Aalto university
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Exercise sheet 2
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 for grading. Deadline Wednesday 28.10 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as one pdf-file with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 29.10 or Friday 30.10.

(1) Express the following in the form $x + iy$:
   (a) $\text{Log}(-e^2)$ (2p)
   (b) $(-1)^i$ (2p)
   (c) $i^{\sqrt{2i}}$ (2p)

(2) Let $A = \{z; \text{Re}(z) > 0\}$ and let $f: A \to \mathbb{C}$ be given by $f(z) = \text{Log}(1 + z^2)$. Determine the range $B = f(A)$. (*Hint:* Look at f as a composition of $g(z) = z^2$, $h(z) = 1 + z$, and $k(z) = \text{Log}\, z$.)
                                                                              (6p)

(3) Show that the locus of points in the complex plane satisfying
$$A z\overline{z} + Bz + \overline{B}\,\overline{z} + C = 0$$
in which A and C are real numbers satisfying
$$|B|^2 - AC > 0$$
is a circle when $A \neq 0$ and a line if $A = 0$. (The converse also holds but you don't need to show this.)

(4) Compute:
   (a) all square roots of $-1 + i\sqrt{3}$.
   (b) all cube roots of -8.