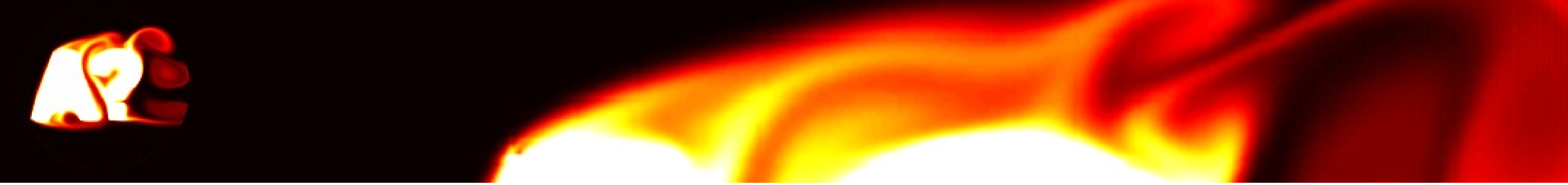


EEN-1020 Heat transfer

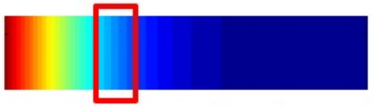
Week 3: Convective Heat Transfer, Internal Flow and Numerical Solution in 2d

Prof. Ville Vuorinen

November 10^{th} - 11^{th} 2020

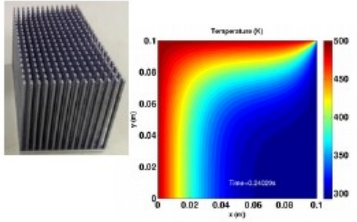
Aalto University, School of Engineering

Week 1: Energy conservation, heat equation, conduction Fourier/Newton

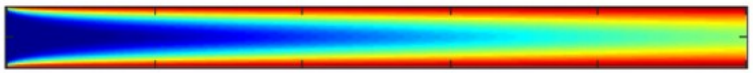


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x}\left(k\frac{\partial T(x+\Delta x/2,t)}{\partial x} - k\frac{\partial T(x-\Delta x/2,t)}{\partial x}\right)$$

Week 2: Fin theory, conduction, intro to convection

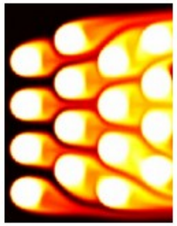


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

Week 5: natural convection, boiling, correlations



In heat transfer course, we have “4 friends” who typically help us to approach and solve any problem

1) Energy conservation

2) Fourier’s law

3) Newton’s cooling law

4) Bonus “helpers”:

    4.1) heat eqn, conv.-diff. eqn (relatives of friend #1)

    4.2) Navier-Stokes eqn (momentum conservation)

    4.3) non-dimensional numbers (follow from the other friends)

Recommended reading: Ch 8 “Internal flow” selective parts from “Principles of Heat and Mass Transfer”, Incropera

Remember: These slides may contain typos or other mistakes so please be cautious when reading.

Remember: Fluid and solid properties depend in reality on thermodynamic conditions so please use always values taken from a proper source (e.g. Incropera Appendix contains some reasonable values)

During weeks 1 and 2 we have mostly envisioned conduction in solid materials (often metals). Also the material properties for solids were often used earlier.

During weeks 3-5, we will focus on convection/conduction heat transfer in fluids (gases and liquids). Also the material properties for fluids are now used mostly.
In fin theory & Newton's law, the convective heat transfer coefficient $\boldsymbol{h}$ was introduced.
Now we study where h actually comes from.

Air: At relatively low velocities (<100m/s) and moderate temperature differences air flow can be assumed to be incompressible (density = constant) which is the most typical assumption also in Incropera text book.

Liquids: Liquids are assumed to be incompressible on this course.

Unless otherwise stated, all fluids (liquids and gases) are assumed incompressible on the present course.

Lecture 3.1 Theory: Flow through a fin system, governing equations and analysis

ILO 3: Student can write the governing equations of fluid/heat flow in a channel, estimate the energy balance and estimate temperature rise for different heating conditions. The student can confirm the channel heat transfer using generated/provided simulation data.

The table below illustrates Nusselt numbers (non-dim.heat trans.coefficient) for different channel types with different boundary conditions.

Relevance: Lecture 3 +HW3 → understand physics beyond the table.

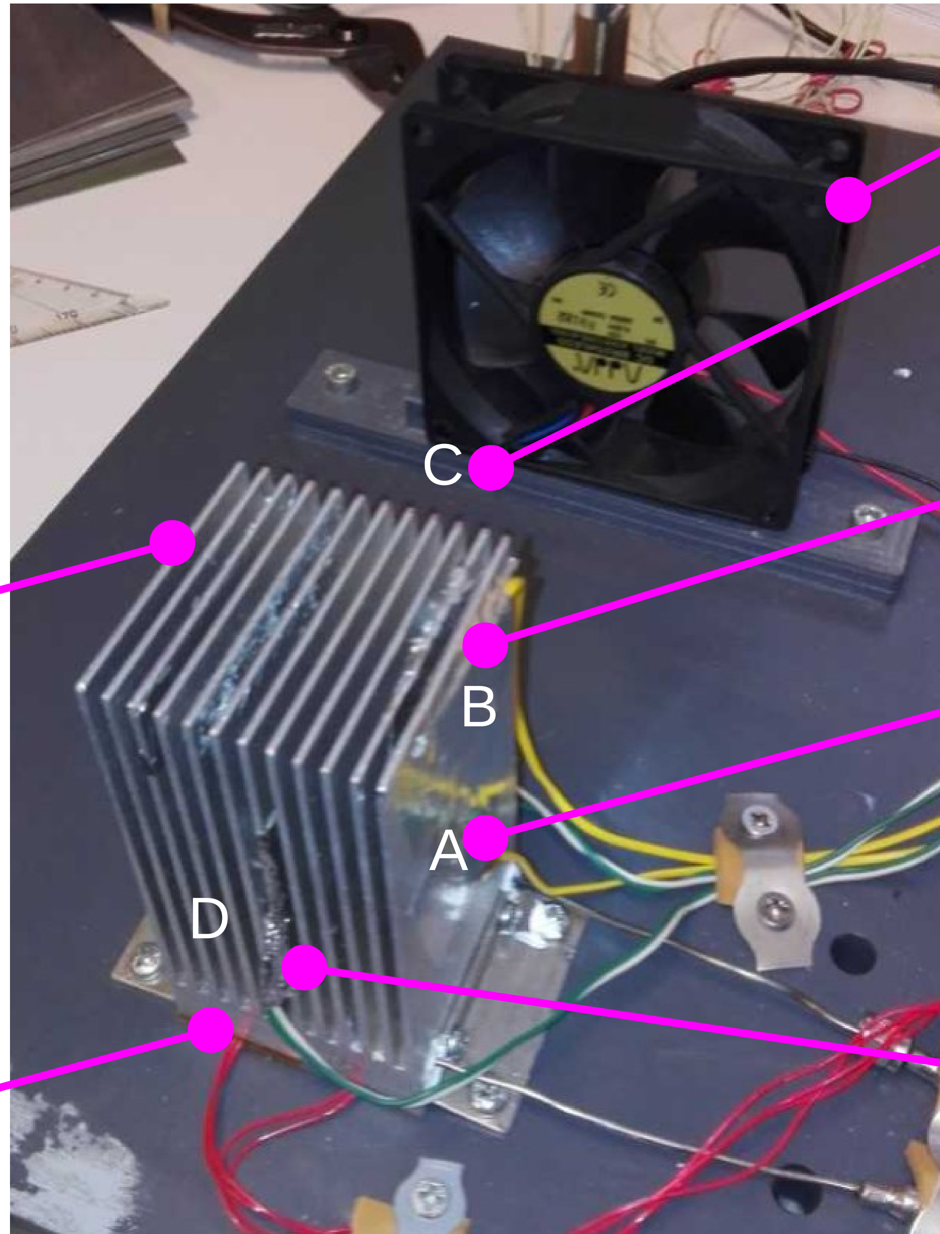
Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	$f\, Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Table 8.1 from Incropera, de Witt (Principles of Heat and Mass Transfer)

In HW3 we want to check if we can get the value Nu = 7.54 from numerical simulation.

Classroom demo heat exchanger (2018)

Acknowledgements:
K.Saari and M.Ahlgren



Fan

Temperature & velocity probe C
(between fins&fan)

Temperature probe B

Temperature probe A

Temperature & velocity probe D
(after fins)

12 fins
Assume 2W heat escapes from each

Heating power
P = 24 W
in the base
Assume uniform heating

Estimate order of magnitude of air heating power (W) based on experimentally measured flow velocity and temperature. **Do you get 2W ?**

Data for air

$P = 24\ W$ 12 fins → **2W**/fin gap $C_p = C_{p,air} = 1.007\ kJ/kgK$

$\Delta T_{CD} \approx 5\ K$ $k = k_{air} = 0.026\ W/mK$

$U_C \approx U_D \approx 1.2\ m/s$ $\rho \approx 1\ kg/m^3$ $A = 0.1m \cdot 0.003m$

Info: The fin gap distance is 3mm

Use the formula:

$$P = C_{p,air}\,\rho U A (T_{out} - T_{in})$$

Power = Energy rate out - Energy rate in

"J/s thinking"

Air temperature distribution in a plate fin heat exchanger (cross section)

Some concepts:
1) Thermal boundary layer
2) Free boundary layer (external flow)
3) Confined boundary layer (internal flow)
4) Local vs global heat transfer coefficient

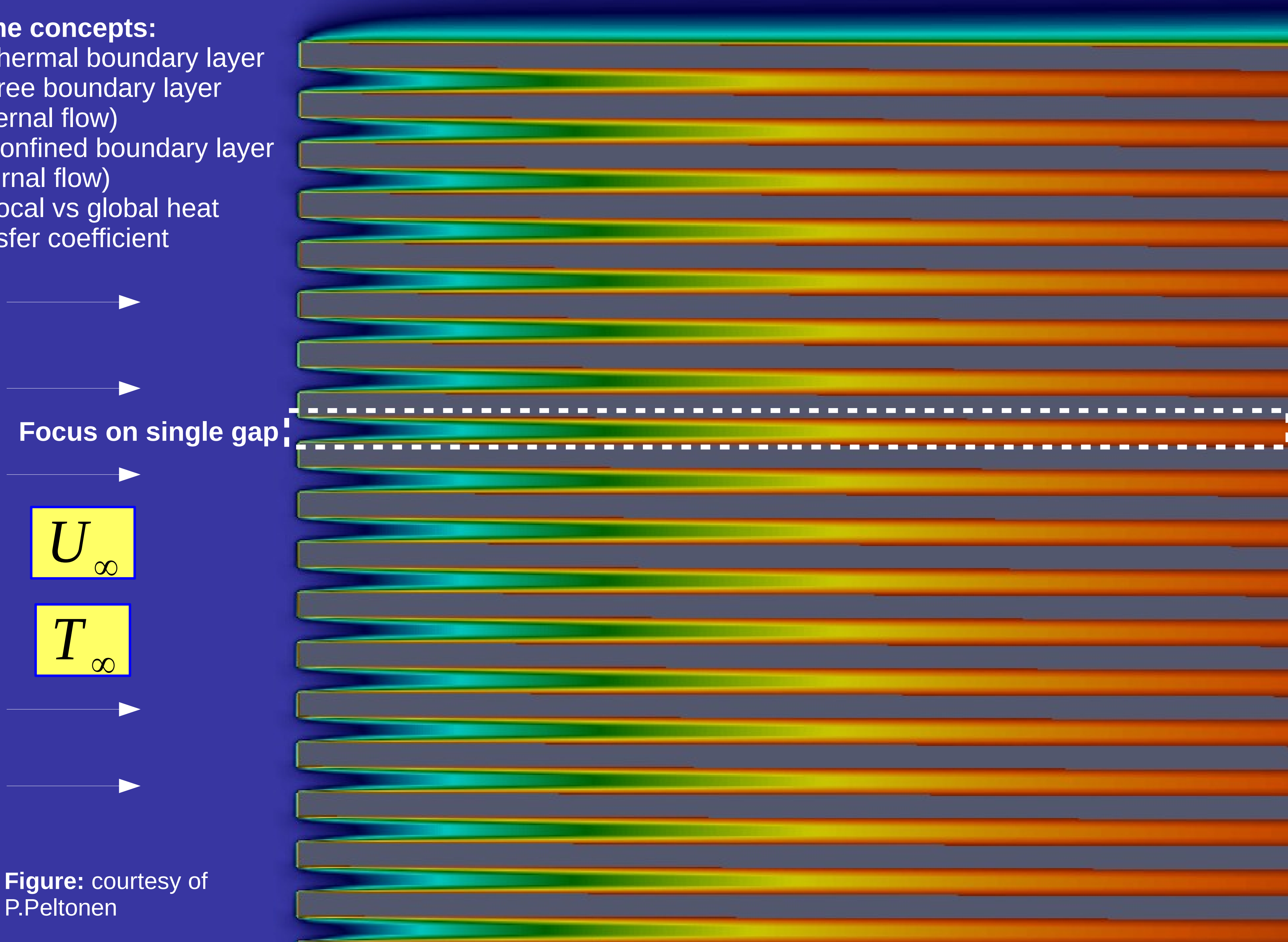


Figure: courtesy of P.Peltonen

Excluding radiation …

Heat transfer (J/s) follows from

1) fluid mechanical behavior of velocity
described by N.S. equation

2) convective and diffusive transport of temperature
described by convection-diffusion equation

Relevance to the course

HW1-HW2 (typically conduction, convection via h if present)
HW3-HW5 (convection and diffusion simultaneously)

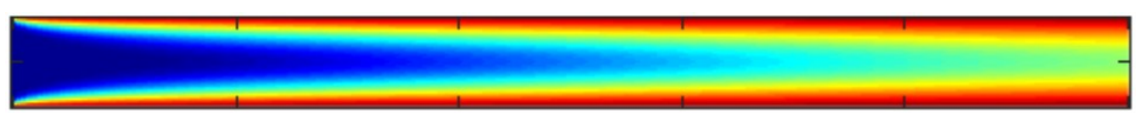
Governing equation 1: Convection-diffusion equation for temperature

In heat transfer, the general transport equation for **temperature** is the convection-diffusion equation which indicates that convective (laminar or turbulent) and diffusive processes dictate outcome of heat transfer problems.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

- $\frac{\partial T}{\partial t}$: T changes in given position in time due to convection and diffusion
- $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$: T is transported by velocity field (convection)
- $\alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$: T is transported by thermal diffusion (diffusion/conduction)

$T=T(x,y)$ in steady state 2d laminar channel flow



Example: CFD solution of instantaneous temperature distribution of unsteady, turbulent fluid flow going from left to right over a heated object – in CFD NS and CD eqn are both solved

Some concepts:
1) Thermal boundary layer
2) Flow separation
3) Local vs global heat transfer coefficient
4) Flow recirculation

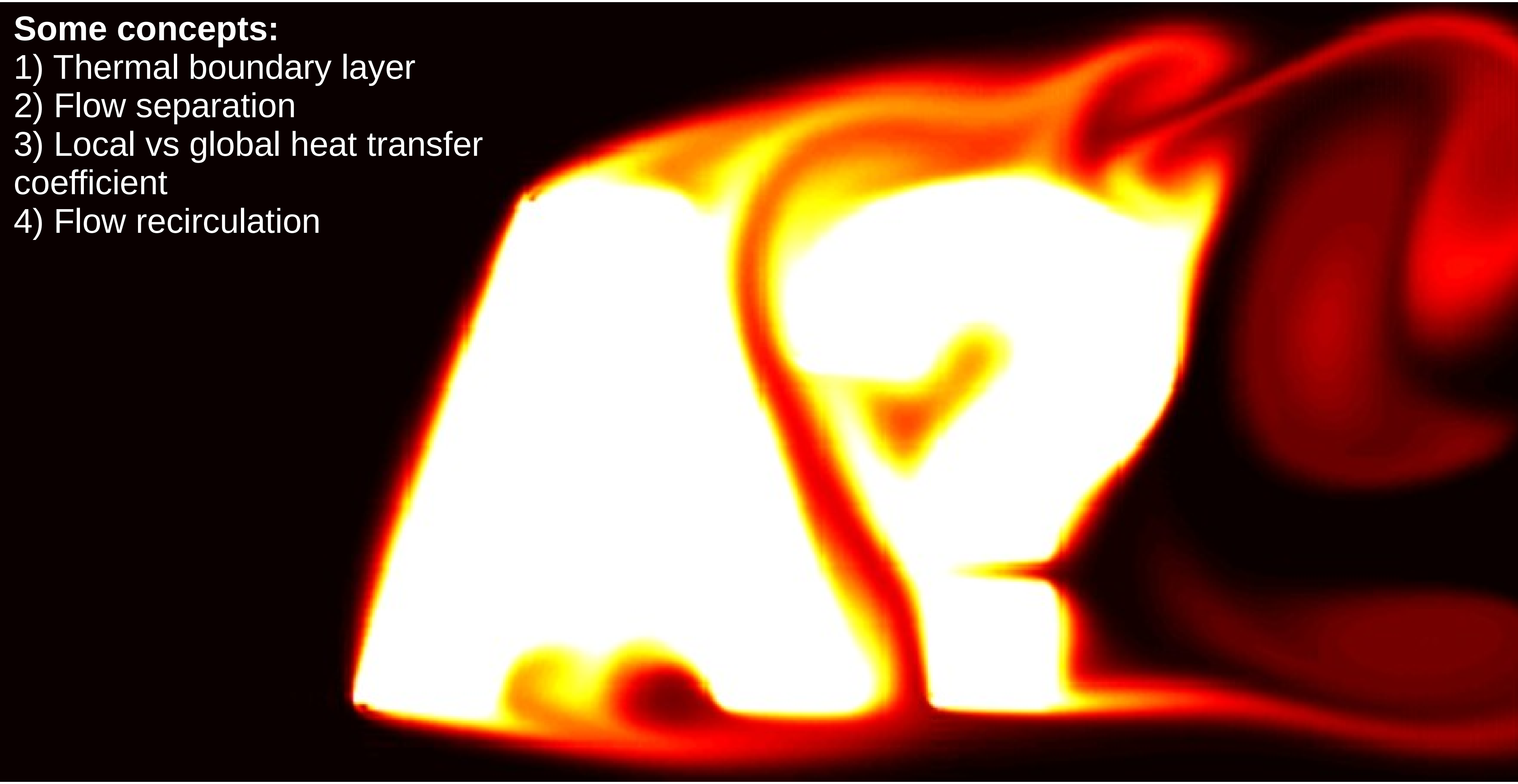


Figure: V.Vuorinen (2017)

Governing equation 2: Navier-Stokes equation for velocity

In heat transfer, the transport equation for **velocity** is the Navier-Stokes equation which is just the convection-diffusion equation for velocity components with pressure gradient. The equation indicates that velocity "self-convects" itself non-linearly and diffuses by molecular viscosity.

Navier-Stokes equation (conservation of momentum)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu_k\frac{\partial^2 u}{\partial x^2}+\nu_k\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu_k\frac{\partial^2 v}{\partial x^2}+\nu_k\frac{\partial^2 v}{\partial y^2}$$

Continuity equation (conservation of mass)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

u,v changes in given position in time due to convection and diffusion and pressure gradient

u,v are transported by velocity field (convection)

pressure gradient

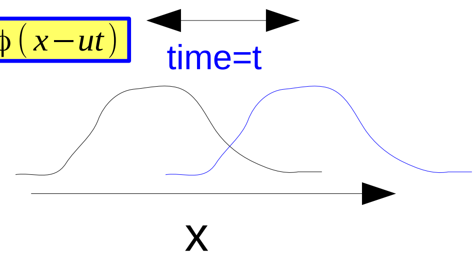
u,v are transported by viscous, molecular diffusion (viscosity)

Kinematic viscosity: $\nu_k=\nu=\mu/\rho, [\nu]=m^2/s$

Explanations on mathematical difference between convection, diffusion, convection-diffusion, and Navier-Stokes equations

1d convection equation

$\phi = \phi(x - ut)$



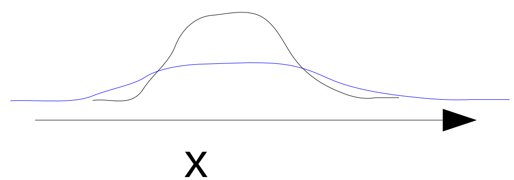
$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} = 0$$

In pure convection

Shape → unchanged
Amplitude → unchanged
Position → moves to the direction of velocity

1d diffusion equation

$$\frac{\partial \phi}{\partial t} = \alpha \frac{\partial^2 \phi}{\partial x^2}$$

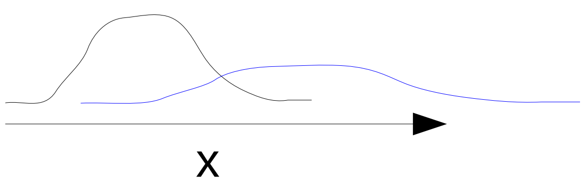


In pure diffusion

Shape → spreads/diffuses
Amplitude → decreases
Position → center fixed but “spreads”

1d convection-diffusion equation

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} = \alpha \frac{\partial^2 \phi}{\partial x^2}$$



In convection+diffusion

Shape → spreads/diffuses
Amplitude → decreases
Position → moves to the direction of velocity

Fluid dynamical and heat transfer conditions

Velocity scale — Reference length scale

$$\text{Reynolds number: } \mathrm{Re} = \frac{U\,L}{\nu}$$

Kinematic viscosity

$$\text{Prandtl number: } \mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\text{Viscous diffusion}}{\text{Thermal diffusion}} = \frac{\mu/\rho}{k/(c_p\,\rho)}$$

Heat transfer coeff. — Reference length scale

$$\text{Nusselt number: } \mathrm{Nu} = \frac{h\,L}{k} = \frac{\text{Total heat transfer}}{\text{Conductive heat transfer}}$$

Estimate Reynolds number for a single "channel" between two plates

Data for air

$P = 24\ W$ 12 fins → **2W**/fin gap $C_p = C_{p,air} = 1.007\ kJ/kgK$

$\Delta T_{CD} \approx 5\ K$ $k = k_{air} = 0.026\ W/mK$

$U_C \approx U_D \approx 1.2\ m/s$ $\rho \approx 1\ kg/m^3$ $A_{plates} = 2 \cdot 0.1m \cdot 0.05m$

$T_{wall} \approx 28\ C$ $D = 0.003m$

$\nu = 1.6*10^{-5}\ m^2/s$ (kinematic viscosity)

Reynolds number

$$Re = UL/\nu = 1.2*0.003/1.6*10^{-5} = 225 << 2000$$

Info: The fin gap distance is 3mm, 2 opposing walls in a channel, height = 0.1m, width=0.05m

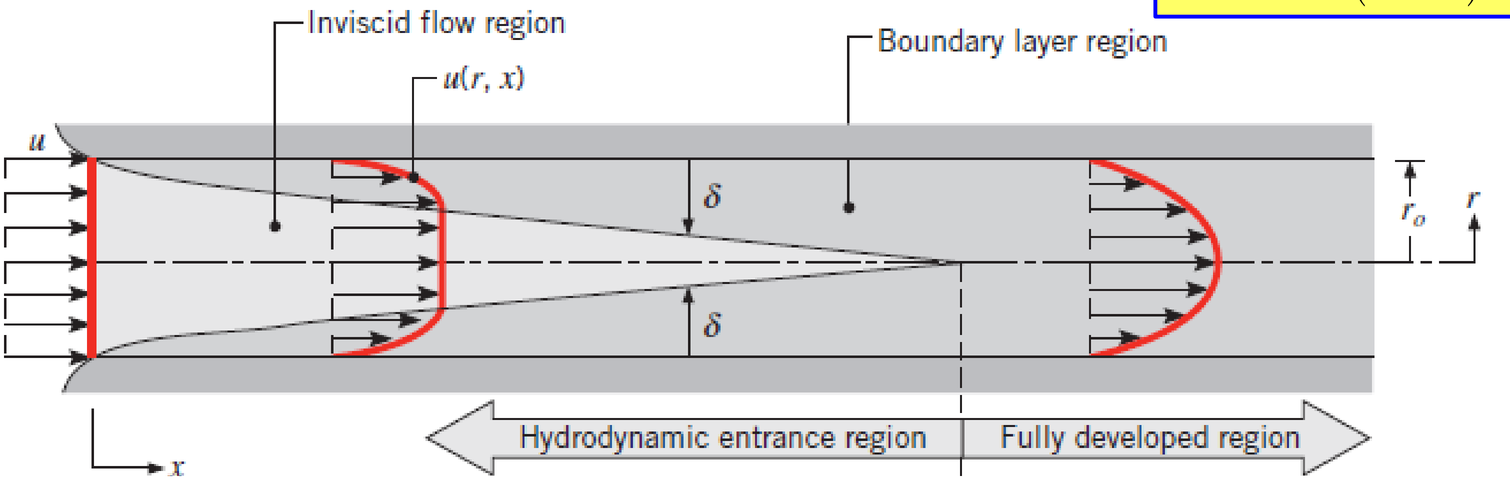
→ Flow is laminar because Re << 2000 (rough limit for turbulence)

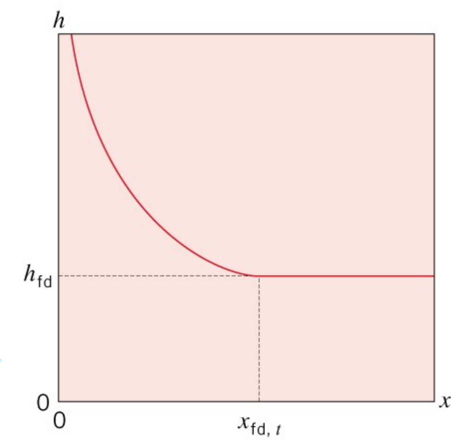
Entry region in laminar pipe/channel flow

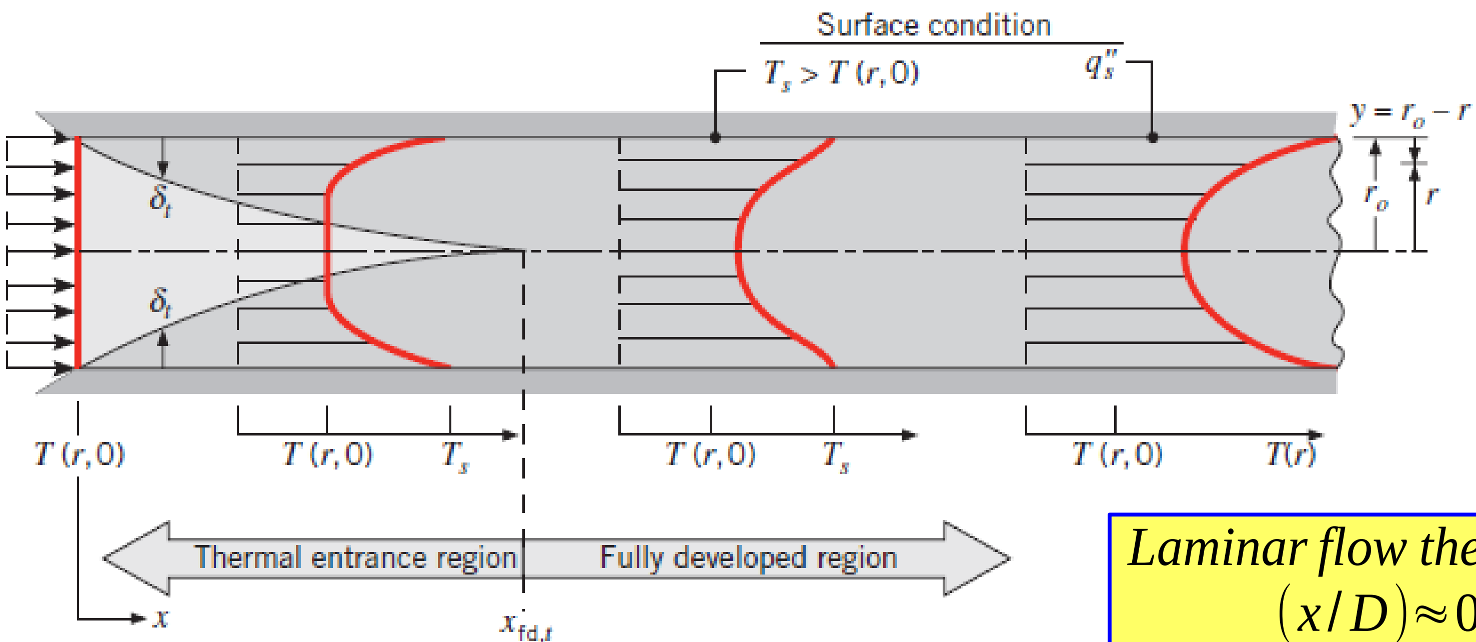
Velocity profile

Laminar flow viscous entry length:

$$(x/D) \approx 0.05\,\mathrm{Re}_D$$







Laminar flow thermal entry length:

$$(x/D) \approx 0.05\,\mathrm{Re}_D\,Pr$$

Temperature profile

Estimate Nusselt number using average heat transfer coefficient in the class room demo system for a single "channel" between two plates

Data for air

$P = 24\ W$ 12 fins → **2W**/fin gap $C_p = C_{p,air} = 1.007\ kJ/kgK$

$\Delta T_{CD} \approx 5\ K$ $k = k_{air} = 0.026\ W/mK$

$U_C \approx U_D \approx 1.2\ m/s$ $\rho \approx 1\ kg/m^3$ $A_{plates} = 2 \cdot 0.1m \cdot 0.05m$

$T_{wall} \approx 28\ C$ $D = 0.003m$

Use formula :

$Nu = hD/k$

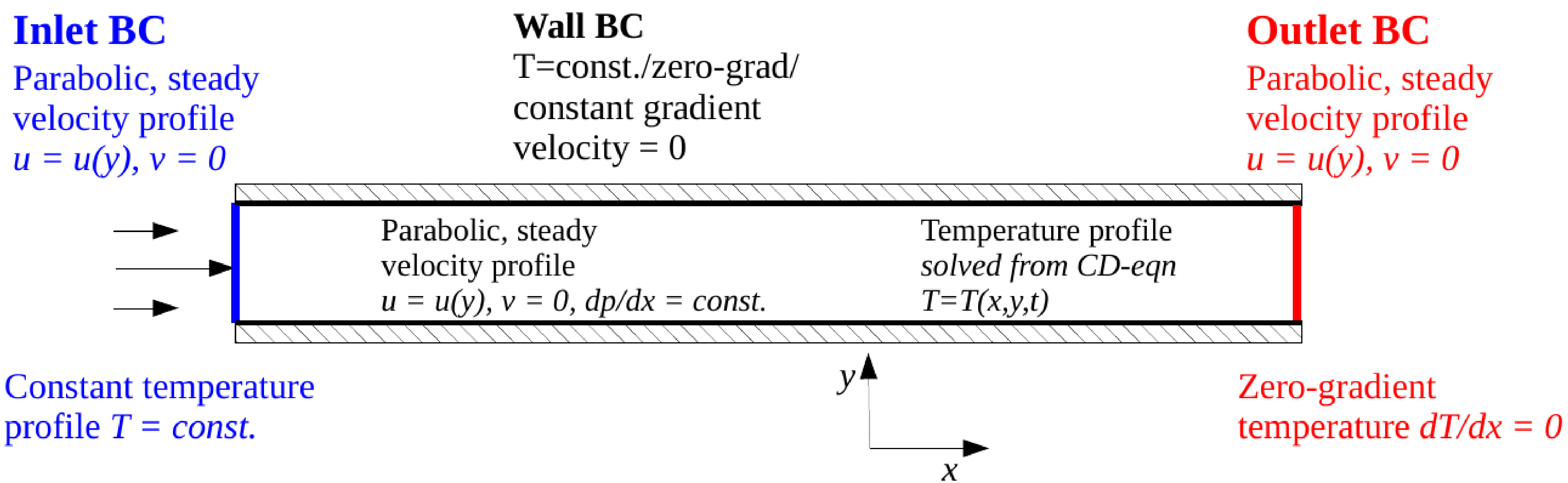
Info: The fin gap distance is 3mm, 2 opposing walls in a channel, height = 0.1m, width=0.05m

Use Newton's law:

$P = q = h_{ave}A(T_{wall} - T_{gas})$

→ You see that a number of assumptions needed!

In HW3 we consider a flow system related to laminar flow between two parallel plates. Assumptions: 1) parabolic velocity profile, 2) different wall BC's for T



Note: In HW2 we consider the same system but with velocity = 0 → pure 2d conduction

Summing up some findings from class room demo problem

$$\mathrm{Re} = \frac{U\,L}{\nu} \approx 225 < 2300 \rightarrow \text{ flow is laminar}$$

$$\mathrm{Pr} = \frac{\alpha}{\nu} \approx 0.7 \rightarrow \text{ thermal and viscous boundary layers grow quite similarly}$$

$$Nu_{ave} \approx 10 - 14 > 7.5$$

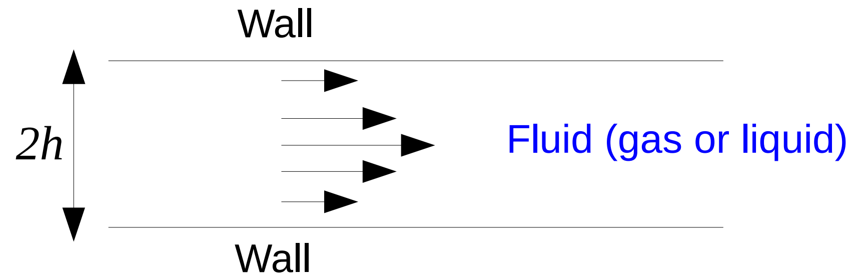
$$\textit{Laminar flow thermal entry length:}$$
$$\left(x/D\right) \approx 8 \rightarrow \text{flow is thermally fully developed in about 2.5cm}$$

The “channel” flow velocity field between two fins can be analytically solved assuming 1) steady state, 2) fully developed laminar flow (Re<2000) with constant pressure gradient, 3) flow is only in x-direction

$$\cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\cancel{\nu\frac{\partial^2 u}{\partial x^2}}+\nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$const.=\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\frac{\partial^2 u}{\partial y^2}$$



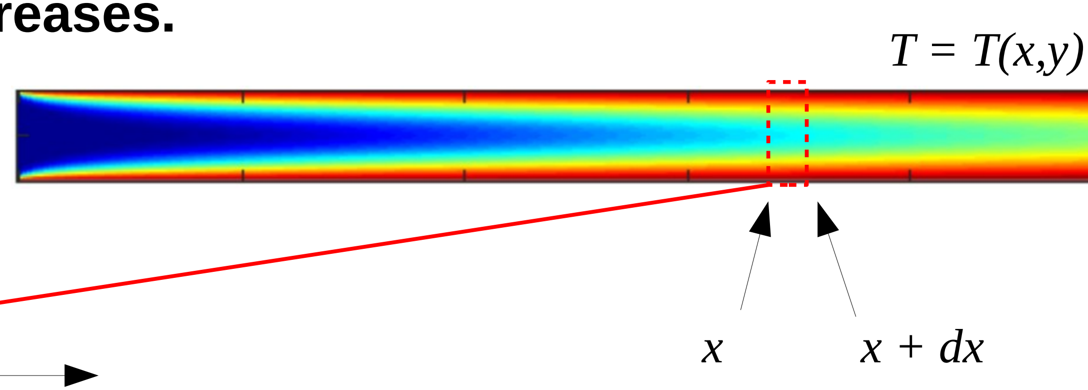
$$u(y)=u_{max}\left(1-\frac{y^2}{h^2}\right)$$

$$u_{max}=-\frac{dp}{dx}\frac{h^2}{2\rho\nu}$$

Wall boundary conditions
Velocity: No-slip wall ($u=v=0$)
Temperature:
T=fixed, insulated, or constant flux

Energy balance for a fluid element between heated parallel plates (**relevance:** finding mean temperature in streamwise direction)

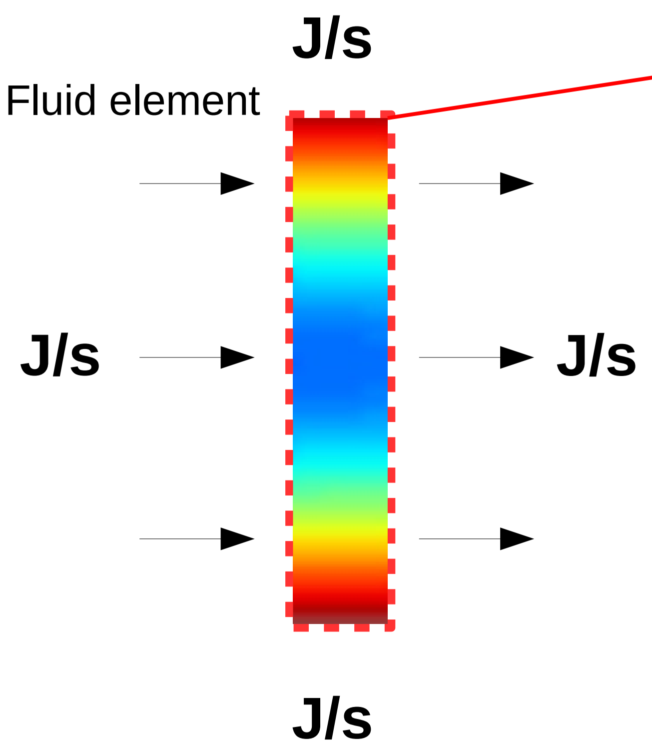
Wall provides a heat flux q [W/m^2] to the fluid so that a fluid element thermal **energy increases.**



L_z = channel depth out of plane

D = channel diameter

T_m = mean temp.



Fluid element energy increase during $x \ldots x+dx$

$$\rho\, c_p\, U\, D\, L_z [T_m(x+dx) - T_m(x)] = 2\, q\, dx\, L_z$$

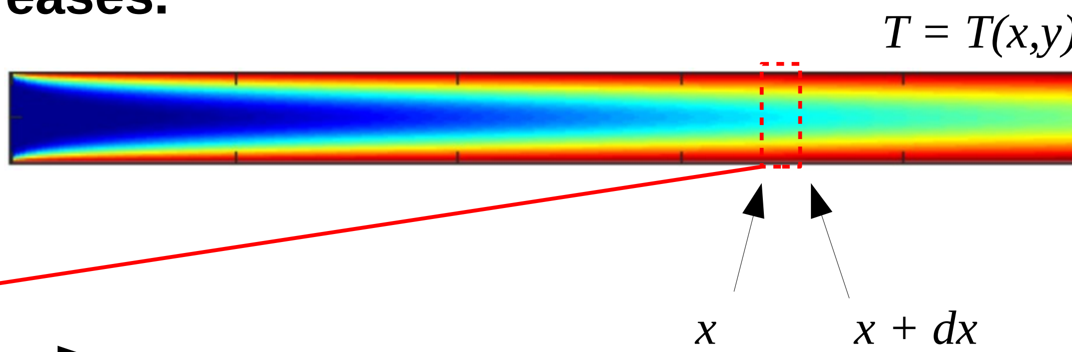
"J/s"="J/s" → units match

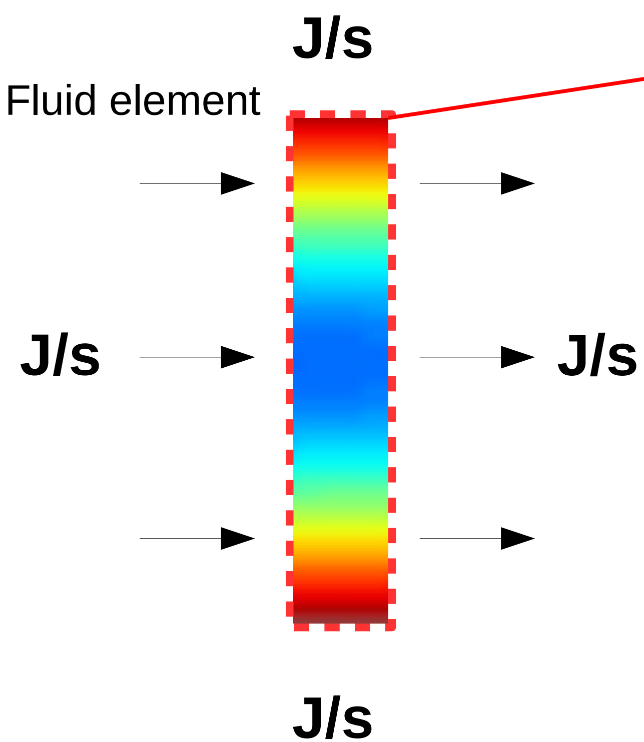
Mean temperature obeys an ordinary diff. eqn.

$$\frac{dT_m}{dx} = \frac{2\,q}{c_p \rho\, U\, D}$$

Energy balance for a fluid element between heated parallel plates (**relevance:** finding mean temperature in streamwise direction)

Wall provides a heat flux q [W/m^2] to the fluid so that a fluid element thermal **energy increases.**





L_z = channel depth out of plane

D = channel diameter

T_m = mean temp.

Fluid element energy increase during $x \ldots x+dx$

$$\rho c_p U D L_z [T_m(x+dx) - T_m(x)] = 2 q \, dx \, L_z$$

"J/s"="J/s" → units match

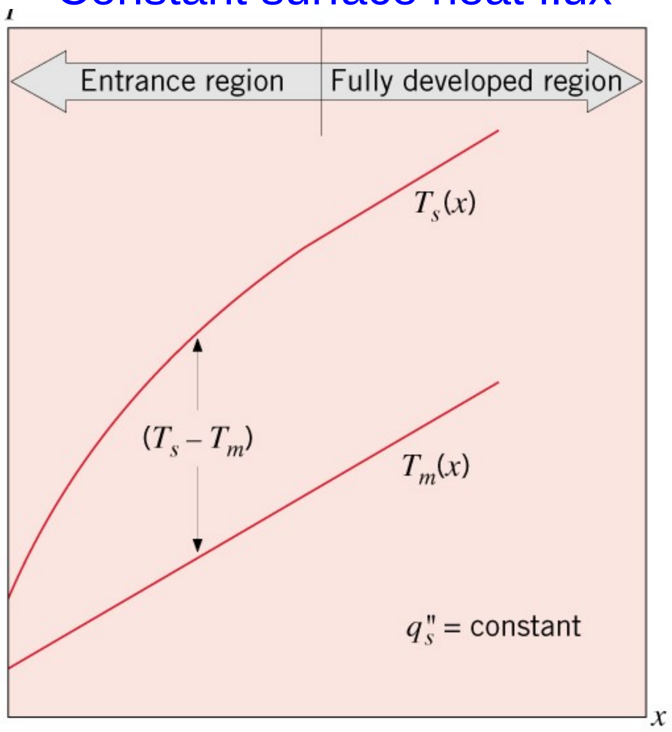
Mean temperature obeys an ordinary diff. eqn.

$$\frac{dT_m}{dx} = \frac{2q}{c_p \rho U D}$$

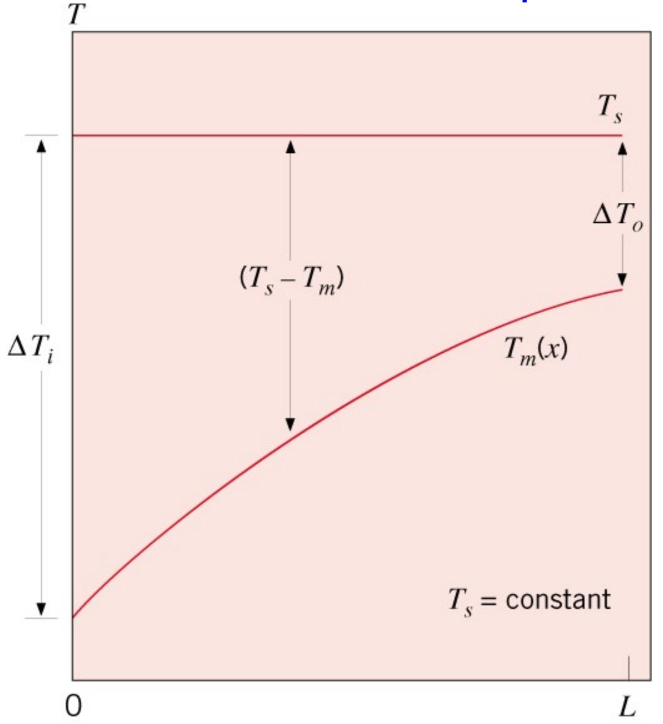
- T_s = fixed → need for Newton's cooling law to get q
- q = fixed → integrate directly

Axial mean temperature in a pipe or channel

Constant surface heat flux



Constant surface temperature



Figs. from Incropera, de Witt (Principles of Heat and Mass Transfer)

For constant surface heat flux

The main points:

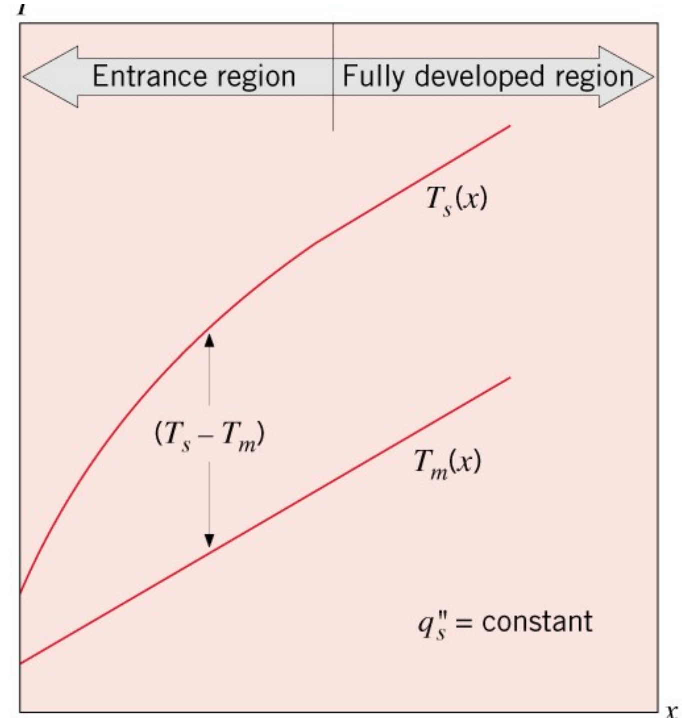
0) We know q_{tot} because q is constant. As q is const. → T_s follows.

1) T_s-$T_m(x)$ can be constant because $T_s = T_s(x)$. Follows from Newton's cooling law + assumption that q=const.

$$\frac{dT_m}{dx} = \frac{2q}{c_p \rho U D} = \text{constant}$$

$$\int_{x=0}^{x} \frac{dT_m}{dx} dx = \int_{x=0}^{x} \frac{2q}{c_p \rho U D} dx$$

$$T_m(x) = T_m^{\text{in}} + \frac{2q}{c_p \rho U D} x$$



→ Linear increase in mean temperature

Intro to log-mean temperature concept: For constant surface temperature at fully developed conditions when *h=const.*

After thermal entry region

$$Nu=\frac{hD}{k_{fluid}}\approx 7.52$$

$$\frac{dT_m}{dx}=\frac{2q(x)}{c_p\rho U D}=\frac{2h(T_s-T_m)}{c_p\rho U D}$$

$$\int_{T_m=T_{in}}^{T_m(x)}\frac{dT_m}{T_s-T_m}=\int_{x=0}^{x}\frac{2h}{c_p\rho U D}dx$$

$$\log\frac{T_m(x)-T_s}{T_{in}-T_s}=\frac{-2h}{c_p\rho U D}x$$

$$\frac{T_m(x)-T_s}{T_{in}-T_s}=\exp\left(\frac{-2h}{c_p\rho U D}x\right)$$

→ Mean temperature increases according to exp function

→ Total heat flux can be calculated based on log mean temperature

$$q_{tot}=h A \Delta T_{lm}$$

The main points:

0) We do not know q_{tot} because when T_s fixed then heat flux follows.

1) T_s-$T_m(x)$ is not constant i.e. $q=q(x)$.

2) Thus, one can not use the average of inlet and outlet temperature in Newton's law directly because mean temp. increases non-linearly.

3) Need for log-mean temperature concept.

See: Incropera Ch. 8 Eqn. (8.43)

Lecture 3.2 Numerical approach: a Matlab solver for the 2d convection-diffusion equation to describe temperature transport

ILO 3: Student can write the governing equations of fluid/heat flow in a channel, estimate the energy balance and estimate temperature rise for different heating conditions. The student can confirm the channel heat transfer using generated/provided simulation data.

In this session we will look mostly into local heat transfer along a channel wall. Hence, we assume that **the mean temperature**, in a channel with heated walls, **depends on x-coordinate** in streamwise direction.

Other assumptions during the session:

1) fin/channel walls at $T_{top} = T_{bot} = T_{wall} = +30$ deg C
2) inflow temperature is $T_{left} = +20$ deg C
3) velocity field is fully developed and laminar
4) pressure gradient is chosen so that the Reynolds number is between 200-250.

Strong relevance to HW3 - Heat flux balance at the surface: Fourier’s law (physics) equals to Newton’s law (engineering)

Diffusive heat flux (Fourier) immediately at the wall on the fluid side = *Heat flux from Newton’s law of cooling*

$$-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall} = h \left(T_s - T_{mean} \right)$$

If temperature gradient in wall-normal direction would be known at each x location → we could calculate h (W/m^{2}K) every single surface point

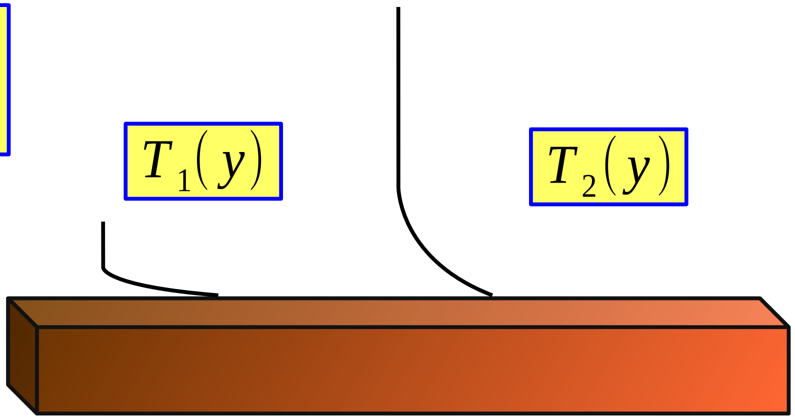


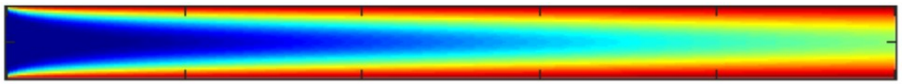
Figure: temperature profiles on bottom wall

Note:
even in convective heat transfer the heat first diffuses i.e. conducts near the wall because $u, v \rightarrow 0$ next to the wall

$$h = \frac{-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall}}{T_s - T_{mean}}$$

$$[h] = W / m^2 K$$

Think:
How can we maximize h ?
How do h and heat flux vary in the flow direction ?



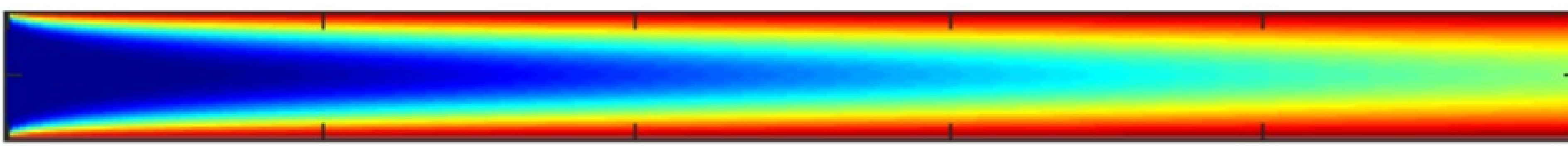
HW3: Convection-diffusion equation for temperature to estimate e.g. local Nu

The Matlab session will focus on adding the convective terms to the 2d heat equation solver. Parabolic flow profile is assumed through the 2d channel. The session is started by assuming constant wall T.

$$\frac{\partial T}{\partial t} + u(y)\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

Ignore y-velocity

$T=T(x,y)$ in steady state 2d channel flow with constant wall temperature BC



QUESTION 1: assuming constant inflow temperature and constant (hot) wall temperature, what will be the temperature level at the outlet ?
QUESTION 2: using the 2d numerical solution for temperature, estimate local heat transfer coefficient and Nusselt number as a function of x-coordinate.

HW3: Flow system and BC's related to laminar flow between two parallel plates (class-room demo system)

Inlet BC

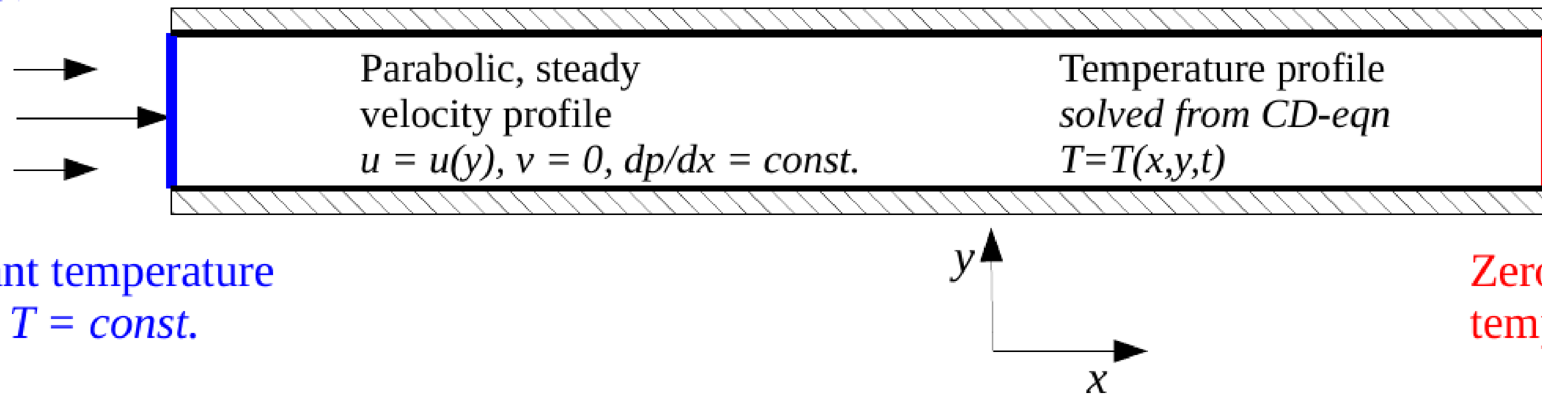
Parabolic, steady velocity profile
$u = u(y),\ v = 0$

Wall BC

T=const./zero-grad/ constant gradient
velocity = 0

Outlet BC

Parabolic, steady velocity profile
$u = u(y),\ v = 0$



Constant temperature profile $T = const.$

Zero-gradient temperature $dT/dx = 0$

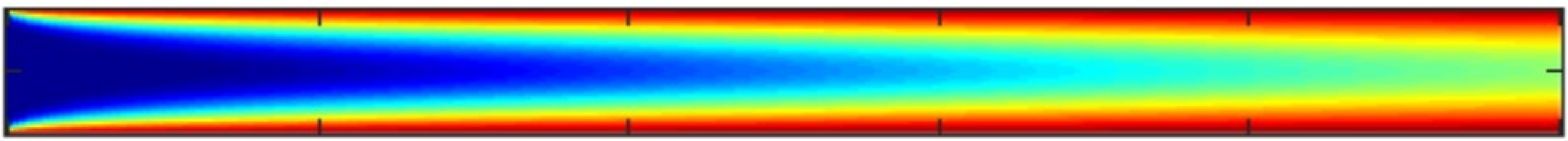
$$\mathrm{Re} = \frac{U\,L}{\nu} \approx 200 < 2300 \rightarrow \text{ flow is laminar}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha} \approx 0.7 \rightarrow \text{ thermal and viscous boundary layers grow quite similarly}$$

STEP 1: download the `heat2d` code from MyCourses and extract – by right clicking mouse - to new folder `Week3`.

STEP 2: let us look into the code structure together from the big screen.

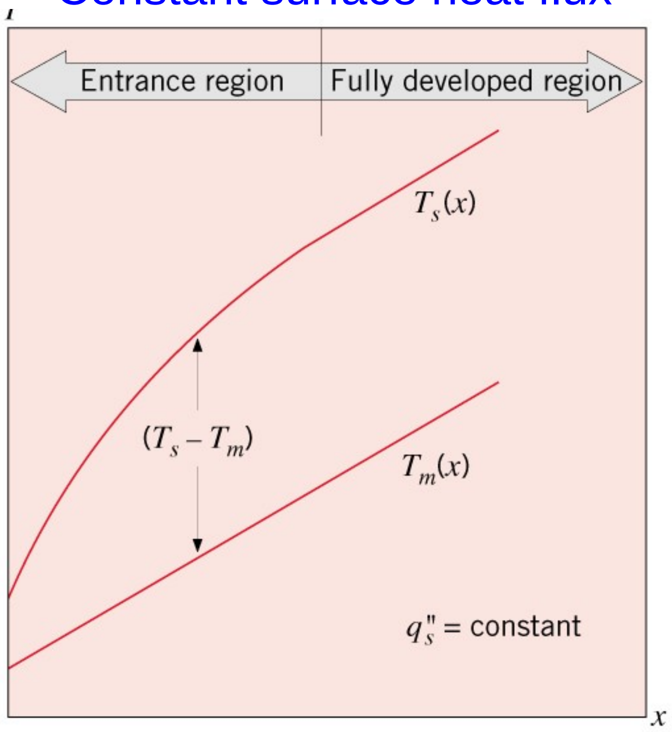
STEP 3: let us visualize the colorful $T(x,y)$ picture together by modifying the `VisualizeResults.m` file



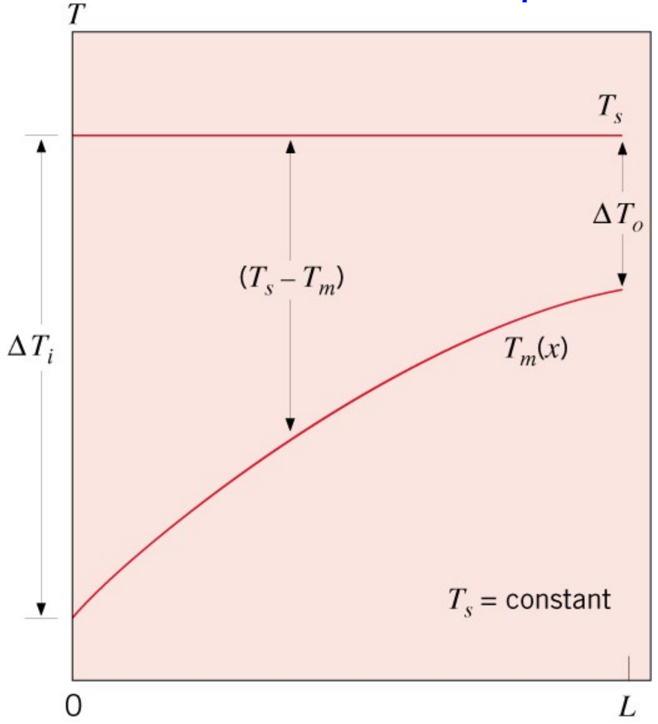
```
figure(1), clf, box
imagesc([min(min(X)) max(max(X))],[min(min(Y)) max(max(Y))], T(iny,inx));
    axis equal
    colormap jet
    axis tight, drawnow
        ylabel('y [m]')
    xlabel('x [m]'), colorbar, pause(0.1)
    print -dpng Temperature2d
```

Axial mean temperature in a pipe or channel

Constant surface heat flux



Constant surface temperature



STEP 4: let us next modify file `visualizeResults.m` and plot axial temperature along x-axis together.

```
figure(4), clf, box
plot(X(1,:),T(Ny/2,inx),'k-','Linewidth',2);
hold on
```

STEP 5: let us next further modify file `visualizeResults.m` and plot the wall heat flux along x-axis together.

$$q = -k_f \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

```
figure(2), clf, box
dTdywall = (T(1,inx)-T(2,inx))/dy; % top
plot(X(1,:), k*dTdywall)

ylabel('Wall heat flux [W/m^2]')
xlabel('x [m]'), pause(0.1)

print -dpng WallHeatFlux
```

STEP 6: let us next futher modify file `visualizeResults.m` and compute the axial mean temperature and Nusselt number together.

```
% Mean temperature in x-direction
% a vector of length Nx-2
% The sum function sums over columns
% of matrix and dy cancels out from
% the integral

Tm = sum(T(iny,inx).*U)./sum(U);
```

$$T_m(x) = \frac{\int_0^{L_y} T(x,y)U(y)\,dy}{\int_0^{L_y} U(y)\,dy}$$

See Incropera Ch. 8.2

```
% Local Nusselt number along the channel
% a vector of length Nx-2

Nu = 2*dTdywall*Ly./(Ttop-Tm); % a vector of length Nx-2

% note 1) ./(Ttop-Tm) because pointwise
% division of two vectors, 2) k cancels out
```

$$Nu(x) = \frac{h(x)L_y}{k_{fluid}}$$

See Incropera Table 8.1

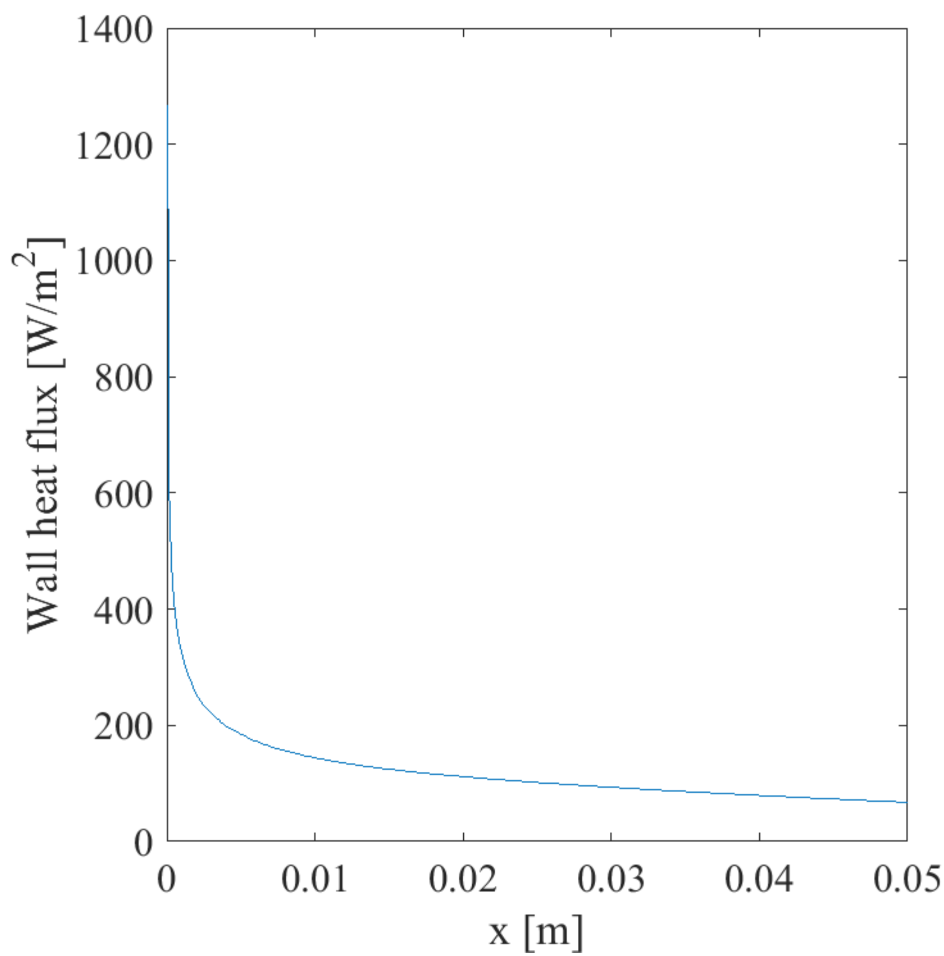
Nusselt numbers for different channel types with different boundary conditions

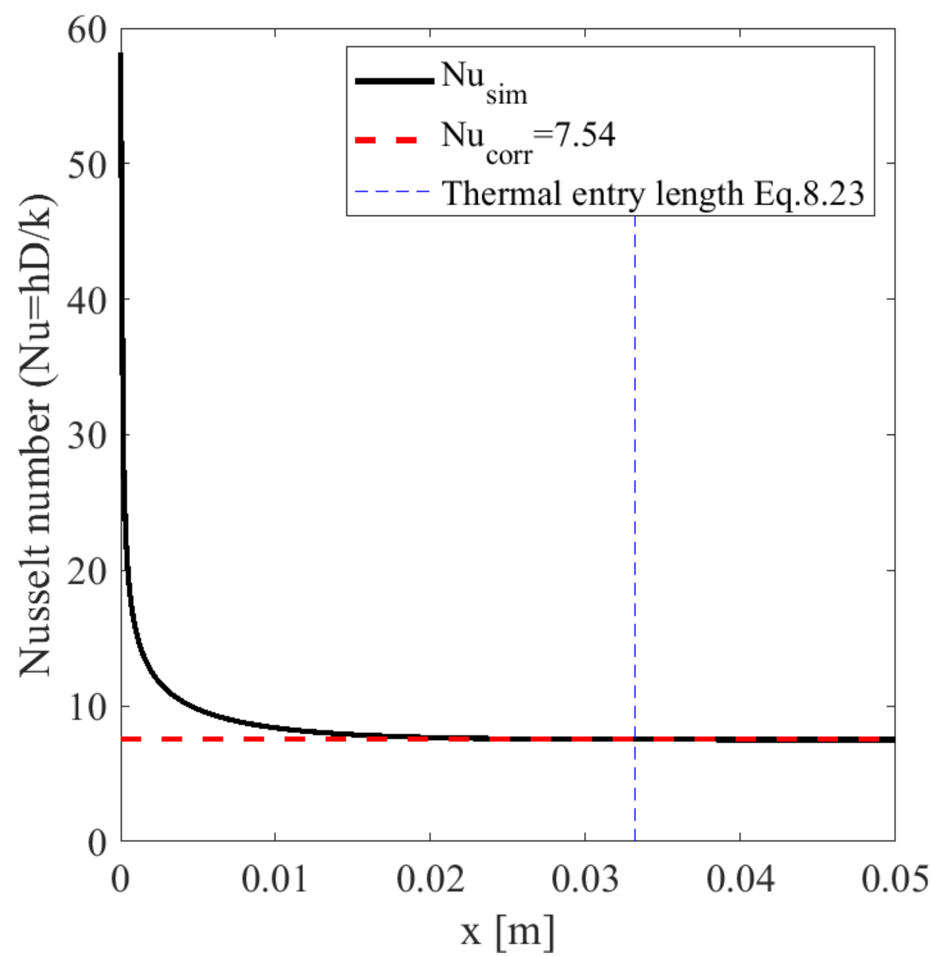
Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	$f\ Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Table 8.1 from Incropera, de Witt (Principles of Heat and Mass Transfer)

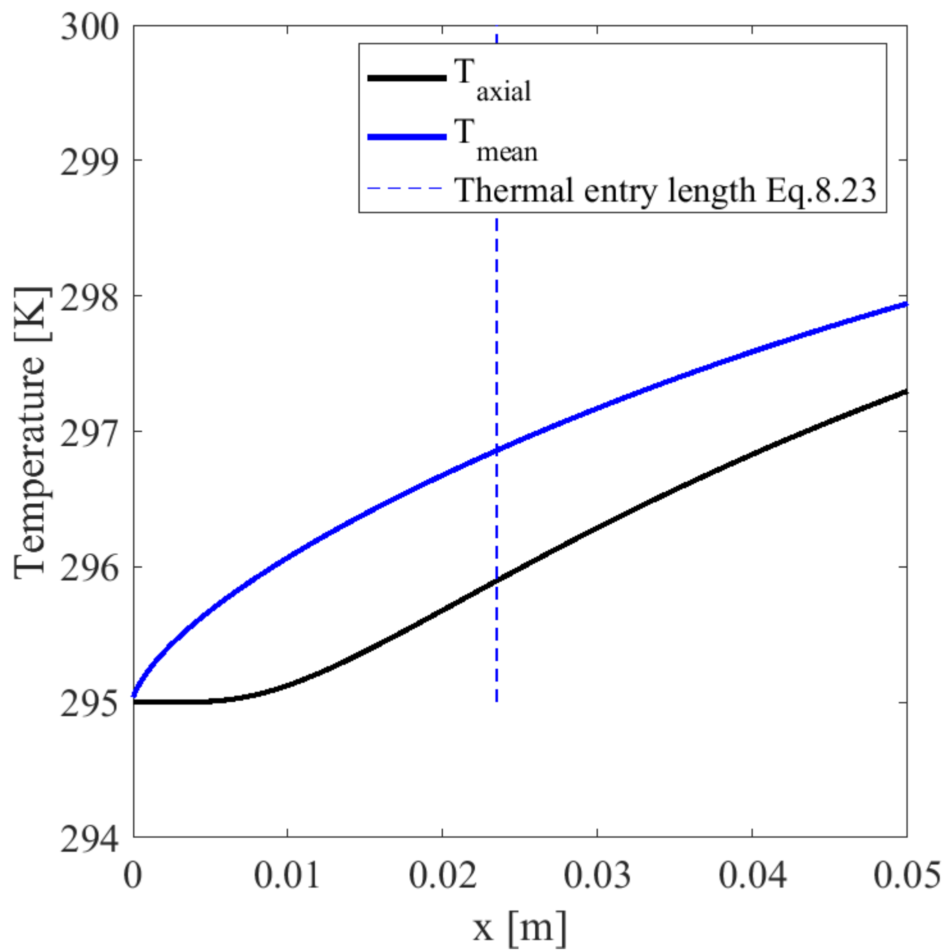
We want to check if we can get the value Nu = 7.54 from numerical simulation.

For constant wall temperature BC some example results using code `heat2d.m`

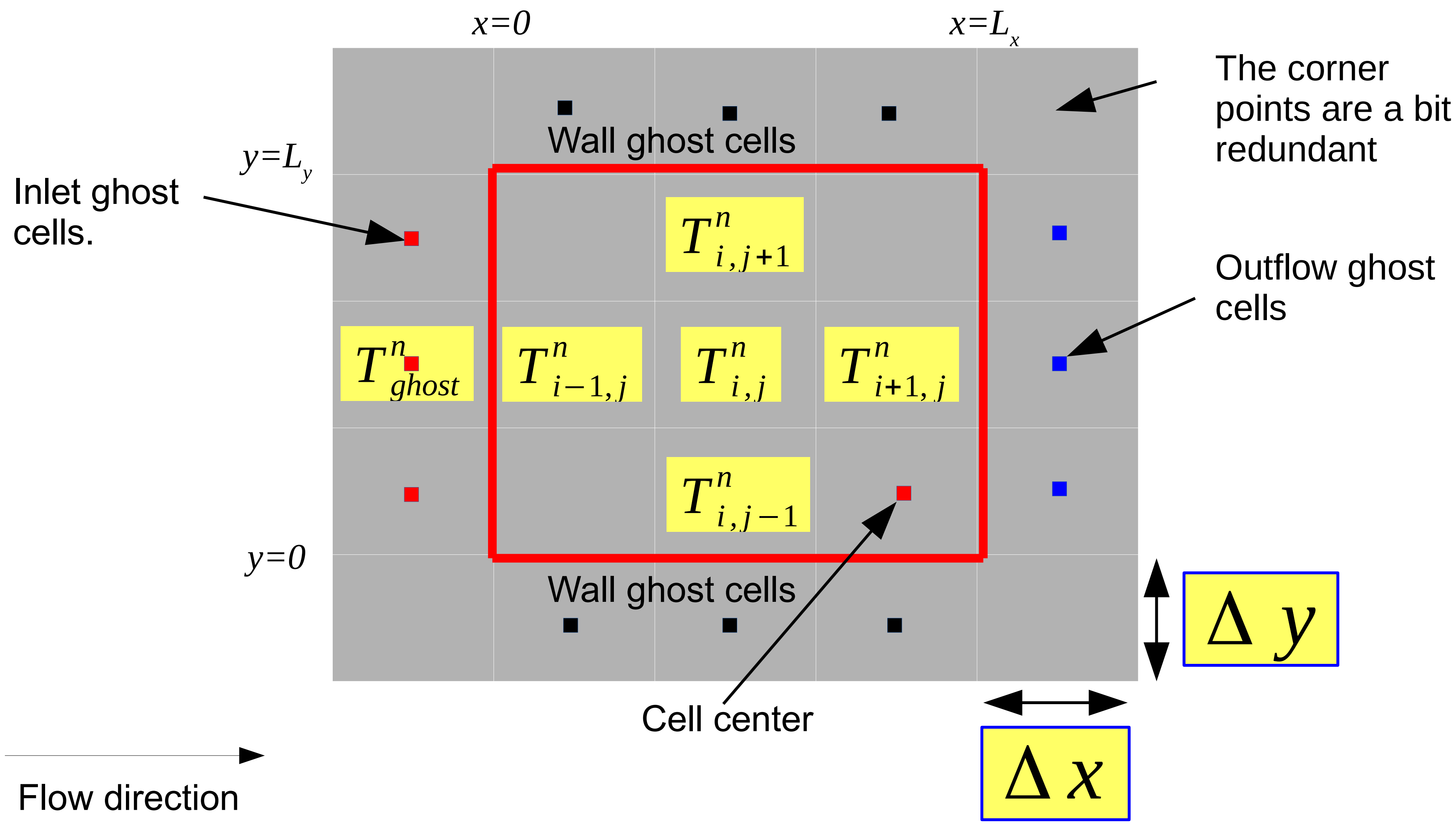




For constant wall temperature BC some example results using code `heat2d.m`



Discretization of a 2d domain in the case of 2d channel flow



2d Convection-Diffusion Heat Equation and Numerical Solution (extension of Week 2): Again, finite difference formulas are used to estimate partial derivatives

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial x^2}+\alpha\frac{\partial^2 T}{\partial y^2}$$

$$\left(\frac{\partial T}{\partial t}\right)_{i,j}^{n}\approx\frac{T_{i,j}^{n+1}-T_{i,j}^{n}}{\Delta t}$$

$$\left(\frac{\partial T}{\partial x}\right)_{i,j}^{n}\approx\frac{T_{i+1,j}^{n}-T_{i-1,j}^{n}}{2\Delta x}$$

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_{i,j}^{n}\approx\frac{T_{i+1,j}^{n}-2T_{i,j}^{n}+T_{i-1,j}^{n}}{\Delta x^2}$$

The numerical scheme to update temperature at points (i,j) is a straightforward extension from Week 2

$$T_{i,j}^{n+1} = T_{i,j}^{n} + \Delta T_{i,j}^{n}$$

$$T_{i,j}^{n+1} = T_{i,j}^{n} - \Delta t\, u_{i,j} \frac{T_{i+1,j}^{n} - T_{i-1,j}^{n}}{2\Delta x} + \alpha \Delta t \frac{T_{i+1,j}^{n} - 2T_{i,j}^{n} + T_{i-1,j}^{n}}{\Delta x^2} + \alpha \Delta t \frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{\Delta y^2}$$

$$= \Delta T_{i,j}^{n}$$

For code stability the timestep must be small enough:

Courant-Friedrichs-Lewy number indicating that diffusion should not transport temperature over longer than cell distance during timestep

$$CFL = \frac{\alpha \Delta t}{\Delta x^2} \ll 0.5$$

Courant number indicating that convection should not transport the temperature over longer distances than cell size during timestep

$$Co = \frac{\Delta t\, u}{\Delta x} \ll 1$$

Code for a 2d convection-diffusion equation solver code heat2d.m will be delivered online via MyCourses. Essential modifications to Week 2 simulation code below:

heat2d.m

```
% initialize temperature
T = 293*ones(Ny,Nx);
CFLmax = 0.1;
Pr = 0.707;          % Prandtl number of air
nu = (1.589e-5);     % nu = kinematic viscosity of air [nu] = m^2/s
alpha = nu/Pr;       % alpha = thermal conductivity of air [alpha] = [nu]
rho = 1.1614; cp = 1.007*1000;
k   = alpha*rho*cp;     % thermal conductivity of air [k] = W/(mK)

% pressure gradient (if convection is used)
dpdx = ...;

% velocity is assumed to point to x-direction only i.e. V=0
U = (dpdx/(2*nu))*( (Ly/2)^2 - (Y-Ly/2).^2);
```

computedT.m

```
dTdx = ( T(iny,east)-T(iny,west))/(2*dx);
dTdy = ( T(north,inx)-T(south,inx))/(2*dy); % not needed if V=0->V*dTdy =0

d2Tdx2 = ( T(iny,east)-2*T(iny,inx)+T(iny,west))/(dx^2);
d2Tdy2 = ( T(north,inx)-2*T(iny,inx)+T(south,inx))/(dy^2);

% evaluate temperature increment dT assuming V.*dTdy = 0
dT = dt*( -U.*dTdx + alpha*d2Tdx2 + alpha*d2Tdy2);
```

Assessment: Take 5 min time to answer the questions in the online query form.

Thank you for your attention!