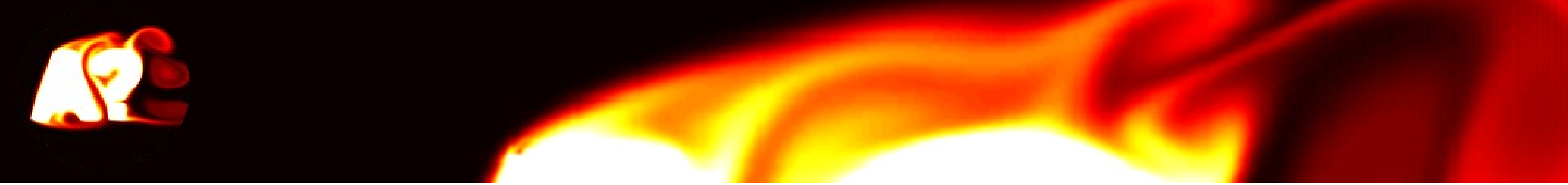
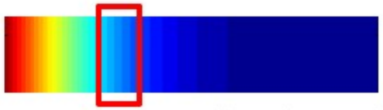


EEN-1020 Heat transfer

Week 4: Convective Heat Transfer External Flow

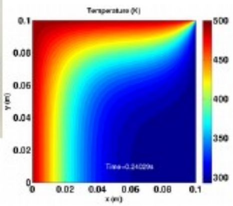
Prof. Ville Vuorinen
November 17th-18th 2020
Aalto University, School of Engineering

Week 1: Energy conservation, heat equation, conduction Fourier/Newton

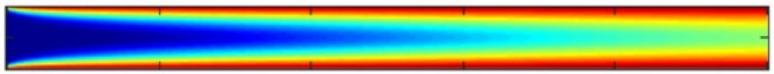


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x}\left(k\frac{\partial T(x+\Delta x/2,t)}{\partial x} - k\frac{\partial T(x-\Delta x/2,t)}{\partial x}\right)$$

Week 2: Fin theory, conduction, intro to convection

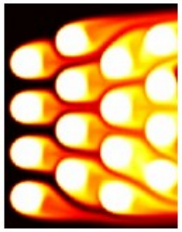


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

Week 5: natural convection, boiling, correlations



In heat transfer course, we have “4 friends” who typically help us to approach and solve any problem

1) Energy conservation

2) Fourier’s law

3) Newton’s cooling law

4) Bonus “helpers”:

   4.1) heat eqn, conv.-diff. eqn (relatives of friend #1)

   4.2) Navier-Stokes eqn (momentum conservation)

   4.3) non-dimensional numbers (follow from the other friends)

Recommended reading: Ch 7 “External flow” selective parts from “Principles of Heat and Mass Transfer”, Incropera

Remember: These slides may contain typos or other mistakes so please be cautious when reading.

Remember: Fluid and solid properties depend in reality on thermodynamic conditions so please use always values taken from a proper source (e.g. Incropera Appendix contains some reasonable values)

Lecture 4.1 Theory and experiment: Heat transfer of flow over a cylinder

ILO 4: Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.

However, we start by a brief recap of heat transfer phenomena near walls.

1) Thermal entry length derivation

2) Features of Prandtl boundary layer analysis

3) Axial mean temperature in a heated channel with constant wall heat flux (see Appendix – needed in HW3)

Strong relevance to HW3 - Heat flux balance at the surface:
Fourier's law (physics) equals to Newton's law (engineering)

Diffusive heat flux (Fourier) immediately at the wall on the fluid side = *Heat flux from Newton's law of cooling*

$$-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall} = h \left(T_s - T_{mean} \right)$$

If temperature gradient in wall-normal direction would be known at each x location → we could calculate *h* (W/m^2K) every single surface point

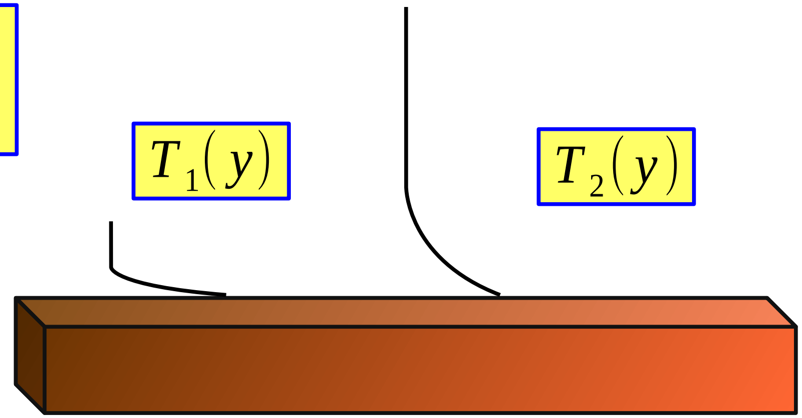


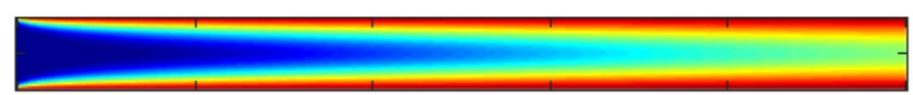
Figure: temperature profiles on bottom wall

Note:
even in convective heat transfer the heat first diffuses i.e. conducts near the wall because $u,v \rightarrow 0$ next to the wall

$$h = \frac{-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall}}{T_s - T_{mean}}$$

$$[h] = W / m^2 K$$

Think:
How can we maximize *h* ?
How do *h* and heat flux vary in the flow direction ?



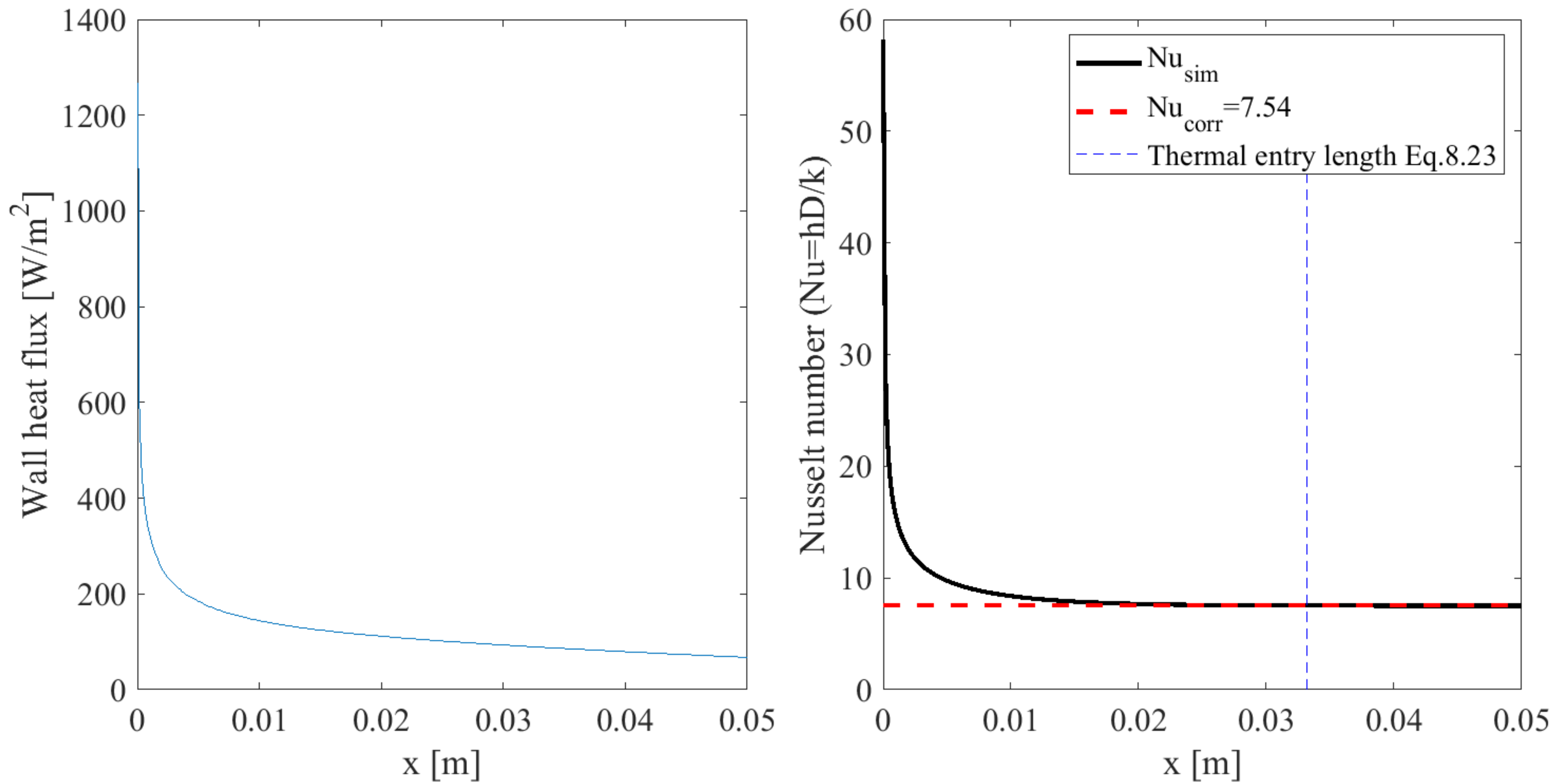
Nusselt numbers for different channel types with different boundary conditions

In HW3 we want
to check if
we can get the value
Nu = 7.54 from
numerical simulation.

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	(Uniform T_s)	$f\ Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated, Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

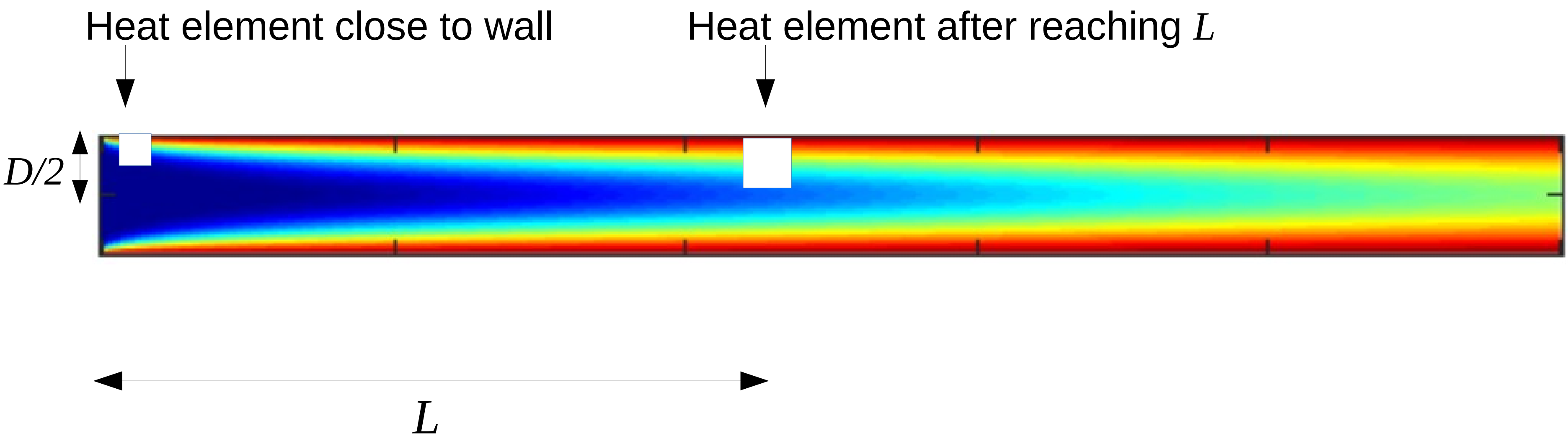
Table 8.1 from Incropera, de Witt (Principles of Heat and Mass Transfer)

For constant wall temperature BC some example results using code `heat2d.m`

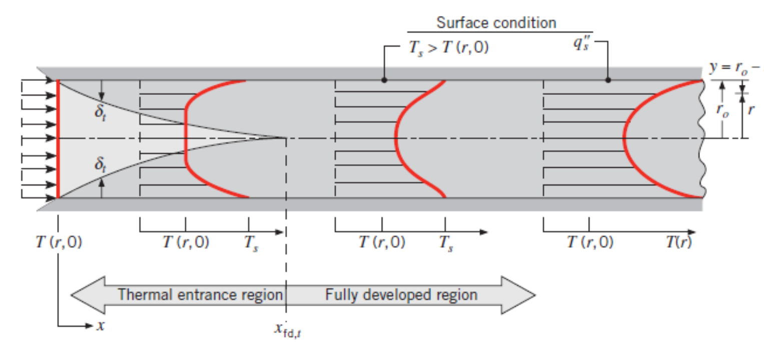


Thermal entry length derivation

Note – the correlation should be only taken as informative and guiding



- ***y*-direction:** In laminar channel flow heat has only diffusion as an option towards the centerline since y-velocity = 0.
- Recall order of magnitude for diffusion time (Week 1): $\tau_{diff} = (D/2)^2/\alpha$
- ***x*-direction:** During diffusion time heat convected in x-direction distance L
- Approximate convection time: $\tau_{conv} = L/U$
- Equate: $\tau_{conv} = \tau_{diff}$



Here, we get:

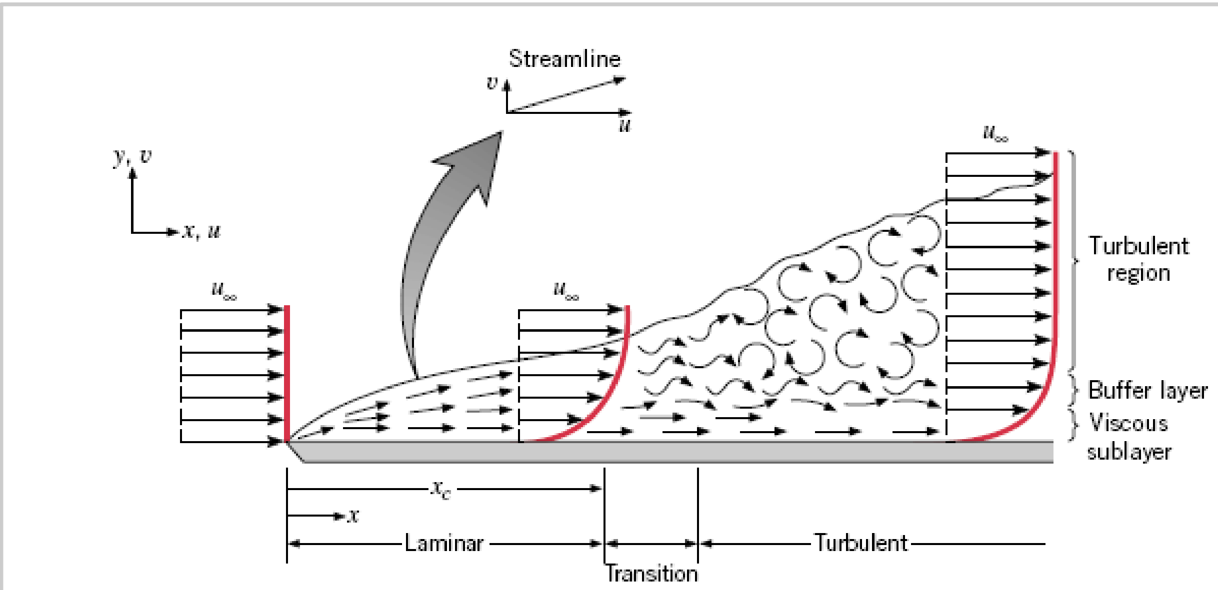
$$(x/D) \approx 0.25 \, \mathrm{Re}_D \, Pr$$

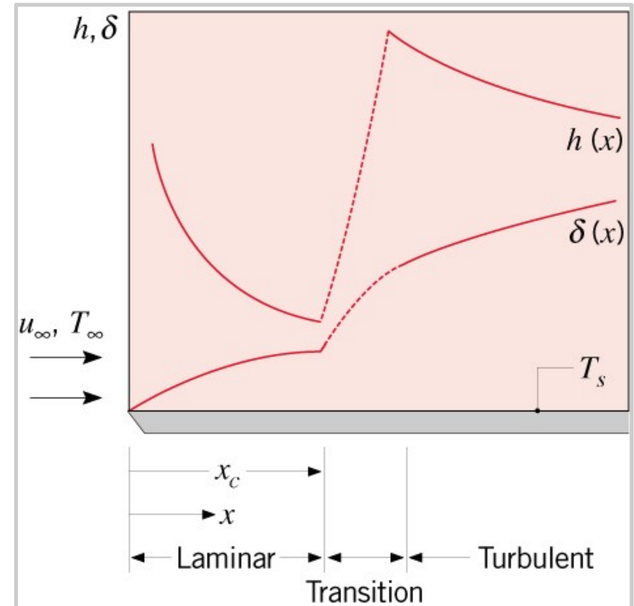
Instead of the thermal entry length:

$$(x/D) \approx 0.05 \, \mathrm{Re}_D \, Pr$$

Summary: physics and way of thinking ok but the prefactor is wrong but explainable and refinable.

Free boundary layers form an important starting point to int./ext.heat transfer analysis





Viscous boundary layer

$$Re_c = \frac{\rho U_\infty x_c}{\mu}$$

Critical Reynolds number at the onset of laminar to turbulence transition.

$$10^5 < Re_c < 3 \cdot 10^6$$

$$U/U_\infty = 0.99$$

$$\delta = \frac{5.0}{\left(\frac{U_\infty}{\nu x}\right)^{1/2}} = \frac{5x}{Re_x^{1/2}}$$

Laminar boundary layer thickness grows along the plate.

Thermal boundary layer

$$\delta/\delta_T = Pr^{1/3}$$

Thickness ratio between viscous/thermal BL's depends on Pr.

$$Nu_x = h_x x/k = 0.332\, Re_x^{1/2} Pr^{1/3}$$

Local Nusselt number scaling depends on Re_x and Pr.

https://www.youtube.com/watch?v=e1TbkLIDWys&t=82s

Convection-diffusion equation for temperature = energy conservation

By simulating the CD-equation below in HW3-HW5 we are in fact exploring the classical boundary layer characteristics stemming back to findings of e.g. Prandtl on previous slides.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

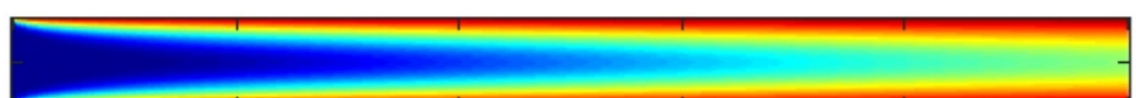
"J/s thinking"

$\frac{\partial T}{\partial t}$: T changes in given position in time due to convection and diffusion

$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$: T is transported by velocity field (convection)

$\alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$: T is transported by thermal diffusion (diffusion/conduction)

$T=T(x,y)$ in steady state 2d laminar channel flow



Observation

→ Starting from week 1 lectures, the CD-equation has been around "all the way" but in a simpler form.

$$u\frac{\partial \rho c_p T}{\partial x} \sim \rho c_p T_{out} u - \rho c_p T_{\text{in}} u$$

→ The CD-equation is simply telling the thermal energy is transported by convection and diffusion in a fluid.

→ The **"J/s thinking"** tells us that the equation reaches steady state if power entering a point equals the power leaving the point.

→ The boundary conditions via heated walls (typically T=const. or q=const.) provide heat to the fluid so that near the walls temperature conducts to the fluid. As always, Fourier's law tells the heat flux.

During the next 30 minutes we will talk about the class room demo heat exchanger system of flow over a heated cylinder with convection i.e. fan turned on

Purpose of empirical heat transfer correlations:

For a broad range of Reynolds numbers and Prandtl numbers, express average heat transfer coefficient h in a non-dimensional form called Nusselt number: $Nu=Nu(Re,Pr)$

Different average Nusselt number correlations for heated cylinders have been developed

$$\overline{Nu}_D = \frac{\bar{h}\,D}{k}$$

Hilpert correlation (see Table 7.2)

$$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

Churchill & Bernstein correlation (broad applicability)

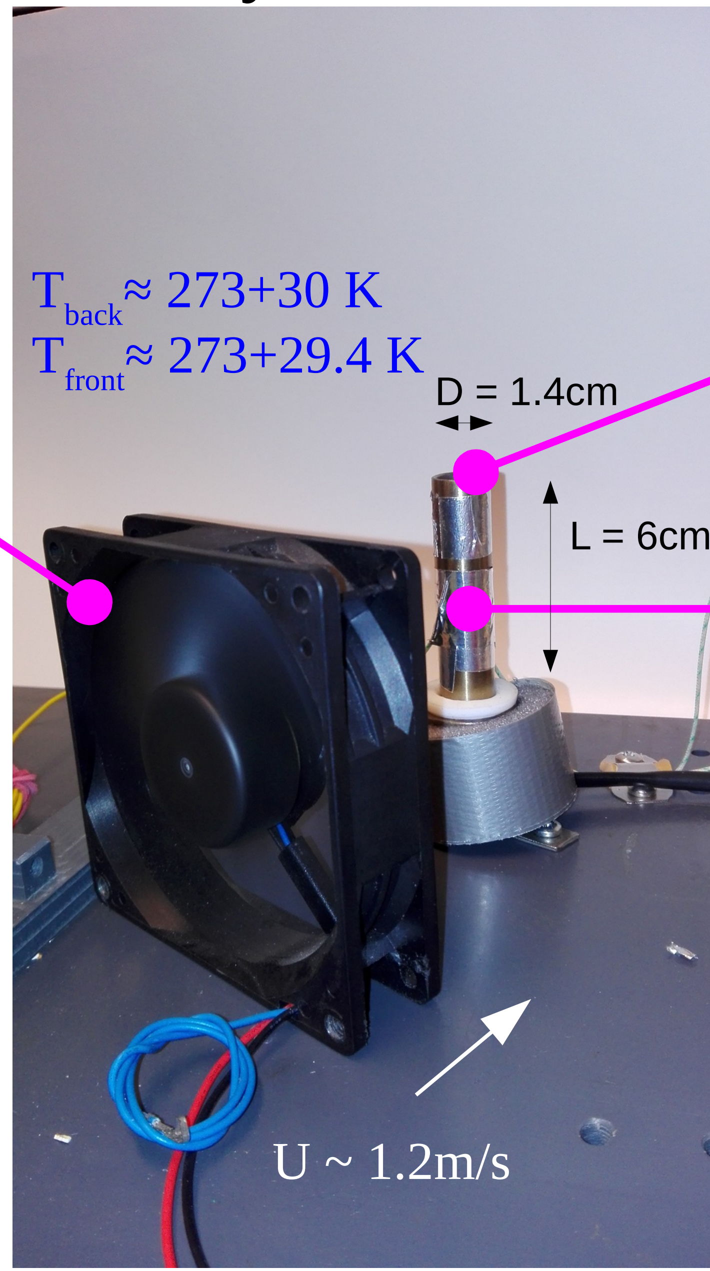
$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\mathrm{Re}_D/282000\right)^{5/8}\right]^{4/5}$$

Zukauskas correlation (broad applicability, see Table 7.4)

$$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,Pr^{1/3}\left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Answers to class-demo
Re =1050
Power =0.54154W
h =28.9033W/m^2K
Nu =15.5633
DT = 0.53 K

Class room demo – external flow over heated cylinder



Estimate cylinder heat transfer coefficient and air heating power from Newton's law (**assume** T_s known)

Data for air

$C_p = 1.007$ kJ/kgK $\quad$ $k = k_{air} = 0.026$ W/mK

Surface temperatures: $T_{front} \approx 273+29.4$ K $\quad$ $T_{back} \approx 273+30$ K

Air temperatures: $T_{air,room} \approx 273+22.6$ K $\quad$ $T_{air,\ after} \approx 273+23.3$ K

$U_C \approx U_D \approx 1.2$ m/s $\quad$ $\rho \approx 1.2$ kg/m^3 $\quad$ $\nu = 1.6e\text{-}5\,m^2/s$ $\quad$ Pr = 0.707

L = 6cm $\quad$ d = 1.4cm $\quad$ $A = A_{cylinder} = L\pi d = 0.0026\ m^2$

Use the following formulas and estimate power from Newton's cooling law:

$$\mathrm{Re} = \frac{U\,D}{\nu}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha}$$

$$\mathrm{Nu}_{ave} = \mathrm{Nu} = \frac{h\,D}{k} = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

$C = 0.683$ and $m = 0.466$

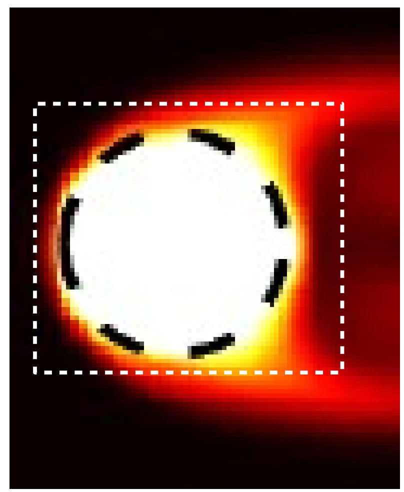
Table 7.2 Incropera

Find order of magnitude for temperature increase from energy balance for heated cylinder (**assume** T_s known)

$$\mathrm{Nu} = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

$Nu = ?$
$h = ?$

D
$A_{proj} = LD$



$A_{cyl} = \pi DL$

$\Delta T = ?$

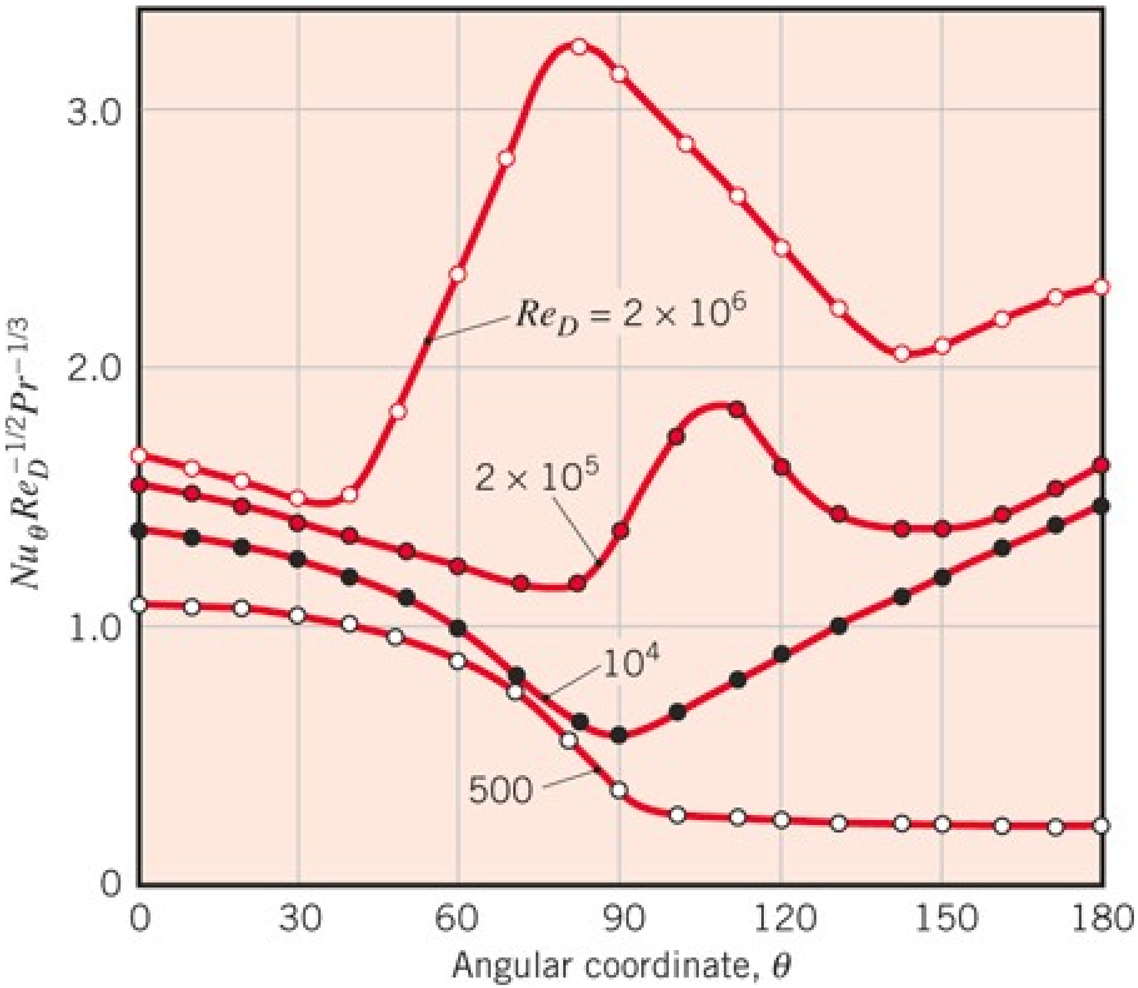
$$q = hA_{cyl}\left(T_s - T_\infty\right)$$

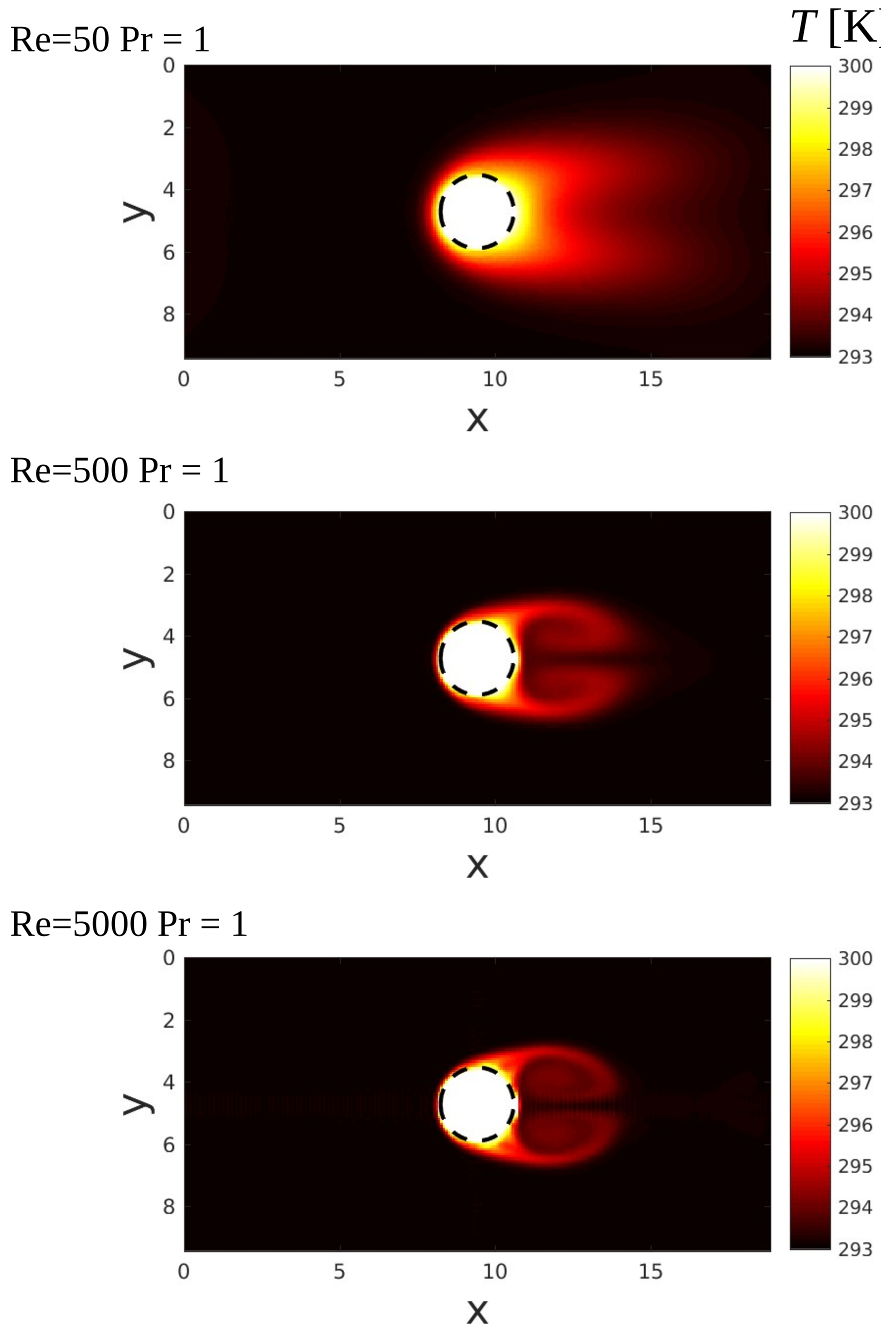
$q = ?$

$$q = \rho\,c_p\,U\,A_{proj}\,\Delta T$$

“J/s thinking”

Local Nusselt number for a heated cylinder: exp. data (Incropera) and demonstration of instantaneous temperature distribution from simulation (right) assuming constant T_s





Different Nusselt number correlations for heated cylinders have been developed

$$\overline{Nu}_D = \frac{\bar{h} D}{k}$$

Hilpert correlation (see Table 7.2)

$$\overline{Nu}_D = C \, \mathrm{Re}_D^m \, Pr^{1/3}$$

Churchill & Bernstein correlation (broad applicability)

$$\overline{Nu}_D = 0.3 + \frac{0.62 \, \mathrm{Re}_D^{1/2} \, Pr^{1/3}}{\left[1 + \left(0.4 / Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\mathrm{Re}_D / 282000\right)^{5/8}\right]^{4/5}$$

Zukauskas correlation (broad applicability, see Table 7.4)

$$\overline{Nu}_D = C \, \mathrm{Re}_D^m \, Pr^{1/3} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Answers to class-demo
Re =1050
Power =0.54154W
h =28.9033W/m^2K
Nu =15.5633
DT = 0.53 K

Last, let us try to think of what the exponents mean in convective heat transfer

- Consider the Hilpert correlation $\overline{Nu}_D = C\,\mathrm{Re}_D^m\,Pr^{1/3}$
- The Prandtl number exponent $1/3$ is the same as for flat plate boundary layer → consistent and reasonable
- The Reynolds number exponent $0<m<1$ and it is expected to be related to flat plate BL exponents for average Nu.
- Case $m>1$: unphysical
- Case $m=1$: unphysical
- Case $0<m<1$: physical

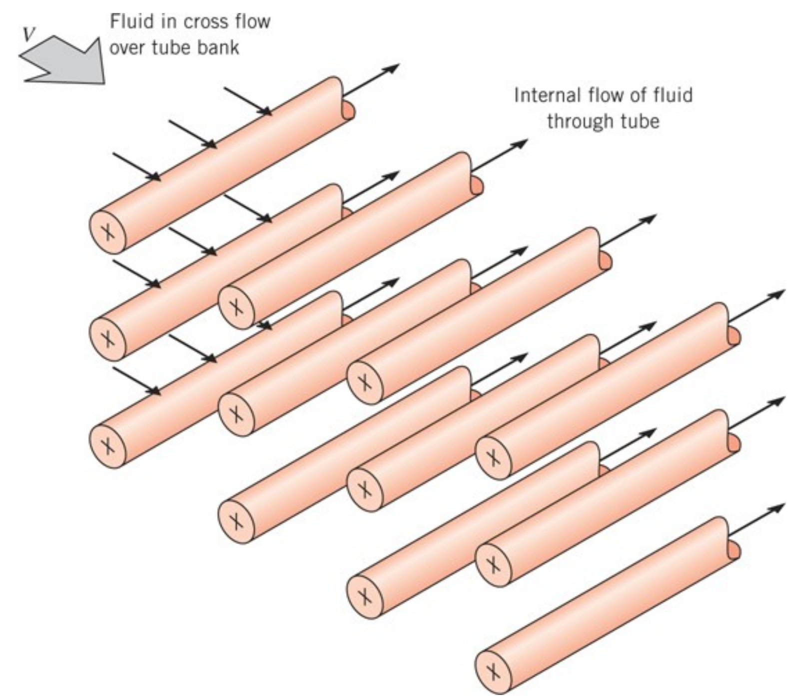
Answers to class-demo
Re =1050
Power =0.54154W
h =28.9033W/m^2K
Nu =15.5633
DT = 0.53 K

Lecture 4.2 Numerical approach: 2d heat transfer over a fin bed using Matlab

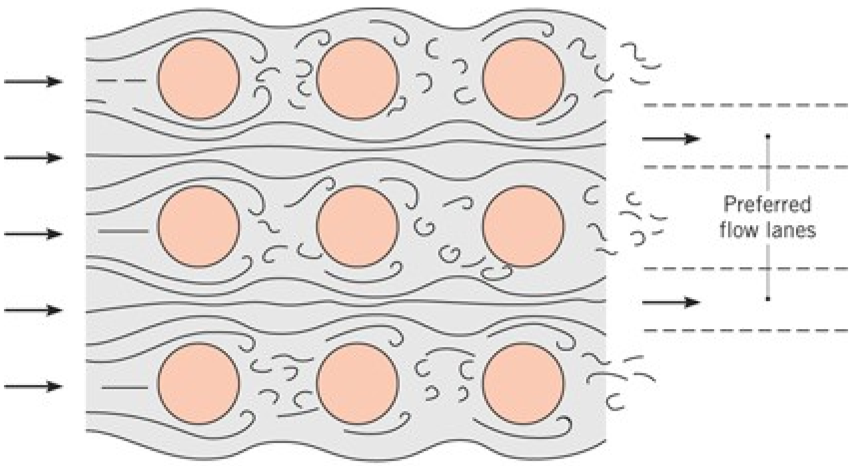
ILO 4: Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.

Background motivation: Fin arrays

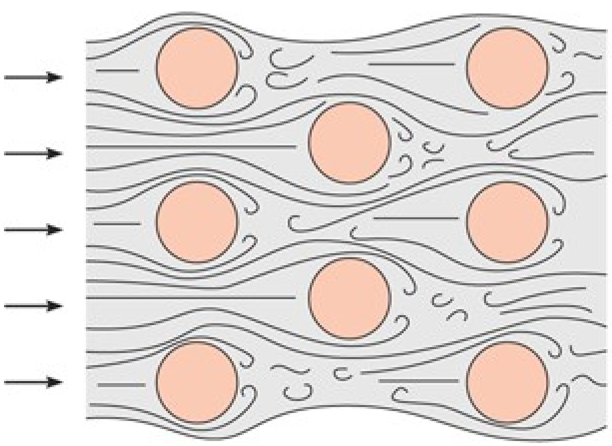
- Cylinder or pin fin beds very commonly used in heating and cooling applications
- E.g. cooling system in a ship where cool sea water is pumped through pipes and air is blown by fan over the array for AC cooling of cabins



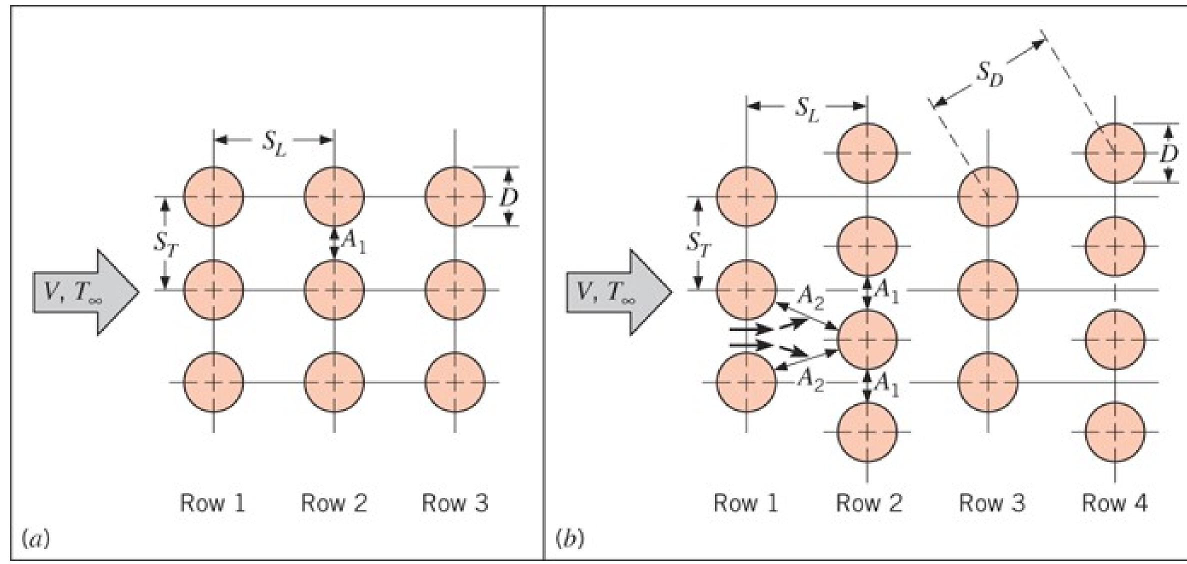
Fin arrays



(a)



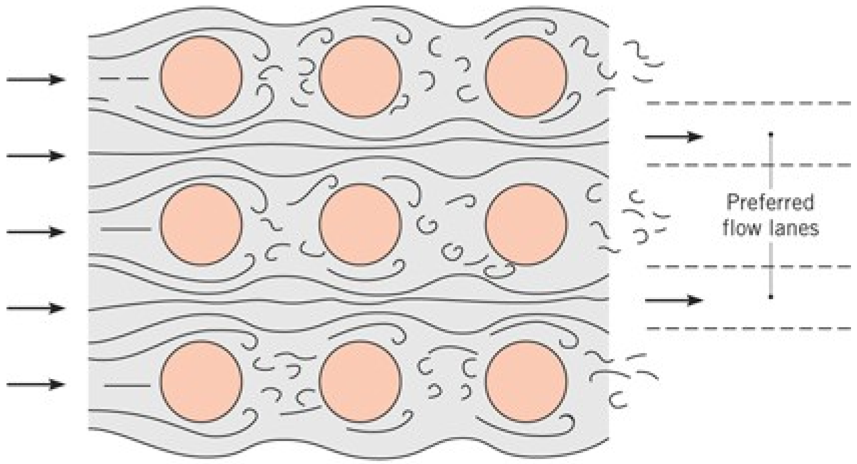
(b)



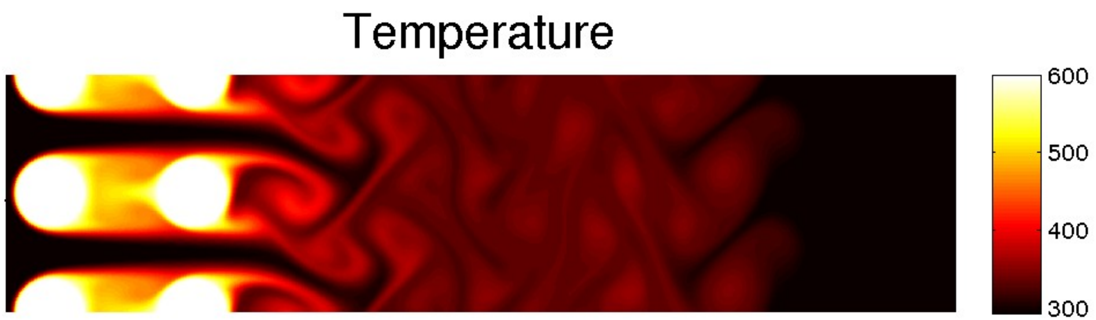
$S_T = transverse\ pitch$

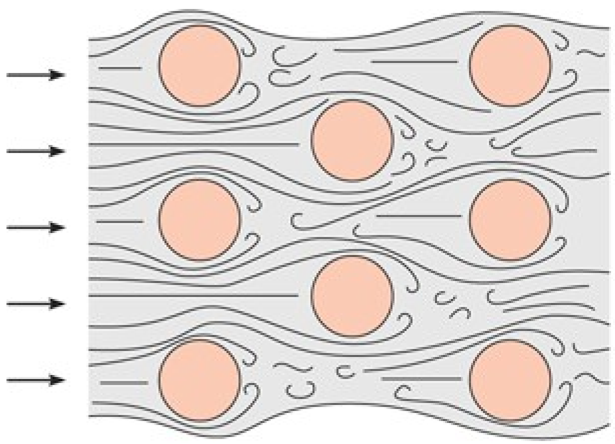
$S_L = longitudinal\ pitch$

Aligned vs staggered configurations



(a)





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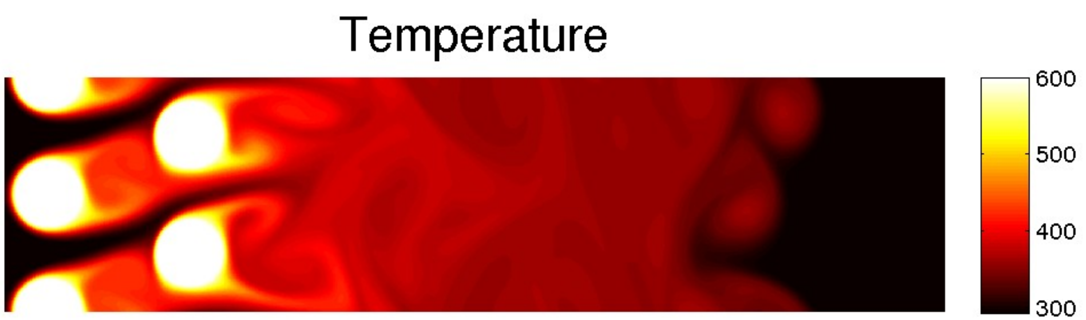
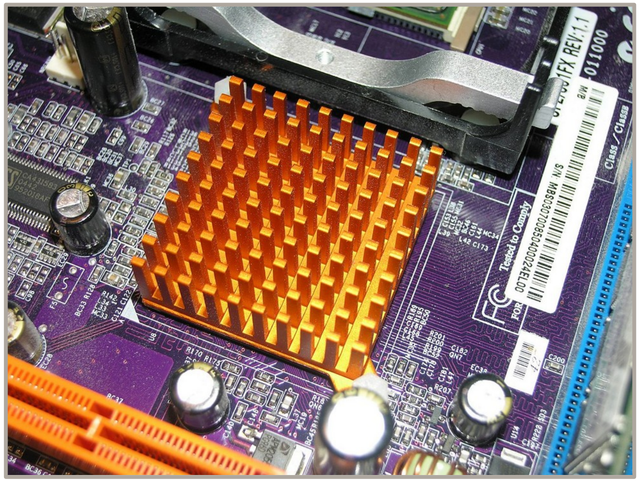


What do you think which one would pose better heat transfer ?



https://pl.wikipedia.org/wiki/Plik:Heatsinkrods.jpg

HW4: Pin fin configuration

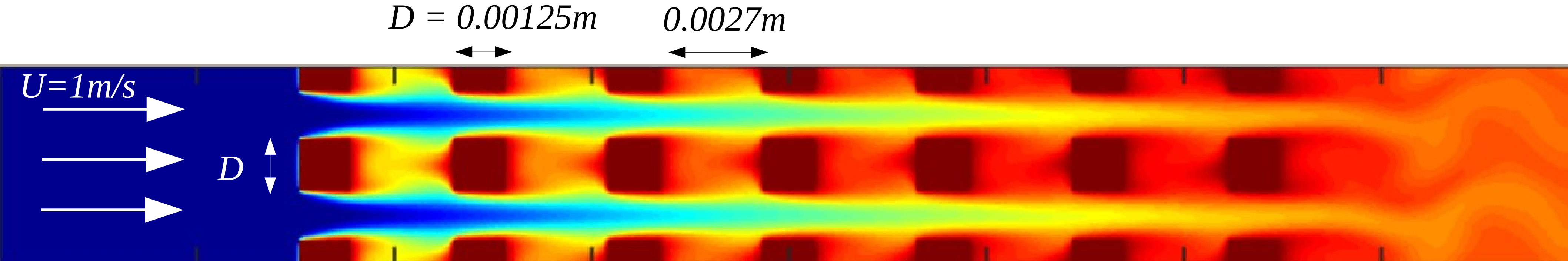
In this session we will look mostly into heat transfer in a pin fin bed. Could occur in electronics cooling or air conditioning system.

Other assumptions during the session:

1) pin fin walls at T_{wall}=+29.6 deg C

2) inflow temperature is T_{left} = +22.6 deg C

3) velocity field is fully developed and laminar and mean inflow velocity is close to 1m/s

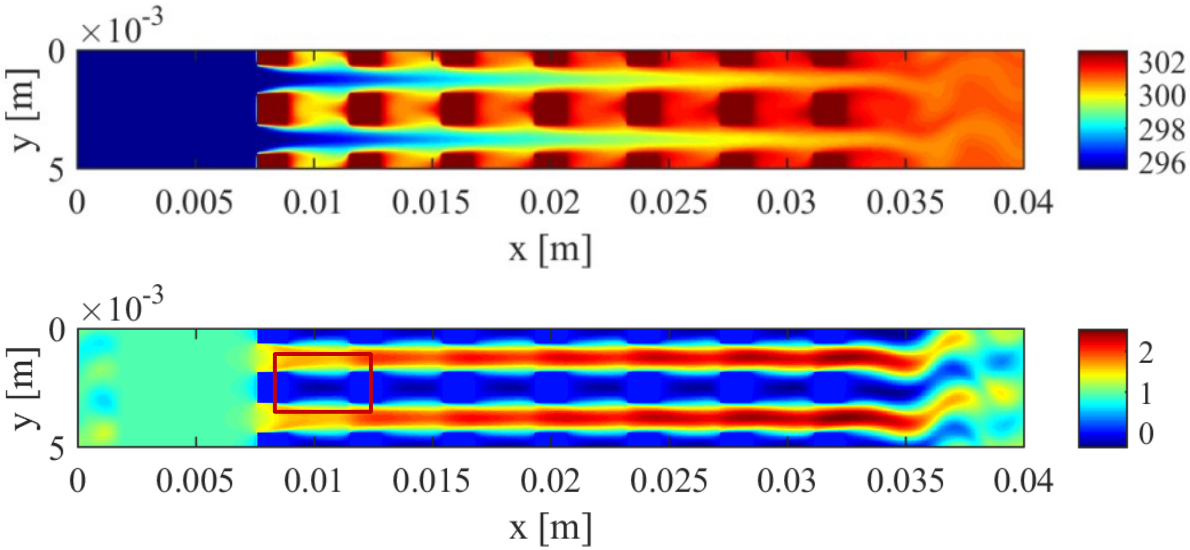


HW4: Convection-diffusion equation for temperature to estimate Nu

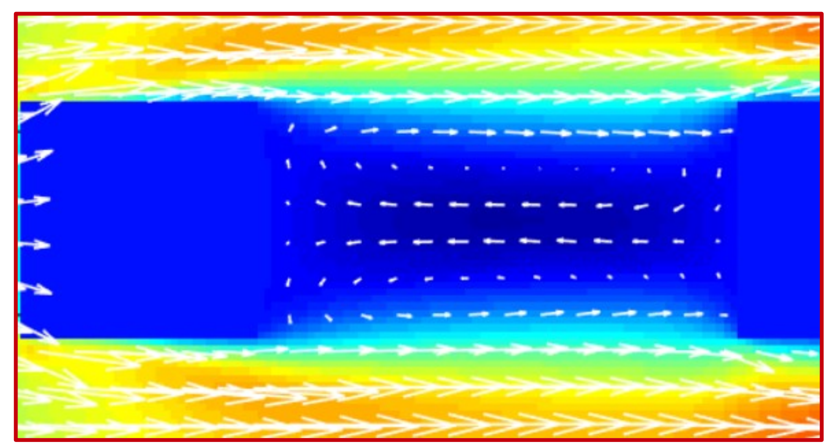
The Matlab session will focus on investigating heat transfer in a 2d fin bed using a provided velocity field which is assumed constant i.e. almost steady state. Constant wall T is assumed.

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}+\alpha\frac{\partial^2 T}{\partial y^2}$$

$T=T(x,y)$ in steady state 2d flow with constant wall temperature BC



Zoom to recirculation zone showing reverse flow (every fifth vector shown)



$u=u(x,y)$ and $v=v(x,y)$ are provided from separate CFD simulation. Assume “frozen flow” and zero velocity on the walls.

STEP 1: download the `heat2dfins` code from MyCourses and extract – by right clicking mouse - to new folder `Week4`.

STEP 2: let us look into the code structure together from the big screen. If you want to study a single fin you may change the line

```
load ../VeldataSingle.mat  % single fin
% or instead
%load ../Veldata.mat       % pin fin array
```

STEP 3: let us visualize the colorful pictures together (velocity, vectors and temperature).

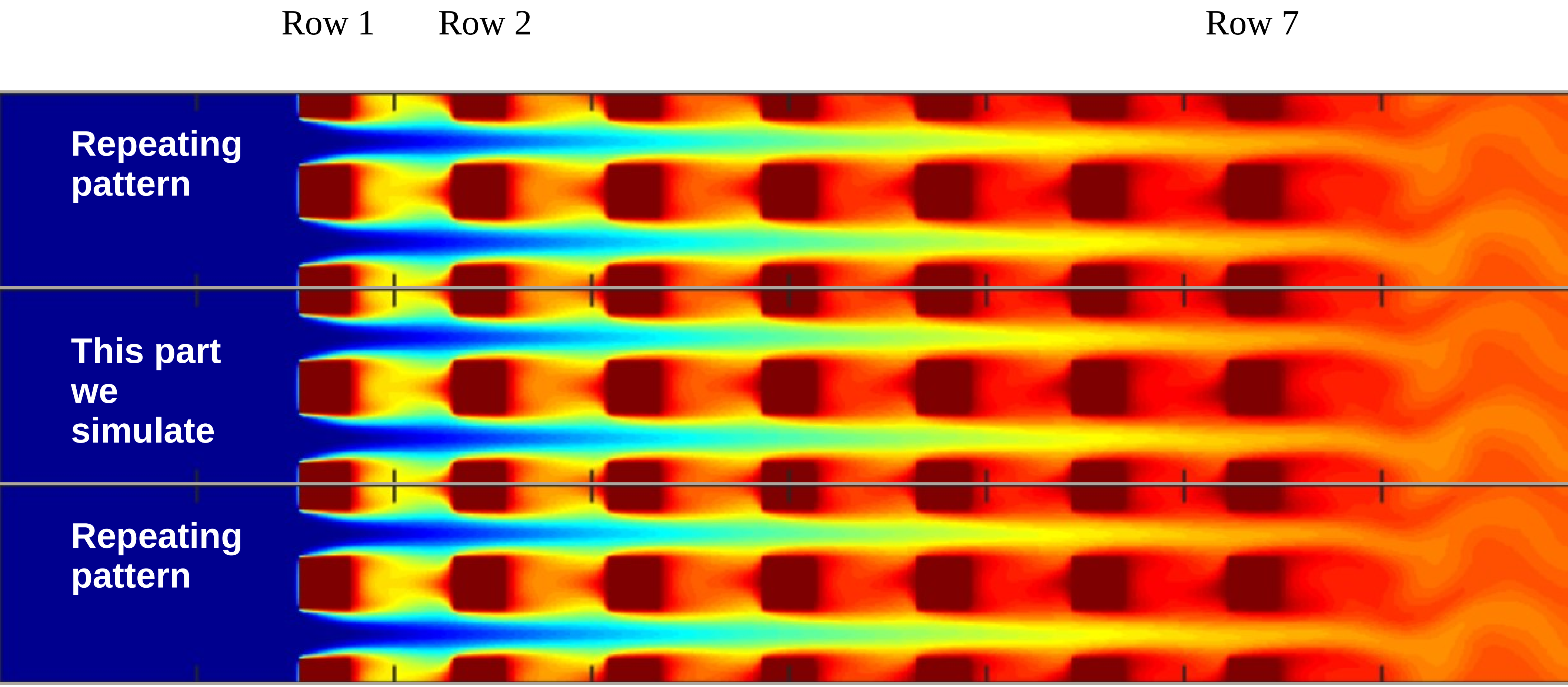
```
figure(2), clf, box
imagesc([min(min(X)) max(max(X))],[min(min(Y)) max(max(Y))], U)
axis equal, hold on

% we want only to plot every 5th vector for visual reasons
qiny = iny(1:5:length(iny)); qinx = inx(1:5:length(inx));
quiver( X(qiny,qinx), Y(qiny,qinx), U(qiny,qinx), V(qiny,qinx),4, 'w')
   colormap jet
   axis tight

   ylabel('y [m]')
   xlabel('x [m]'), colorbar,  drawnow
   axis tight
   axis([0,Lx,0,Ly])

   print -dpng Velocity2dfins
```

STEP 4: use the Hilpert correlation (for cylindrical fins) to estimate Nusselt number under these conditions.

STEP 5: can you get similar order of magnitude for **simulation result** for Nusselt number from **the single fin** case ?

$$\mathrm{Nu}_{ave} = \mathrm{Nu} = \frac{h\,D}{k} = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

We can think we are in fact modeling an “infinite”, repeating pattern of fins (T_s = *const.*)



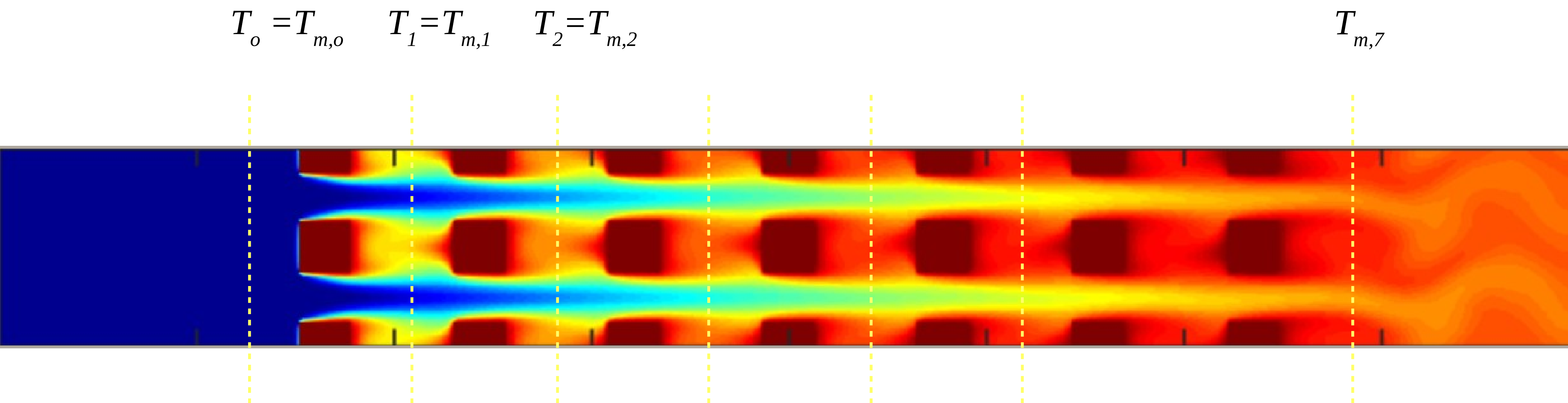
Question: How to use Newton’s cooling law correctly and consistently in this situation ?
Is h constant ? What is T_m ?

$$q = hA(T_s - T_m)$$

Things we are interested in:

ΔT, h, $T_m(x)$

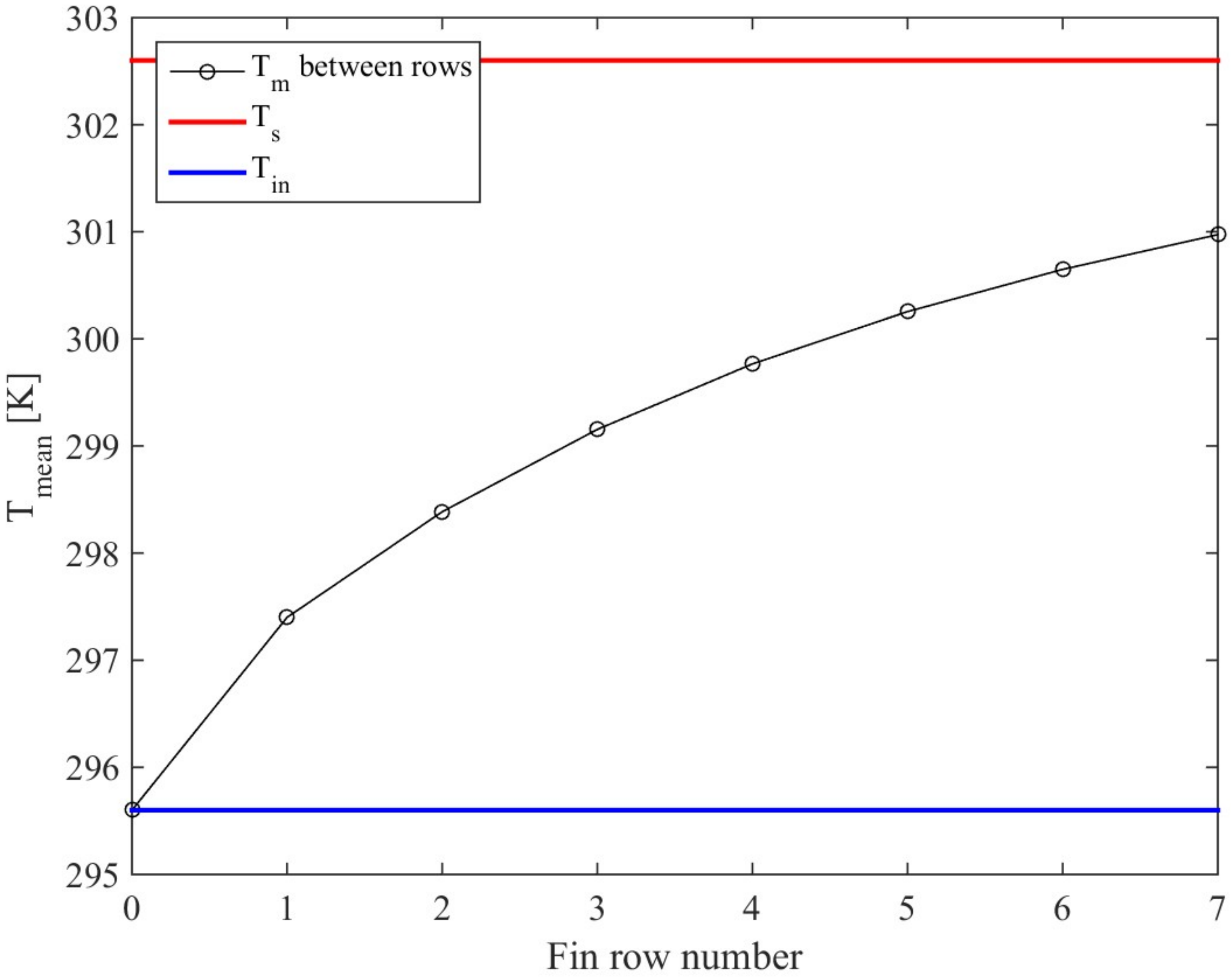
$$q = h_n A_{fin}(T_s - T_n) = c_p \rho U A_{in}(T_{n+1} - T_n)$$



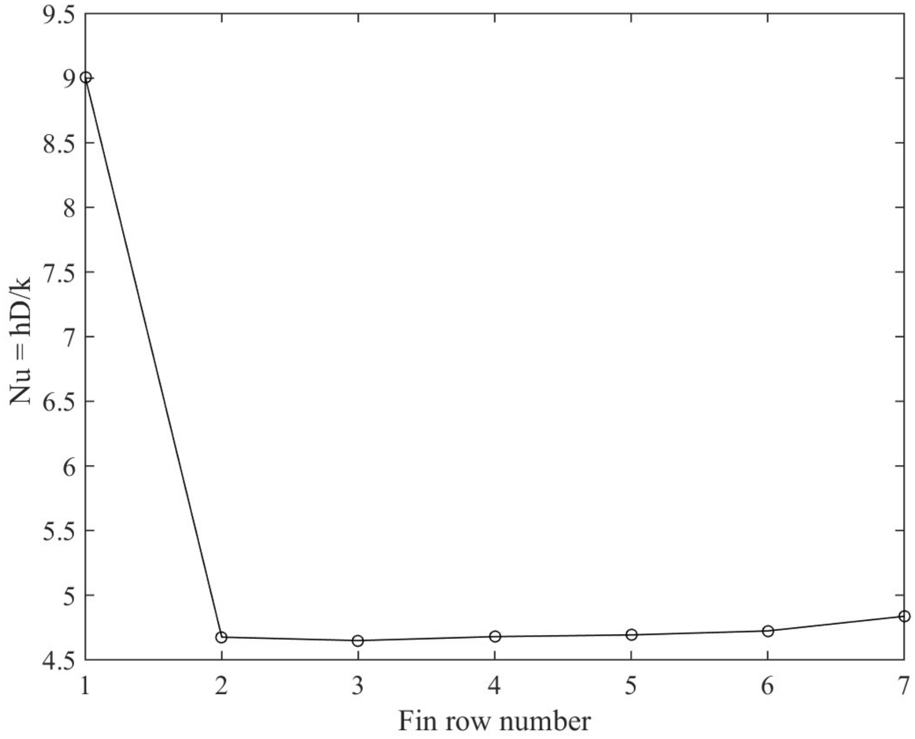
Note:
- we do not know if h is constant row-wise.
- we do not know if temperature jump per row is constant

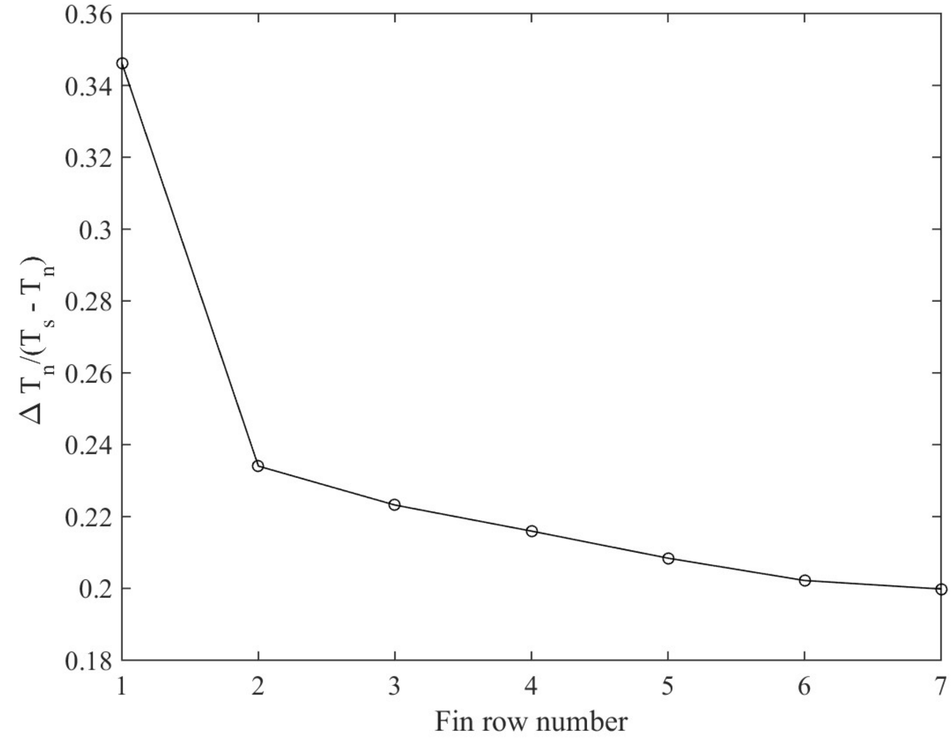
```
%Calculation of mean temperature and velocity
Tm = sum(T(iny,inx).*U)./(sum(U)); Um = mean(U);
```

Mean temperature after n fin rows



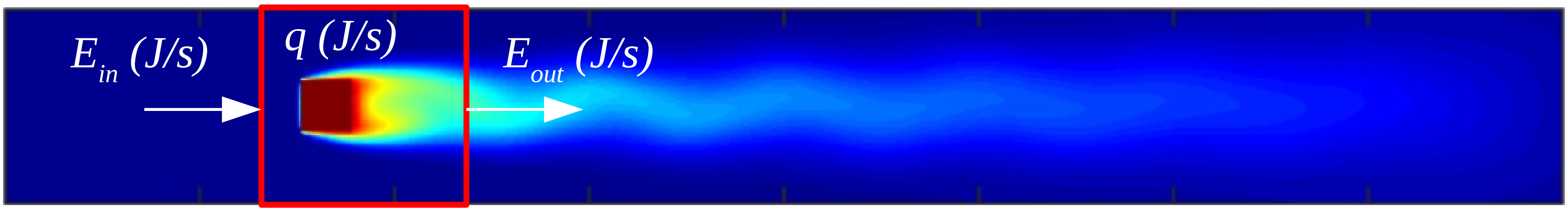
Relative temperature jump for fin array and Nusselt number





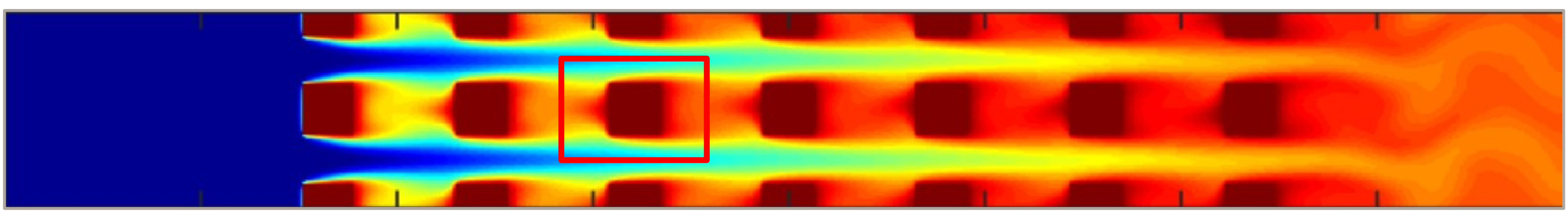
Control volume for energy conservation analysis:

In HW4 one needs to evaluate Nusselt numbers for single fin and multiple fin cases



$$A_{\text{in}} = L_y \cdot L_z \text{ and } A_{fin} = 4\,DL_z$$

$$q = h_n A_{fin}(T_s - T_n) = c_p \rho U A_{\text{in}}(T_{n+1} - T_n)$$



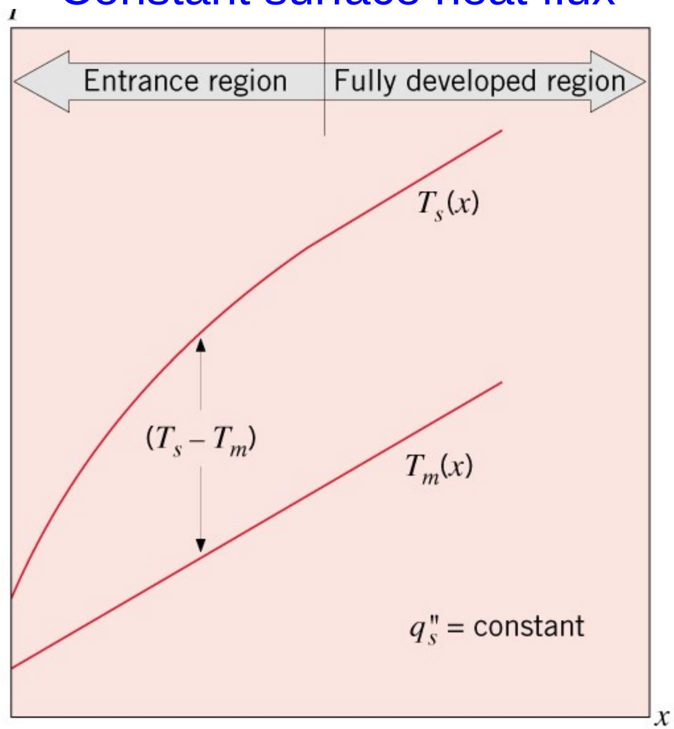
$$A_{\text{in}} = \frac{L_y}{2} \cdot L_z \text{ and } A_{fin} = 4\,DL_z$$

Assessment: Take 5 min time to answer the questions in the online query form.

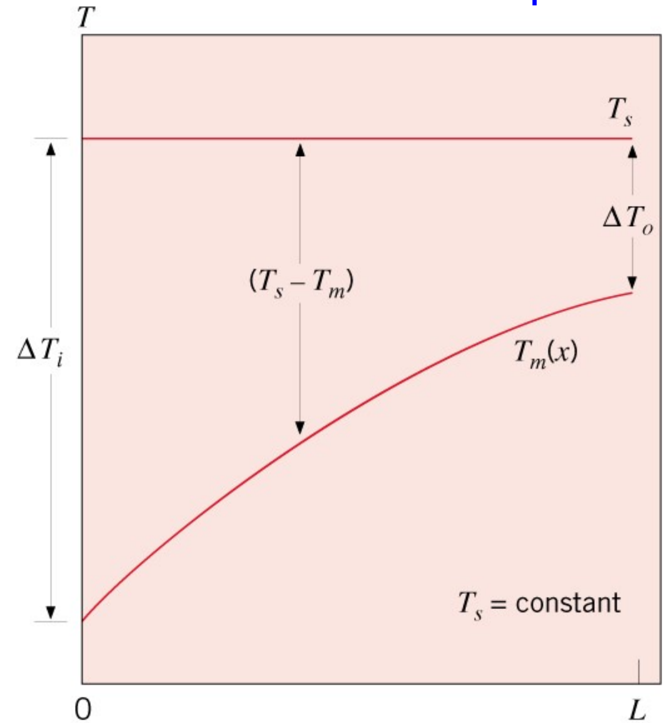
Thank you for your attention!

Appendix I: Axial mean temperature in a pipe or channel (see Incropera Ch. 8)

Constant surface heat flux



Constant surface temperature

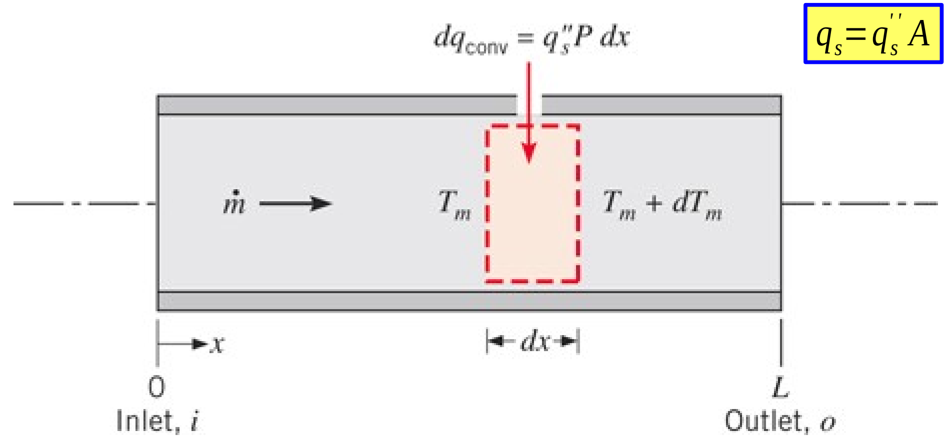


Energy balance for a fluid element in a pipe flow

During a small time interval dt, fluid element dm becomes heated by dT_m (mean temperature) because wall heat flux is conducted to the fluid. **Energy increase** (in Joules):

$$dm\, c_p\, dT_m = q_s''\, P\, dx\, dt$$

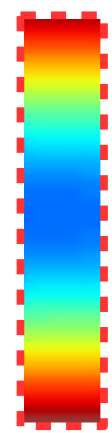
$$q_s = q_s''\, A$$



$P =$ surface perimeter of the tube or spanwise length of channel

The fluid element absorbs energy with certain rate corresponding to the heating power dq_{conv} (in Watts)

$$dq_{conv} = \dot{m}\, c_p\, dT_m = q_s''\, P\, dx$$



The mean temperature obeys the equation

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}\, c_p} = \frac{P}{\dot{m}\, c_p} h\left(T_s - T_m\right)$$

For constant surface heat flux

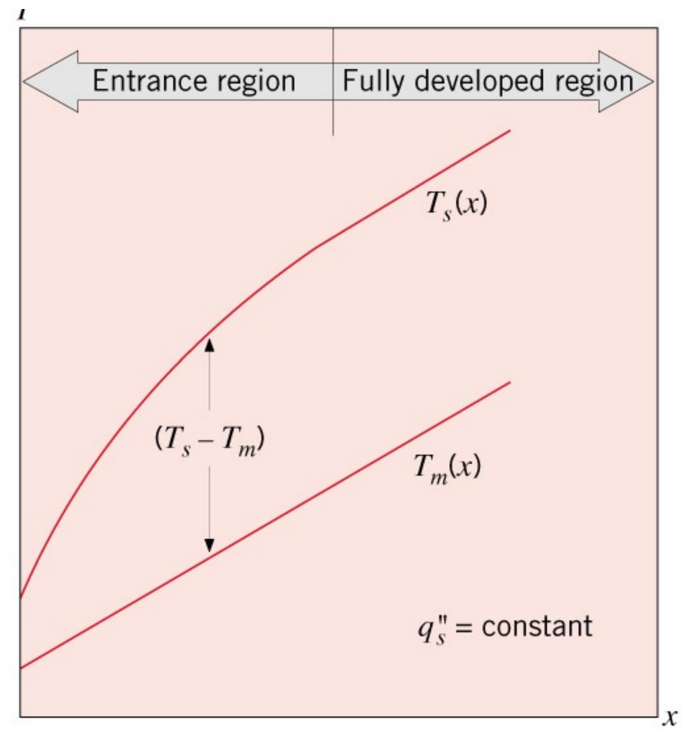
$$\frac{dT_m}{dx}=\frac{q_s^{''}P}{\dot{m}c_p}=\frac{P}{\dot{m}c_p}h(T_s-T_m)$$

$$\frac{dT_m}{dx}=\frac{q_s^{''}P}{\dot{m}c_p}\neq f(x)$$

$$\frac{q_s^{''}P}{\dot{m}c_p}=const.$$

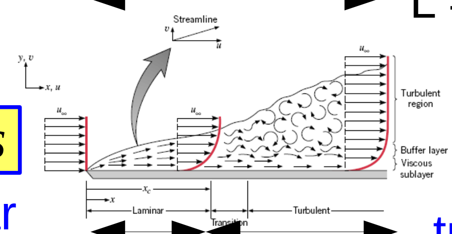
$$T_m(x)=T_{m,i}+\frac{q_s^{''}P}{\dot{m}c_p}x,\, q_s^{''}=const.$$

→ Linear increase in mean temperature



Appendix II: Example 6.4 – Laminar and turbulent water flow – flat plate

L = 0.6m



laminar

turbulent

$$[x]=m$$

$$T_\infty = 300\,K/350\,K$$

$$U_\infty = 1\,m/s$$

$$h_{lam}(x) = C_{lam}\,x^{-0.5}$$

$$h_{turb}(x) = C_{turb}\,x^{-0.2}$$

@T=300K

$$C_{lam,300} = 395\,W/m^{1.5}\cdot K,\; C_{turb,300} = 2330\,W/m^{1.8}\cdot K,$$

$$[h] = W/m^2\cdot K$$

@T=350K

$$C_{lam,350} = 477\,W/m^{1.5}\cdot K,\; C_{turb,350} = 3600\,W/m^{1.8}\cdot K,$$

$$\mathrm{Re}_c = \frac{\rho\,U_\infty\,x_c}{\mu}$$

$$\bar{h} = ?$$

Representative value

$$\mathrm{Re}_c \approx 5\cdot 10^5$$

$$T = 300\,K : \rho = 997\,kg/m^3, \mu = 855 \cdot 10^{-6}\,Ns/m^2$$

$$x_c = \frac{\mathrm{Re}_c \mu}{\rho U_\infty} = 0.43\,m$$

$$T = 350\,K : \rho = 974\,kg/m^3, \mu = 365 \cdot 10^{-6}\,Ns/m^2$$

$$x_c = \frac{\mathrm{Re}_c \mu}{\rho U_\infty} = 0.19\,m$$

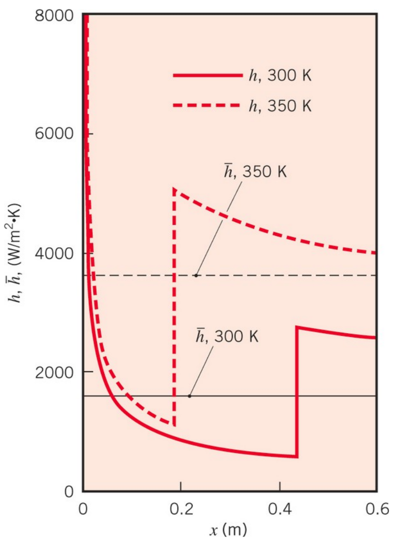
$$\bar{h} = \frac{1}{L}\left[\int_0^{x_c} h\,dx + \int_{x_c}^{L} h\,dx\right]$$

$$\bar{h}_{300} = 1620\,W/m^2 \cdot K$$

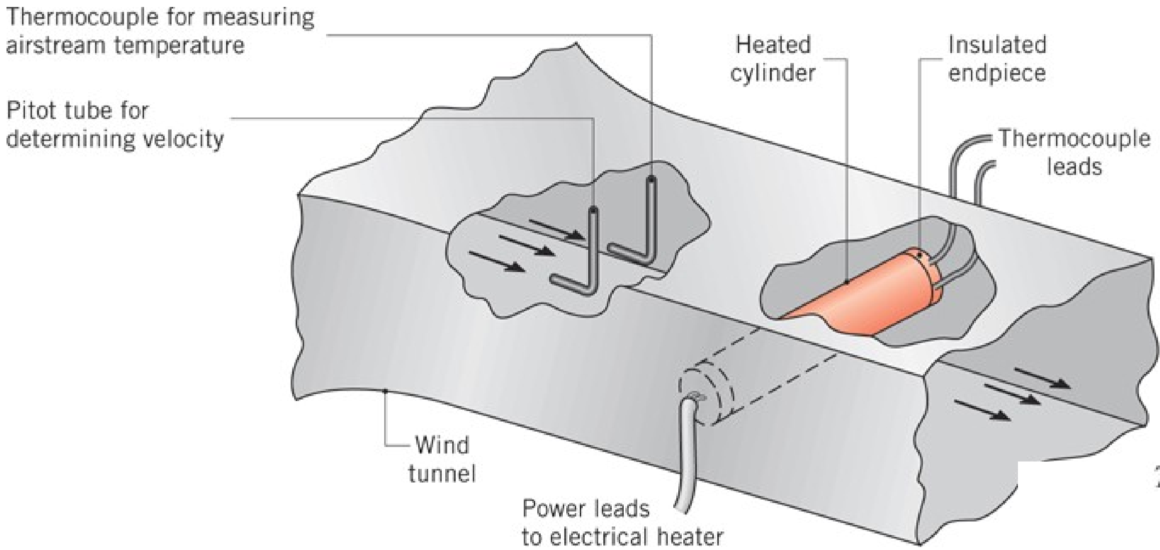
$$\bar{h}_{350} = 3710\,W/m^2 \cdot K$$

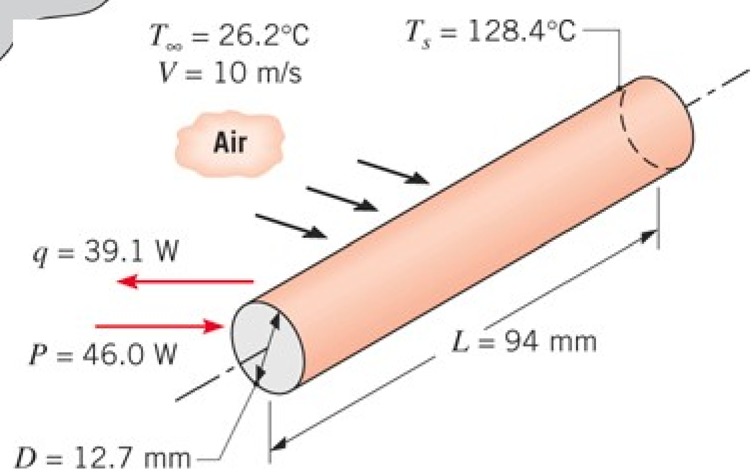
For flat plate

$$\bar{h} = \frac{1}{L}\int_{x=0}^{L} h\,dx$$



Appendix III: Example 7.4 – Heated cylinder in cross-flow





$$\bar{h} = ?$$

+ compare the result to correlation equation

$$\bar{h} = \frac{q}{A\left(T_s - T_\infty\right)} = 102\, W/m^2 \cdot K$$

$$\overline{Nu}_D = C\, \mathrm{Re}_D^m\, Pr^n \left(Pr/Pr_s\right)^{1/4}$$

$$\mathrm{Re}_D = 7992 \rightarrow C = 0.26, m = 0.6, Pr \;\; 0.7 < 10 \rightarrow n = 0.37$$

Table 7.2

$$\overline{Nu}_D = 50.5$$

$$\bar{h} = \overline{Nu}_D\, k/D = 105\, W/m^2 \cdot K$$

$\bar{h} = 105\, W/m^2 \cdot K$ → close to first observed value $\bar{h} = 102\, W/m^2 \cdot K$