tan^{-1}\left(\frac{L\omega}{R}\right) = tan^{-1}\left(\frac{37.7}{100}\right) = tan^{-1}(0.377) = 0.36\, rad = 20.64^{\circ}$$

$$\mathrm{i}(\omega\mathrm{t}) = \frac{100}{106.87} \sin(\omega\mathrm{t} - 0.36) + \frac{100 * \sin(0.36)}{106.87} \mathrm{e}^{-\omega t/0.377}$$

$\beta = 3.5$ rad is given by the problem, so

$$\mathrm{i}(\omega\mathrm{t}) = 0.936\sin(\omega\mathrm{t} - 0.36) + 0.33\mathrm{e}^{-\omega t/0.377} \qquad for \quad 0 \le \omega\mathrm{t} \le 3.5$$

$$\mathrm{i}(\omega\mathrm{t}) = 0 \qquad for \quad 3.5 \le \omega\mathrm{t} \le 2\pi$$

Part (b):

$$\langle \mathrm{i}(\omega t)\rangle = \frac{1}{2\pi}\int_0^{3.5}[0.936\sin(\omega t - 0.36) + 0.33e^{-\omega t/0.377}]d(\omega t)$$

$$= \frac{1}{2\pi}[-0.936(\cos(3.5 - 0.36) - \cos(-0.36)) + 0.33(-0.377)(e^{-3.5/0.377} - e^{0})]$$

$$= \frac{1}{2\pi}[(-0.936)(-1.935) + 0.33(-0.377)(-1)] = 0.308\ A$$

Part (c):

$$\boldsymbol{I_{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{3.5}[0.936\sin(\omega t - 0.36) + 0.33e^{-\omega t/0.377}]^2 d(\omega t)} = 0.474\ A$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{3.5}\frac{(0.936)^2}{2}(1 - \cos(2\omega t - 0.72)) + (0.33)^2 e^{-2\omega t/0.377} + 2(0.33)(0.936)e^{-\omega t/0.377}\sin(\omega t - 0.36)d(\omega t)}$$

$$I_{rms} = 0.474\quad A$$

Note that:

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2 + b^2}(b\sin bx + a\cos bx)$$

$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

Part (d): the power absorbed by the resistor is

$$\mathrm{P} = \mathrm{R} * \mathrm{I}_{\mathrm{rms}}^2 = 100 * (0.474)^2 = 22.47\ W$$

Note that average power absorbed by the inductor is zero.

Part (e):

$$Pf = \frac{P}{V_{in-rms} * I_{in-rms}} = \frac{22.47}{\frac{100}{\sqrt{2}} * 0.474} = 0.67$$

Problem 5:

For a half-wave rectifier with R-L load with $V_{in} = 300\sin(377t)$ and R=12Ω and L=12 mH. Determine

(a) an expression for load current,

(b) the average current,

(c) the power absorbed by the resistor, and

(d) the power factor.

Solution:

Part (a):

For the given problem:

$$Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{(12)^2 + (0.012 * 377)^2} = 12.82$$

$$\theta = tan^{-1}\left(\frac{L\omega}{R}\right) = tan^{-1}\left(\frac{4.524}{12}\right) = tan^{-1}(0.377) = 0.36\,rad = 20.64^{\circ}$$

$$\mathrm{i}(\omega \mathrm{t}) = \frac{300}{12.82}\sin(\omega\mathrm{t} - 0.36) + \frac{300 * \sin(0.36)}{12.82}\mathrm{e}^{-\omega t/0.377}$$

$$\mathrm{i}(\omega \mathrm{t}) = 23.4\sin(\omega\mathrm{t} - 0.36) + 8.24\mathrm{e}^{-\omega t/0.377} \qquad for \quad 0 \le \omega\mathrm{t} \le 3.5$$

$$\mathrm{i}(\omega \mathrm{t}) = 0 \qquad for \quad 3.5 \le \omega\mathrm{t} \le 2\pi$$

Part (b):

$$\langle \mathrm{i}(\omega \mathrm{t})\rangle = \frac{1}{2\pi}\int_0^{3.5}[23.4\sin(\omega\mathrm{t} - 0.36) + 8.24\mathrm{e}^{-\omega t/0.377}]d(\omega\mathrm{t})$$

$$= \frac{1}{2\pi}[-23.4(\cos(3.5 - 0.36) - \cos(-0.36)) + 8.24(-0.377)(\mathrm{e}^{-3.5/0.377} - \mathrm{e}^{0})]$$

$$= \frac{1}{2\pi}[-23.4(-1.935) + 8.24(-0.377)(-1)] = 7.7 \ A$$

Part (c): the power absorbed by the resistor is $P = R * I_{rms}^2$

So, first we need to calculate I_{rms}

$$\boldsymbol{I_{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{3.5}[23.4\sin(\omega t - 0.36) + 8.24e^{-\omega t/0.377}]^2 d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}(880.75)} = 11.84\ A$$

$$\int_0^{3.5}[\frac{(23.4)^2}{2}(1 - \cos(2\omega t - 0.72)) + (8.24)^2 e^{-2\omega t/0.377} + 386e^{-\omega t/0.377} sin(\omega t - 0.36)]\, d(wt)$$

$$= \frac{(23.4)^2}{2}\left(3.5 - \frac{1}{2}\sin(0.72)\right) + (8.24)^2\left(\frac{-0.377}{2}\right)(-1) + 0 = 880.75$$

$$P = R * I_{rms}^2 = 12 * (11.84)^2 = 1682.23$$

Part (d):

$$Pf = \frac{P}{V_{in-rms} * I_{in-rms}} = \frac{1682.23}{\frac{300}{\sqrt{2}} * 11.84} = 0.67$$