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EXERCISE 3

ELEC-E8409 HIGH VOLTAGE ENGINEERING

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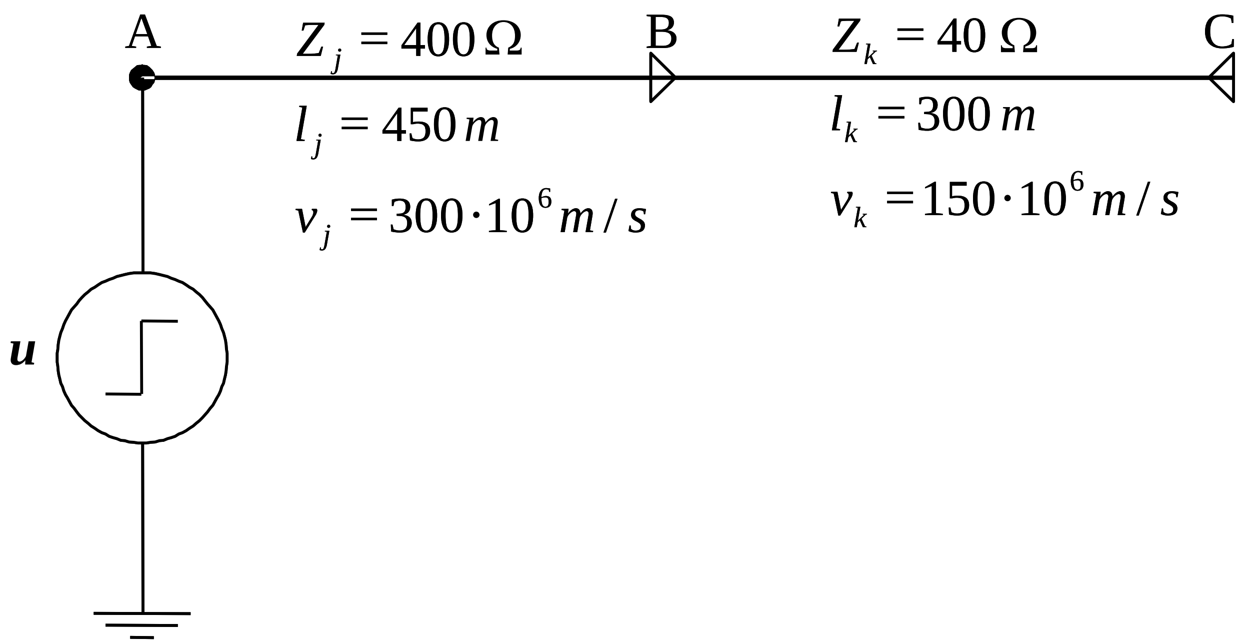
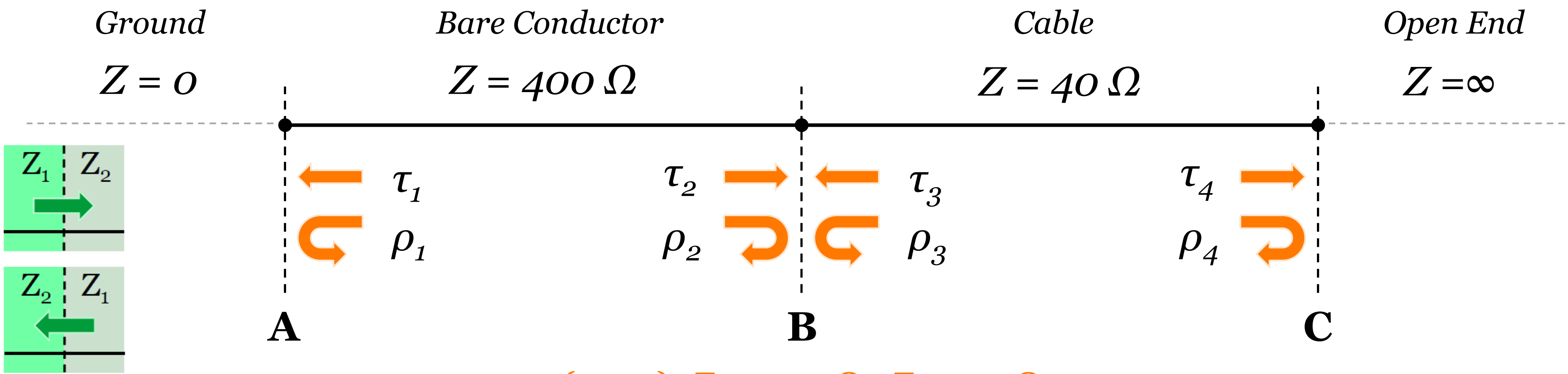
Question 1

- A step wave of amplitude u is applied to the open line illustrated in the figure. Study voltage as a function of time at points A, B and C.



Transmission coefficient: $\tau = \dfrac{2Z_2}{Z_1 + Z_2}$ $\quad (\tau = \rho + 1)$

Reflection coefficient: $\rho = \dfrac{Z_2 - Z_1}{Z_1 + Z_2}$



(τ_1, ρ_1): Z_1 = 400 Ω, Z_2 = 0

$$\tau_1 = \frac{2Z_2}{Z_1 + Z_2} = \frac{2(0)}{400 + 0} = 0$$

$$\rho_1 = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{0 - 400}{400 + 0} = -1$$

(τ_2, ρ_2): Z_1 = 400 Ω, Z_2 = 40 Ω

$$\tau_2 = \frac{2Z_2}{Z_1 + Z_2} = \frac{2(40)}{400 + 40} = \frac{80}{440} = 0.18$$

$$\rho_2 = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{40 - 400}{400 + 40} = \frac{-360}{440} = -0.82$$

(τ_3, ρ_3): Z_1 = 40 Ω, Z_2 = 400 Ω

$$\tau_3 = \frac{2Z_2}{Z_1 + Z_2} = \frac{2(400)}{40 + 400} = \frac{800}{440} = 1.82$$

$$\rho_3 = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{400 - 40}{40 + 400} = \frac{360}{440} = 0.82$$

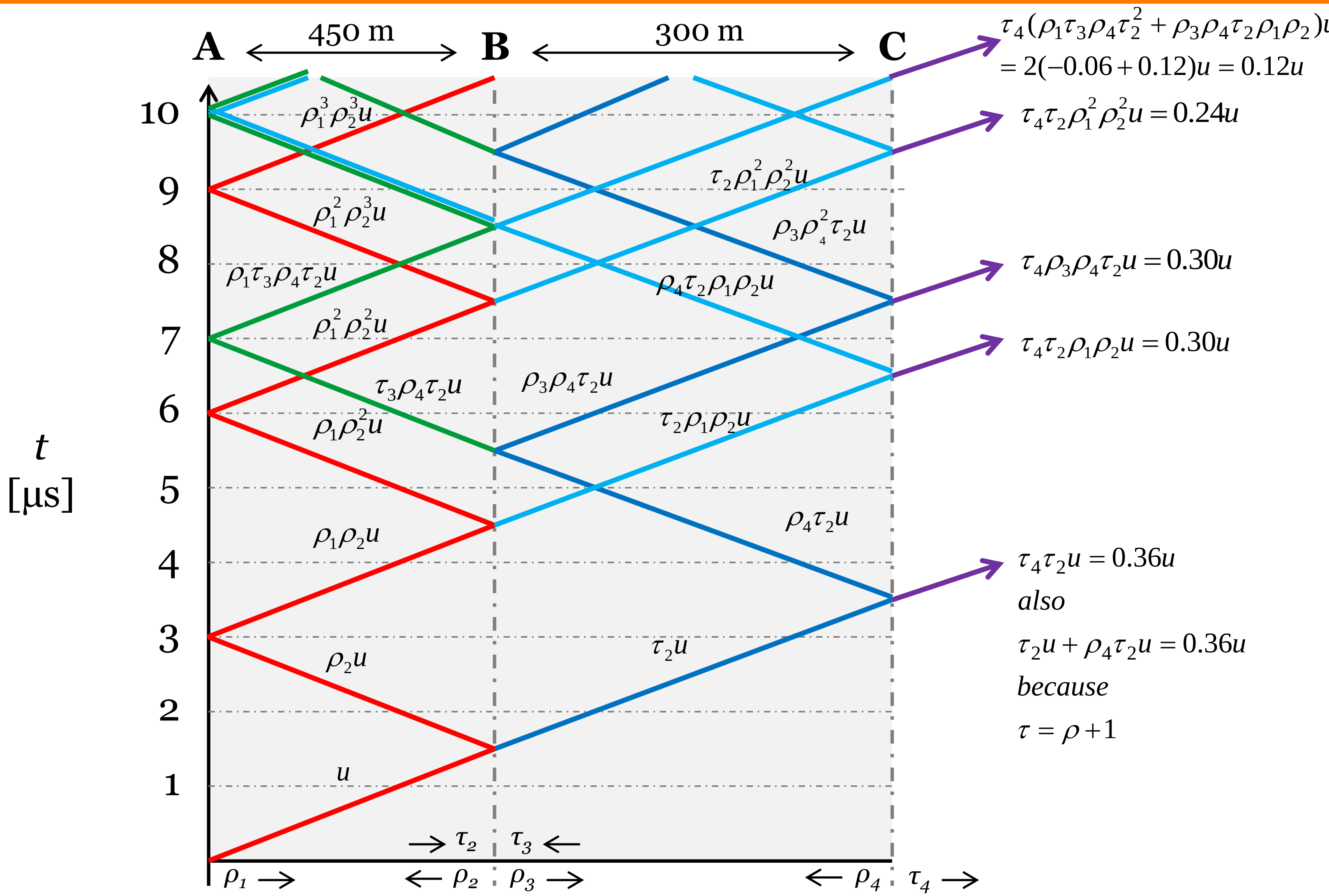
(τ_4, ρ_4): Z_1 = 40 Ω, Z_2 = ∞

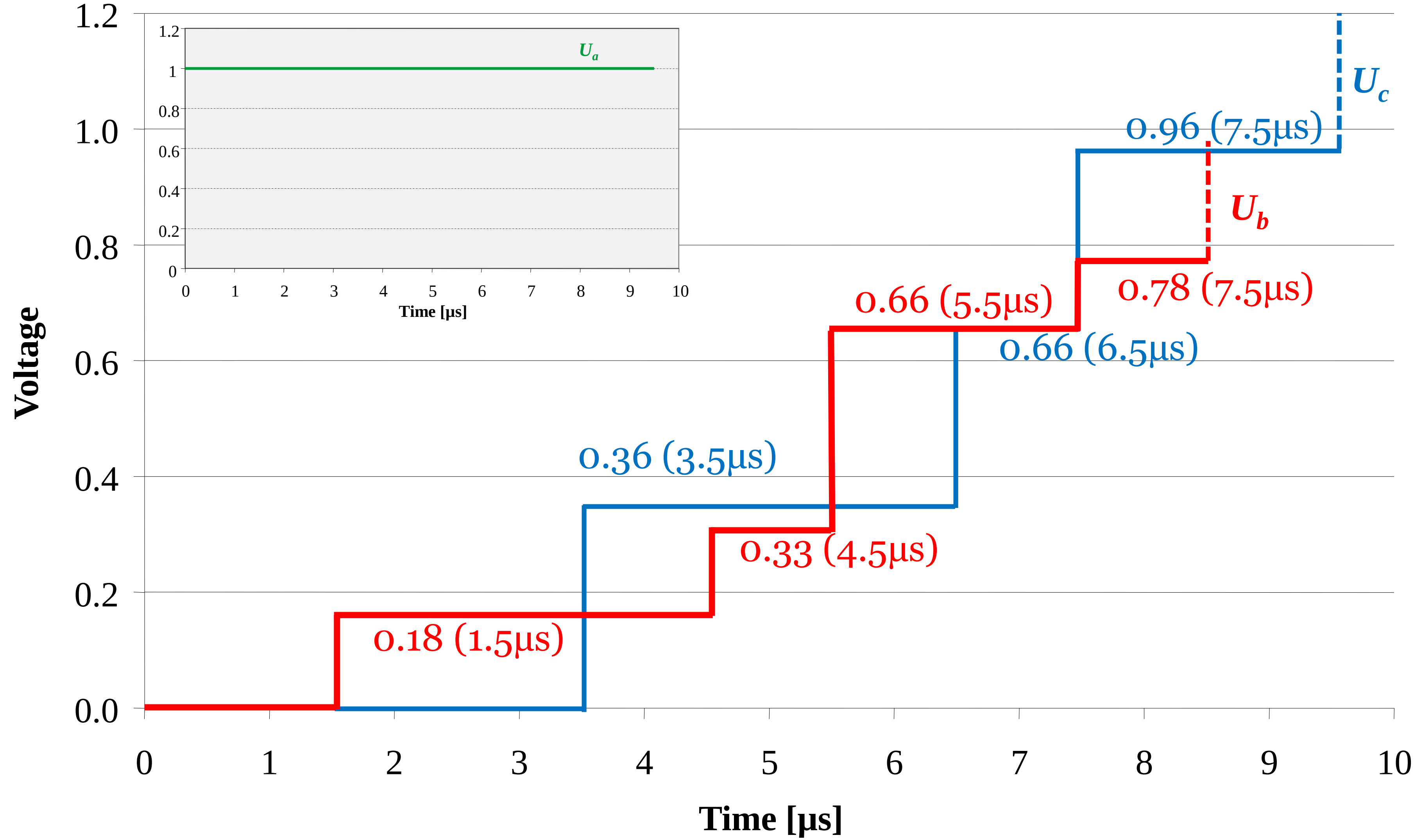
$$\tau_4 = \frac{2Z_2}{Z_1 + Z_2} = \frac{2(\infty)}{40 + \infty} = \frac{2\infty}{\infty} = 2$$

$$\rho_4 = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{\infty - 40}{40 + \infty} = \frac{\infty}{\infty} = 1$$

Propagation of waveform:

$$t_{AB} = \frac{l_j}{v_j} = 1.5\,\mu s \qquad t_{BC} = \frac{l_k}{v_k} = 2.0\,\mu s$$

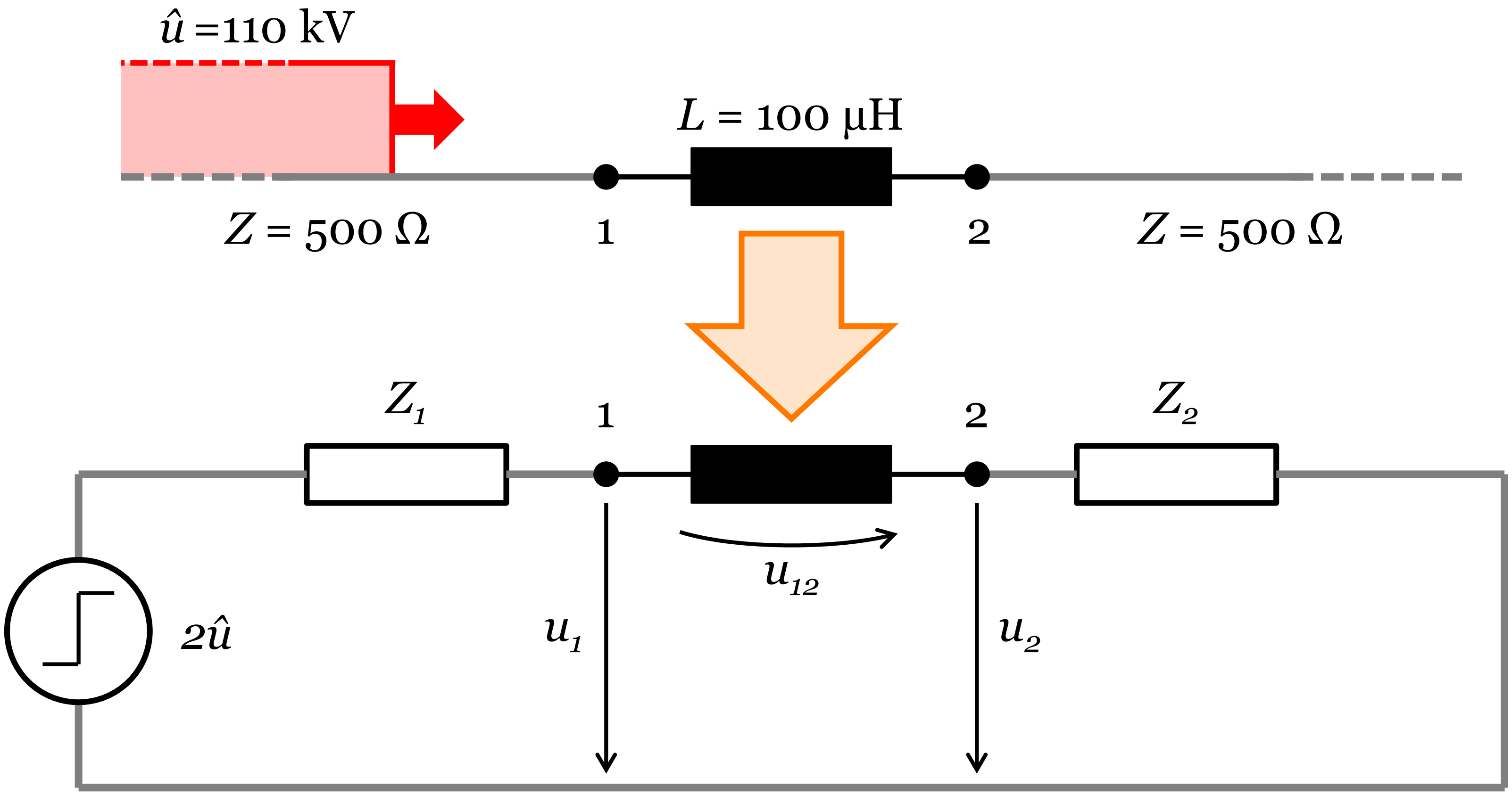




1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
Time [µs]
U_a
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Voltage
0 1 2 3 4 5 6 7 8 9 10
Time [µs]
U_c
0.96 (7.5µs)
U_b
0.78 (7.5µs)
0.66 (5.5µs)
0.66 (6.5µs)
0.36 (3.5µs)
0.33 (4.5µs)
0.18 (1.5µs)

Question 2

- A current transformer is linked to a 20-kV bare conductor line. The primary coil inductance is 100 μH. Wave impedance of the line is 500 Ω. A step wave of amplitude 110 kV arrives at the current transformer. Show, using equations and approximate drawings, both of the transformer's 20-kV terminals against ground and also the voltage between the terminals.



Easy Solution

Lecture slides:

$$u_1(t) = 2\hat{u} - \frac{Z_1}{Z_1 + Z_2} 2\hat{u}\left(1 - e^{-t/\tau}\right)$$

$$u_2(t) = \frac{Z_2}{Z_1 + Z_2} 2\hat{u}\left(1 - e^{-t/\tau}\right)$$

$$u_{12}(t) = 2\hat{u}e^{-t/\tau}$$

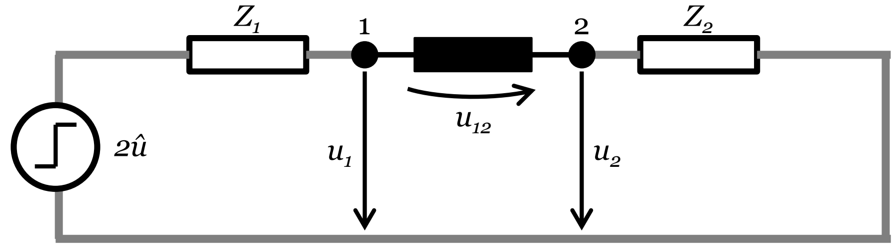
$$\tau = \frac{L}{Z_1 + Z_2}$$

$$u_1(t) = 110\left(1 + e^{-t/0.1\mu s}\right) \text{kV}$$

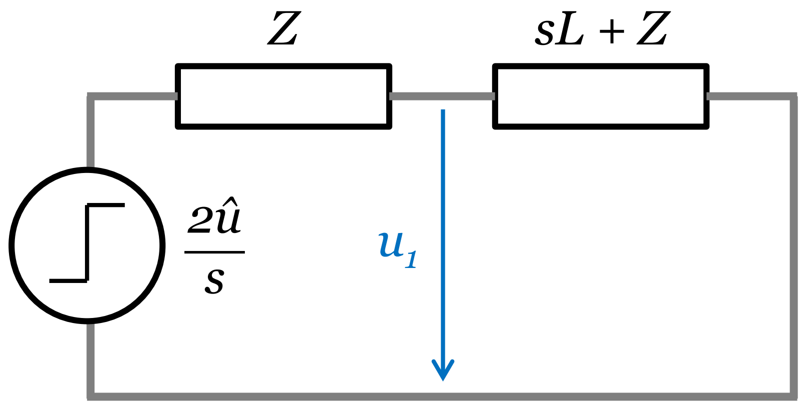
$$u_2(t) = 110\left(1 - e^{-t/0.1\mu s}\right) \text{kV}$$

$$u_{12}(t) = 220e^{-t/0.1\mu s} \text{ kV}$$

Derive the equations using Laplace transformation:



u_1:



$$u_1 = \frac{sL+Z}{sL+2Z}\cdot\frac{2\hat{u}}{s}$$

$$u_1 = \left(\frac{sL+2Z}{sL+2Z}-\frac{Z}{sL+2Z}\right)\frac{2\hat{u}}{s} = \left(1-\frac{Z}{sL+2Z}\right)\frac{2\hat{u}}{s} = \frac{2\hat{u}}{s}-\frac{2\hat{u}Z}{sL\left(s+\frac{2Z}{L}\right)} \rightarrow 2\hat{u}\frac{1}{s}-\hat{u}\frac{\frac{2Z}{L}}{s\left(s+\frac{2Z}{L}\right)}$$

Laplace

$$\frac{1}{s}\Rightarrow u_s(t)$$

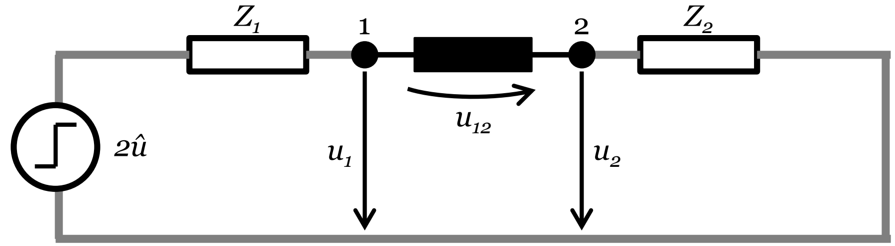
Laplace

$$\frac{a}{s(s+a)}\Rightarrow 1-e^{-at}$$

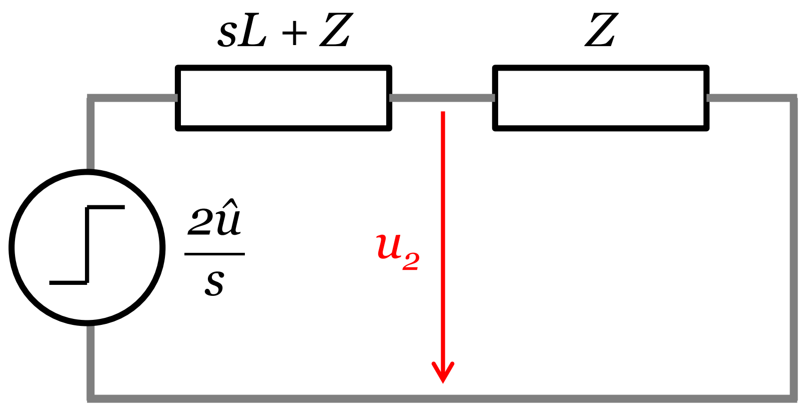
$$\left[\tau = \frac{L}{Z_1+Z_2} = \frac{L}{2Z}\right]$$

$$L^{-1} = 2\hat{u}-\hat{u}(1-e^{-\frac{2Z}{L}t}) = \hat{u}(2-1+e^{-\frac{2Z}{L}t}) = \hat{u}(1+e^{-\frac{t}{\tau}}) = 110(1+e^{-t/0.1\mu s})\ \text{kV}$$

Derive the equations using Laplace transformation:



u_2:



$$u_2 = \frac{Z}{sL+2Z}\cdot\frac{2\hat{u}}{s}$$

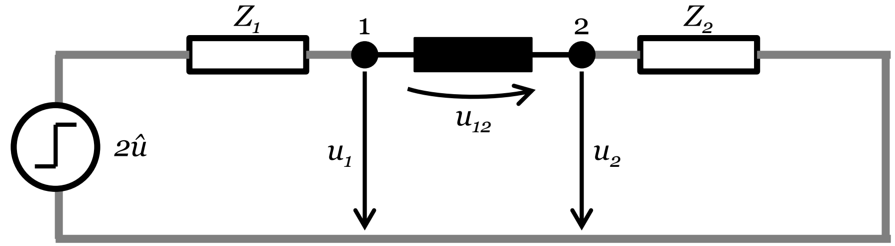
$$u_2 = \frac{2\hat{u}Z}{s(sL+2Z)} = \frac{2\hat{u}Z}{sL\left(s+\frac{2Z}{L}\right)} = \hat{u}\frac{\frac{2Z}{L}}{s\left(s+\frac{2Z}{L}\right)}$$

Laplace

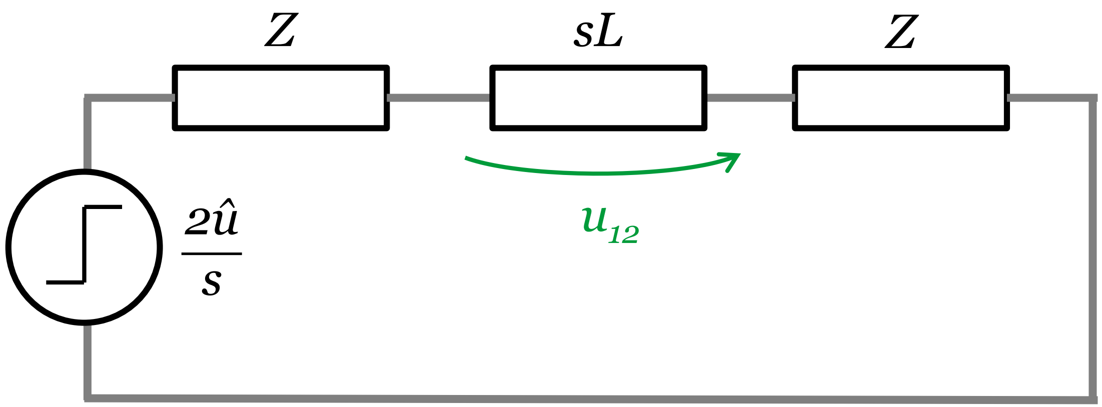
$$\frac{a}{s(s+a)} \Rightarrow 1-e^{-at}$$

$$L^{-1} = \hat{u}\left(1-e^{-\frac{2Z}{L}t}\right) = \hat{u}\left(1-e^{-\frac{t}{\tau}}\right) = 110\left(1-e^{-t/0.1\mu s}\right)\ \text{kV}$$

Derive the equations using Laplace transformation:



u_{12}:



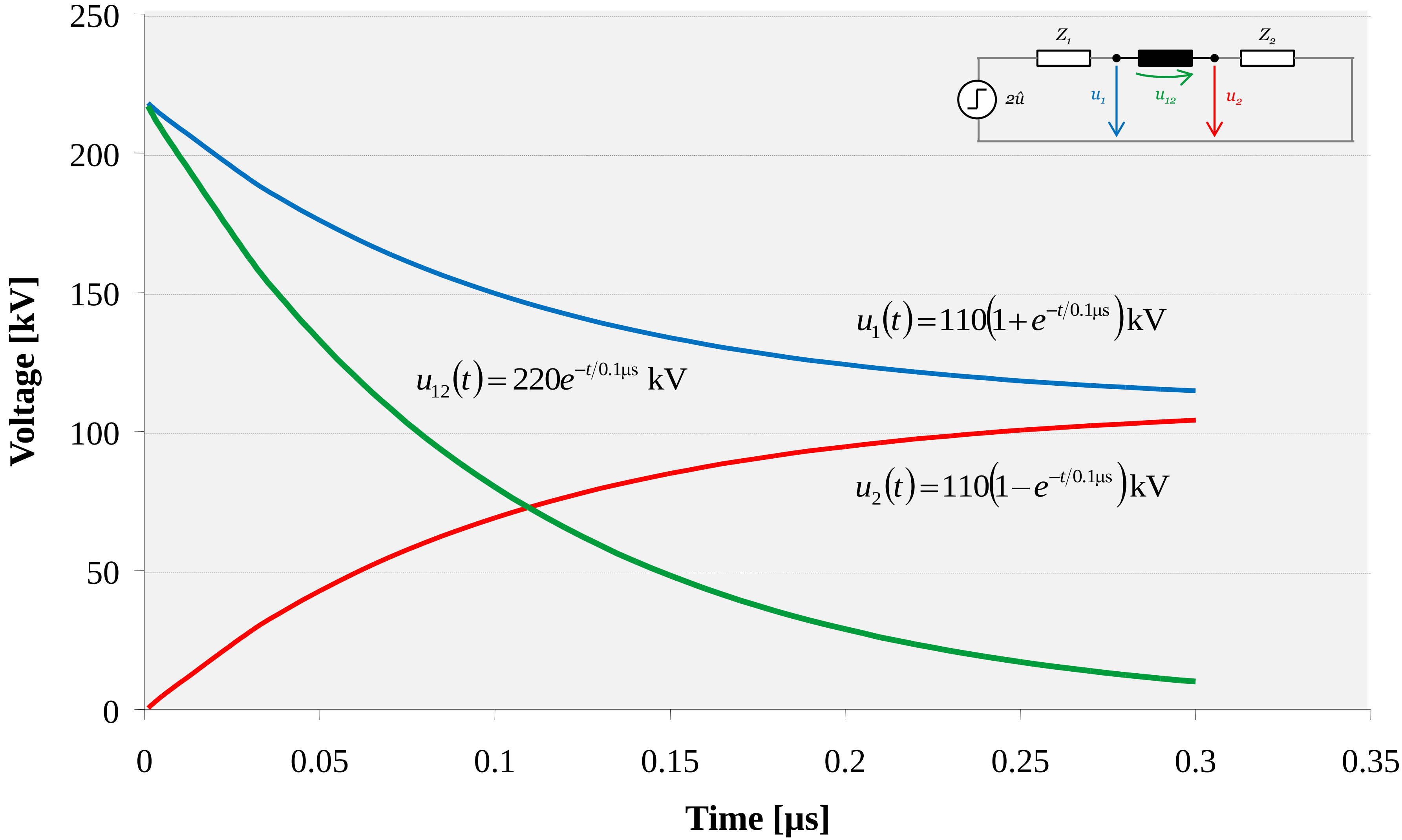
$$u_{12} = \frac{sL}{sL+2Z} \cdot \frac{2\hat{u}}{s}$$

$$u_{12} = \frac{2\hat{u}L}{sL+2Z} = \frac{2\hat{u}L}{L\left(s+\frac{2Z}{L}\right)} = 2\hat{u}\frac{1}{s+\frac{2Z}{L}}$$

Laplace

$$\frac{1}{s+a} \Rightarrow e^{-at}$$

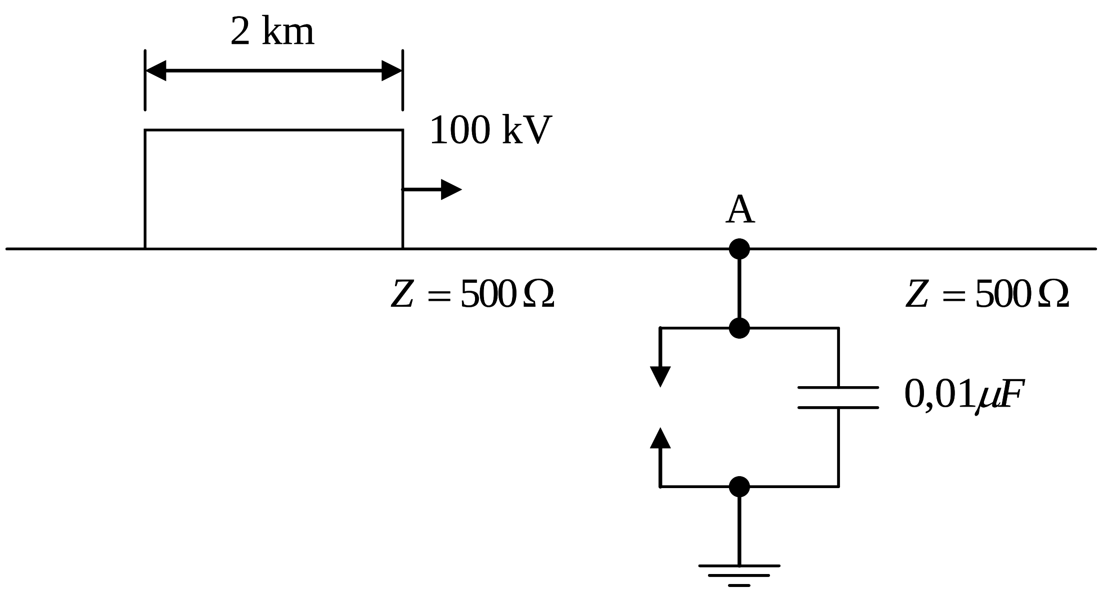
$$L^{-1} = 2\hat{u} \cdot e^{-\frac{2Z}{L}t} = 2\hat{u}e^{-\frac{t}{\tau}} = 220e^{-t/0.1\mu s}\ \text{kV}$$

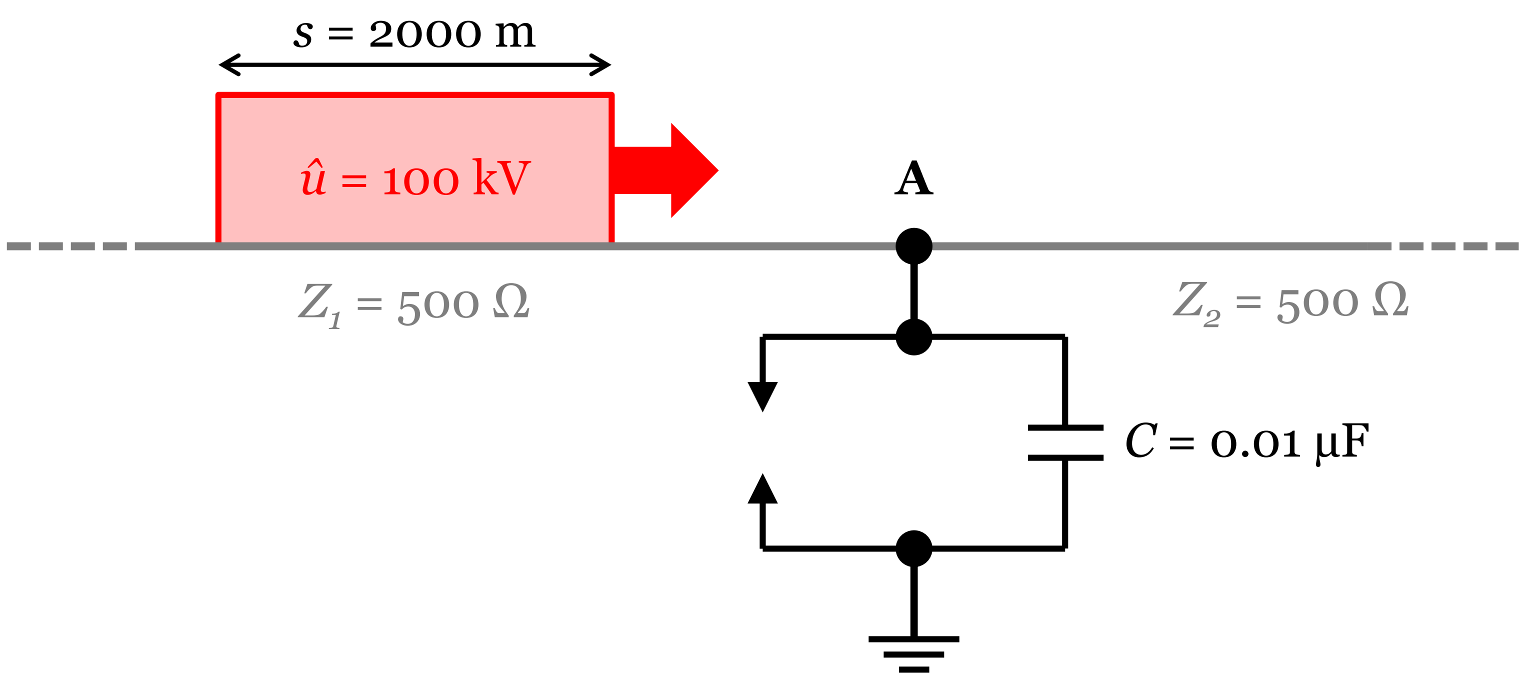


Voltage [kV]
250
200
150
100
50
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Time [µs]
$u_1(t) = 110\left(1 + e^{-t/0.1\mu s}\right)$ kV
$u_{12}(t) = 220e^{-t/0.1\mu s}$ kV
$u_2(t) = 110\left(1 - e^{-t/0.1\mu s}\right)$ kV
Z_1
Z_2
$2\hat{u}$
u_1
u_{12}
u_2

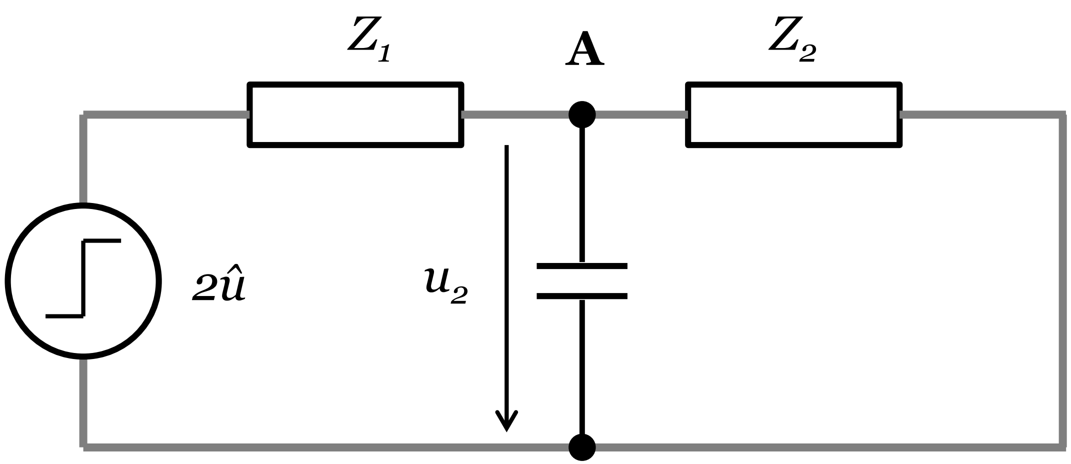
Question 3

- An overhead line receives a square impulse of amplitude 100 kV and length 2 km at point A. Point A is connected to a spark gap parallel with a 0.01 μF capacitor. The inception voltage of the gap is 50 kV. Draw the reflecting waveform at point A and define the waveform parameters.





Equivalent Circuit



$$u_2 = \frac{Z_2}{Z_1 + Z_2} 2\hat{u}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{Z_1 Z_2}{Z_1 + Z_2} C$$

$$u_2 = 100(1 - e^{-t/\tau})kV$$

$$u_2 = 100(1 - e^{-t/\tau})kV$$

Inception voltage of spark gap = 50 kV:

$$50 = 100\left(1 - e^{-t/\tau}\right) \Rightarrow 0.5 = 1 - e^{-t/\tau} \Rightarrow 0.5 = e^{-t/\tau} \Rightarrow \ln 0.5 = -\frac{t}{\tau}$$

$$\tau = \frac{Z_1 Z_2}{Z_1 + Z_2} C$$

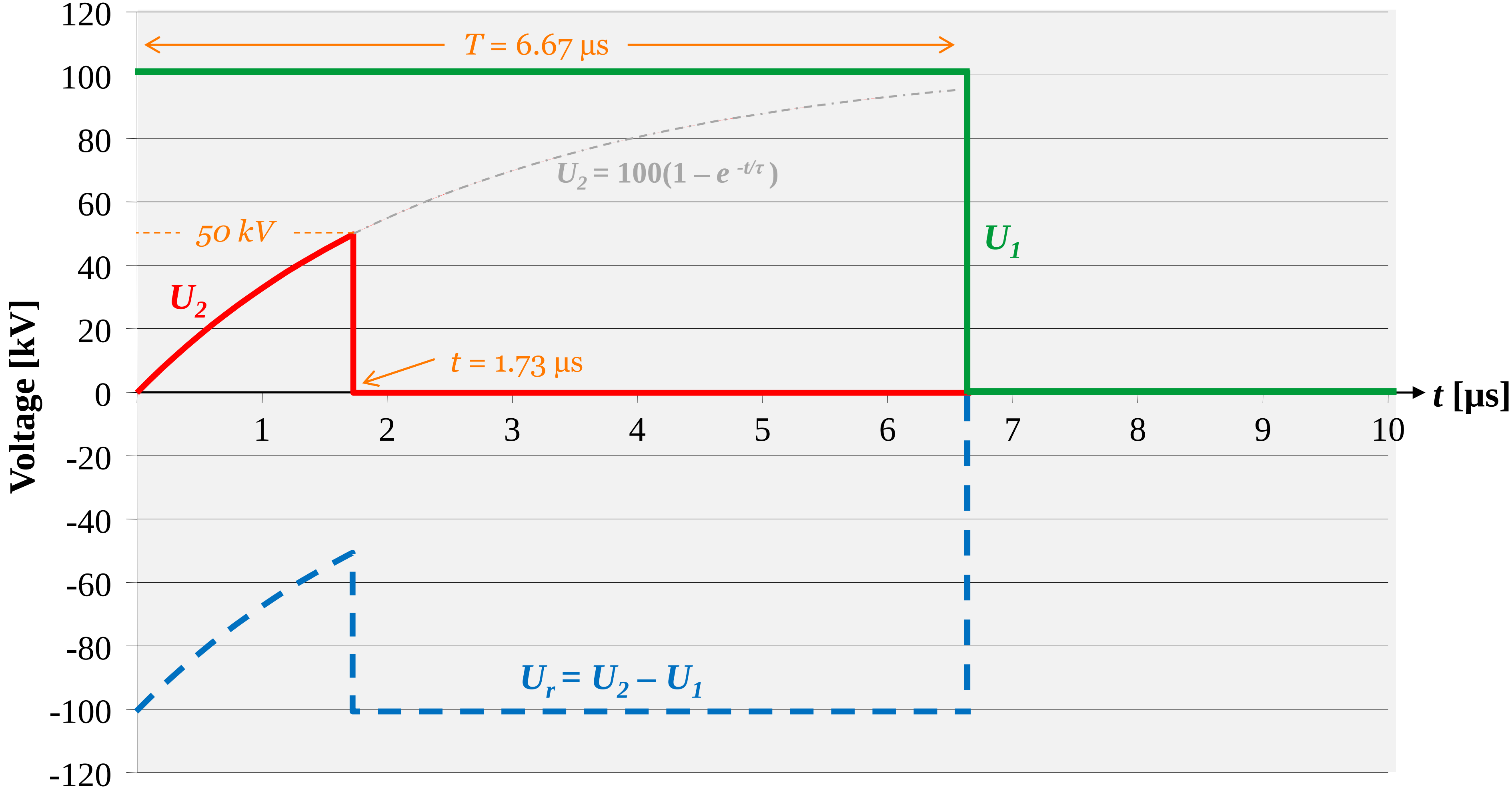
Inception time
(moment of ignition)

$$t = -\tau \ln 0.5 = -\frac{500 \cdot 500}{500 + 500}\left(0.01 \cdot 10^{-6}\right) \ln 0.5 = 1.73\ \mu s$$

Duration of impulse

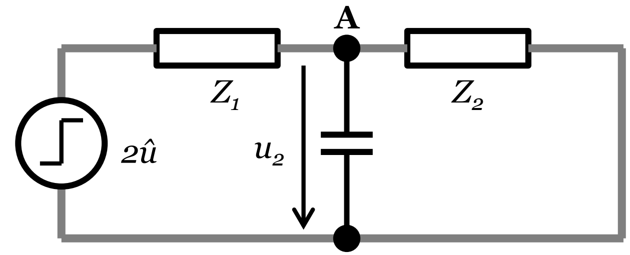
$$T = \frac{s}{v} = 6.67\ \mu s$$

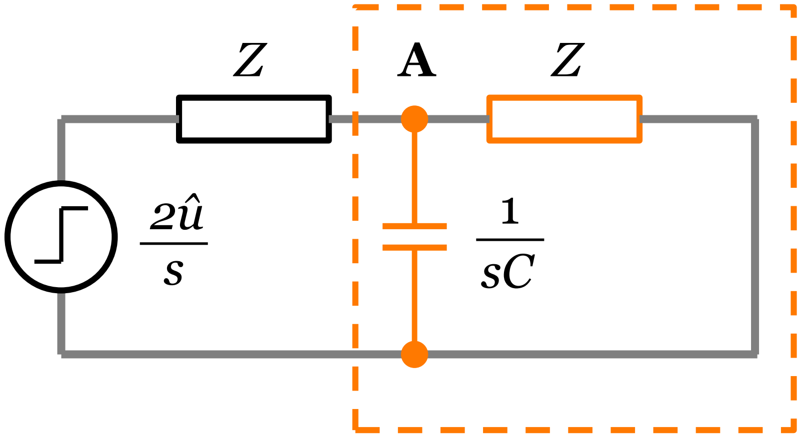
(v = speed of light c)

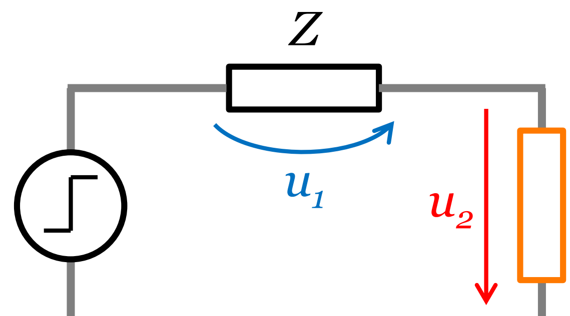


Reflected wave: $u_{1r} = u_2 - u_1$

Derive the equations using Laplace transformation:







$$\frac{Z\cdot\frac{1}{sC}}{Z+\frac{1}{sC}}=\frac{Z}{ZCs+1}$$

$$u_2=\frac{\frac{Z}{ZCs+1}}{Z+\frac{Z}{ZCs+1}}\cdot\frac{2\hat{u}}{s}=\frac{Z}{Z\left(ZCs+1\right)+Z}\cdot\frac{2\hat{u}}{s}=\frac{1}{\left(ZCs+1\right)+1}\cdot\frac{2\hat{u}}{s}=\frac{1}{ZCs+2}\cdot\frac{2\hat{u}}{s}=\frac{2\hat{u}}{s\left(ZCs+2\right)}=\frac{2\hat{u}}{sZC\left(s+\frac{2}{ZC}\right)}=\hat{u}\,\frac{\frac{2}{ZC}}{s\left(s+\frac{2}{ZC}\right)}$$

Laplace

$$\frac{a}{s(s+a)}\Rightarrow 1-e^{-at}$$

$$\tau=\frac{ZC}{2}$$

$$L^{-1}\Rightarrow \hat{u}\left(1-e^{-\frac{2}{ZC}t}\right)=100\left(1-e^{-t/\tau}\right)\text{kV}$$