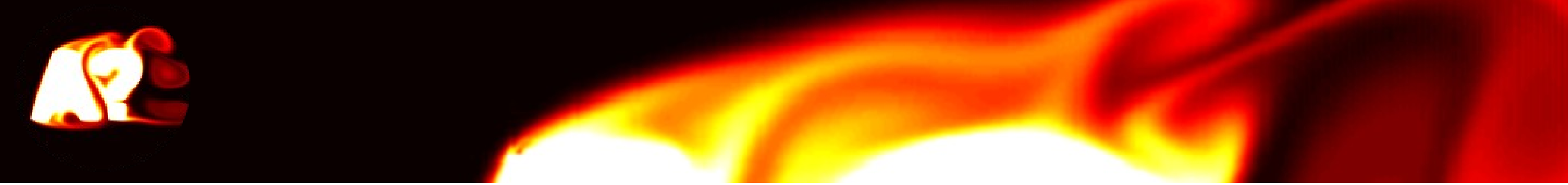


EEN-1020 Heat transfer

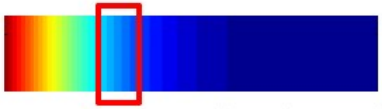
Week 5: Natural Convection

Prof. Ville Vuorinen

November 24th-25th 2019

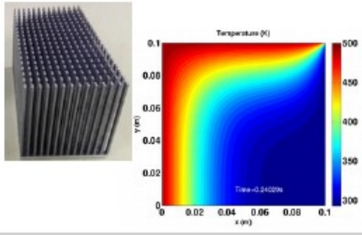
Aalto University, School of Engineering

Week 1: Energy conservation, heat equation, conduction Fourier/Newton

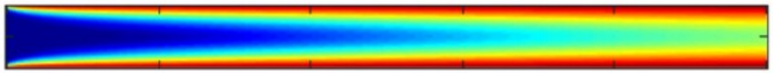


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x}\left(k\frac{\partial T(x+\Delta x/2,t)}{\partial x} - k\frac{\partial T(x-\Delta x/2,t)}{\partial x}\right)$$

Week 2: Fin theory, conduction, intro to convection

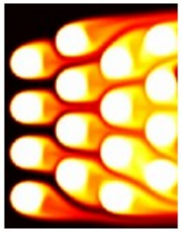


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

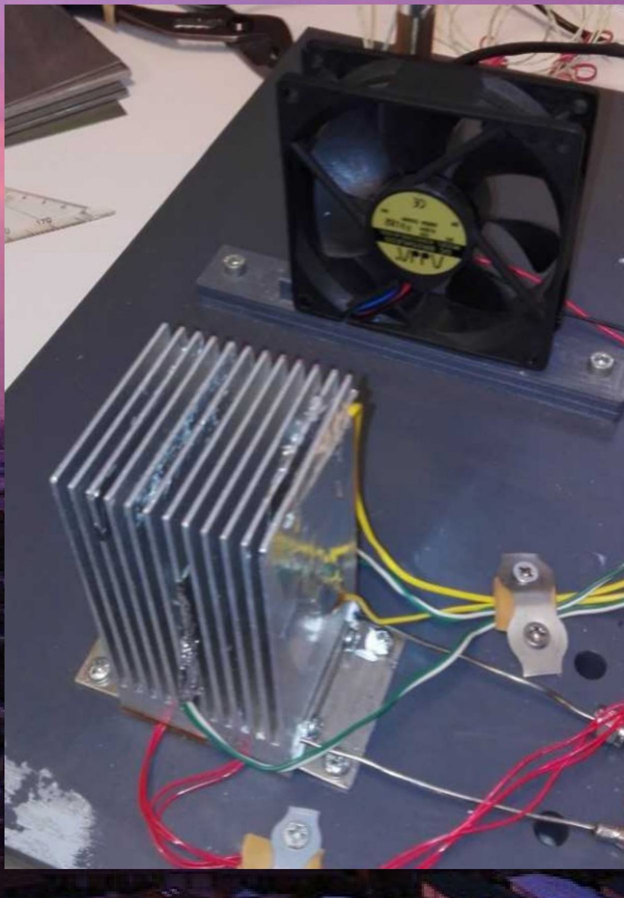
Week 5: natural convection, boiling, correlations



Warm fluids are lighter than cold fluids → ability to rise against gravity.

In 2018, we switched off the fan in the class room demo system. We could consider that natural convection (or radiation) is the remaining cooling mechanism. **Consequence:** the heat exchanger became extremely hot! From energy conservation, it is easy to calculate

$dT/dt = q/mc_p \rightarrow$ e.g. 0.25K/s !



Recommended reading: Ch 9 “Free Convection” selective parts from “Principles of Heat and Mass Transfer”, Incropera

Remember: These slides may contain typos or other mistakes so please be cautious when reading.

Remember: Fluid and solid properties depend in reality on thermodynamic conditions so please use always values taken from a proper source (e.g. Incropera Appendix contains some reasonable values)

In heat transfer course, we have “4 friends” who typically help us to approach and solve any problem

1) Energy conservation (**“J/s” thinking**)

2) Fourier’s law

3) Newton’s cooling law

4) Bonus “helpers”:

   4.1) heat eqn, conv.-diff. eqn (relatives of friend #1)

   4.2) Navier-Stokes eqn (momentum conservation)

   4.3) non-dimensional numbers (follow from the other friends)

Recap on CD-equation i.e. thermal energy conservation under convection/diffusion

$$\boxed{\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}+\alpha\frac{\partial^2 T}{\partial y^2}}$$

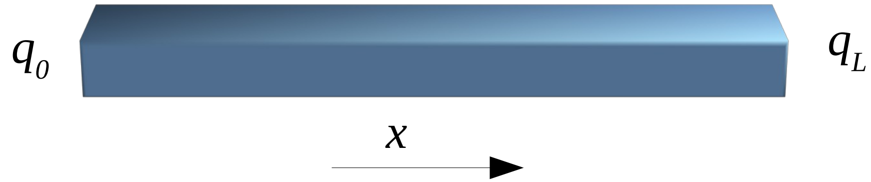
On week 4 we discussed that Fourier's law is essentially connected to the diffusion terms

Consider 1d conduction in a rod heated from both ends

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

Cooled end

Heated end



Considering evolution of mean temperature T_m we integrated 1d heat eqn over rod volume $V = AL$ noting that the differential volume element $dV = Adx$

$$\rho c_p AL \frac{1}{L} \frac{\partial}{\partial t} \int_0^L T(x,t)\, dx = kA \int_0^L \frac{\partial^2 T}{\partial x^2} dx$$

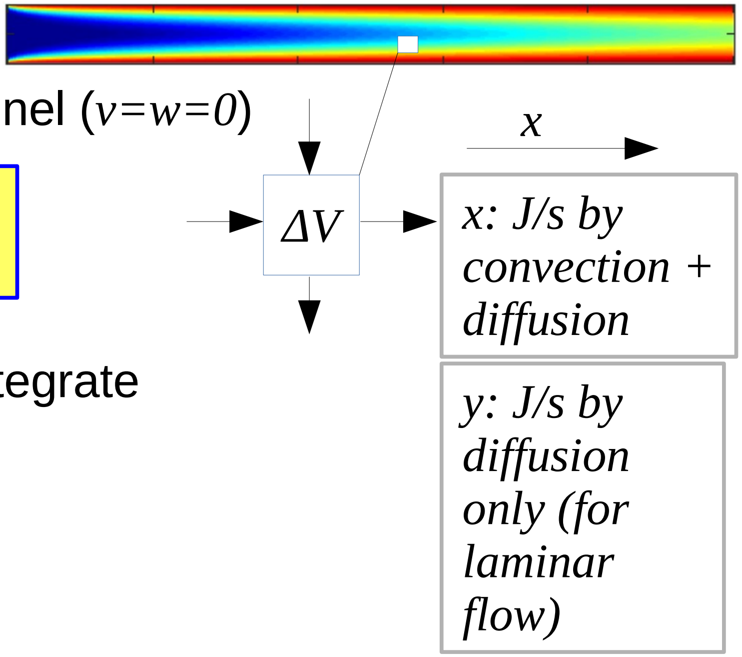
Now, the RHS can be integrated directly which gives:

$$c_p m \frac{dT_m(t)}{\partial t} = A[q_L - q_0]$$

- Rate of change of total thermal energy = *heat rate in – heat rate out*
- "Friend #1" (J/s) observed
- "Friend #2" (Fourier) observed

$$\text{Fourier: } q = k \frac{\partial T}{\partial x}$$

What remains unanswered is the convection terms – do they really represent energy conservation ?



Consider for a moment pure **convection** in a channel (*v=w=0*)

$$\boxed{\rho c_p \frac{\partial T}{\partial t} + \rho c_p \frac{\partial u T}{\partial x} + \cancel{\rho c_p \frac{\partial v T}{\partial y} + \rho c_p \frac{\partial w T}{\partial z}} = 0}$$

Considering evolution of temperature *T(x,y,t)* we integrate over a small volume $\Delta V = \Delta x \Delta y \Delta z$

$$\boxed{\rho c_p \frac{\partial}{\partial t} \int_{\Delta V} T(x,t)\, dV = \rho c_p \left[\int_{\Delta V} u \frac{\partial T}{\partial x} dV \right]}$$

We see that for mean temperature T_m of a volume ΔV with mass m

$$\boxed{c_p m \frac{dT_m}{dt} = c_p \rho \left[u_{m,out} T_{m,out} - u_{m,\mathrm{in}} T_{m,\mathrm{in}} \right] \Delta y \Delta z}$$

+Diffusion terms from previous slide

$$\boxed{+\left[Q_{x,out} - Q_{x,\mathrm{in}} + Q_{y,out} - Q_{y,\mathrm{in}} \right]}$$

Conclusions:
- Rate of change of total thermal energy = *heat rate in – heat rate out*
- **"Friend #1" (J/s) observed**
- Indeed, energy transported by convection and diffusion

Recall week 1

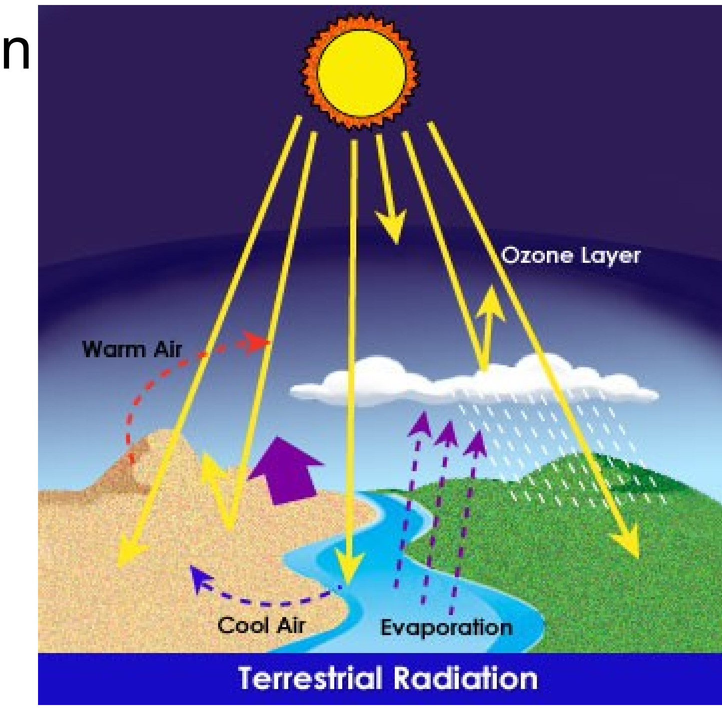
$$\boxed{q = c_p \rho U A \Delta T}$$

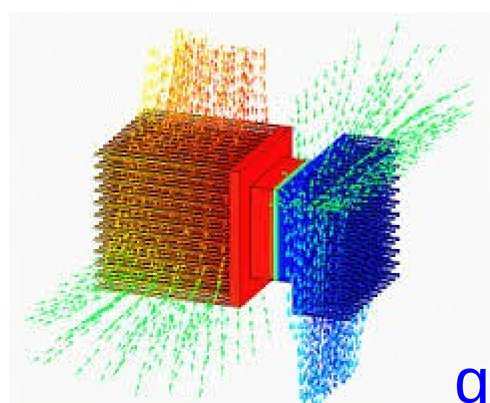
Lecture 5.1 Theory: Natural convection (free convection)

ILO 5: Student can choose Nusselt number correlation equations for different situations including natural convection.

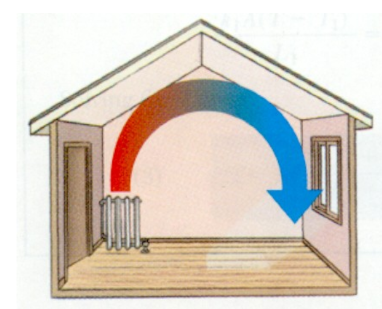
Topics covered

- Governing equations in natural convection
- Non-dimensional numbers
- Stable vs unstable configurations
- Boundary layers in natural convection
- Velocity order of magnitude in natural convection





gravity



Two sides of natural convection

***Advantages*:**

- **Cooling**: if NC provides enough cooling then no extra fans needed
- **Heating**: air may circulate naturally in pipelines; often pumping is still needed
- **Heating**: heated room air can become distributed by slow currents

***Disadvantages*:**

- **Cooling**: NC is a slow cooling mechanism (e.g. thick thermal boundary layers, low Nusselt numbers)
- **Ventilation**: NC can lead to unsteady behavior of air if large human crowds provide extensive heat (e.g. 1000*100W = 100kW) → must be accounted for in ventilation design
- **Energy production:** "It is always there" → Always somehow present as $\sim L^3$ dependent parameter → system size L dependent, sensitive phenomenon

RADIATION: NOT CONSIDERED HERE

Why a drink can cools ? Faster cooling horizontally or vertically ? What difference would it make ?

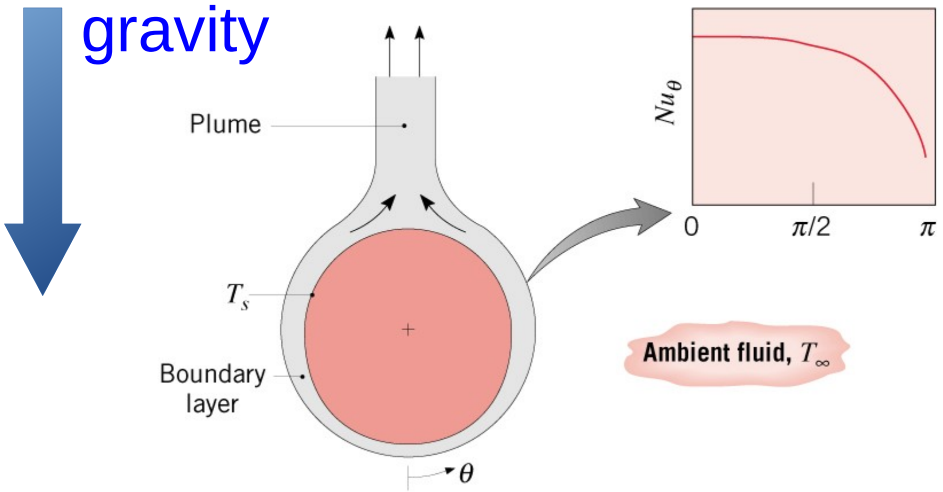
Newton

$$\boxed{q=hA_s(T_s-T_\infty)}$$

Physical mechanism behind h ?

Nusselt number correlation for a horizontal cylinder

Horizontal cylinder



$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{Buoyancy\ force}{Viscous\ force}$$

$$Ra_D < 10^{12}$$

$$\bar{Nu}_D = 0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}$$

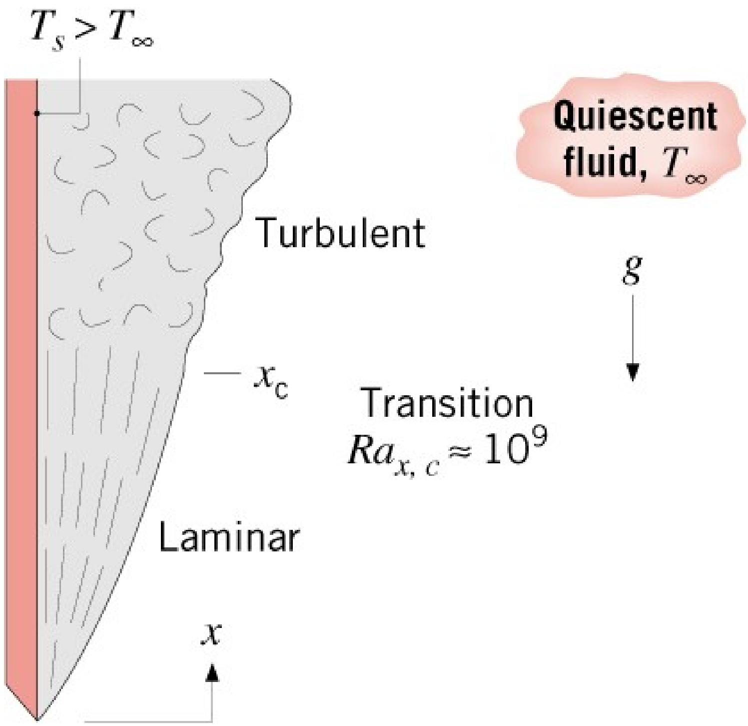
$$What\ happens\ when\ Ra_D \rightarrow 0\ ?$$

$$\bar{Nu}_D = \frac{\bar{h}\, D}{k}$$

Figure: V.Vuorinen (2016)

Nusselt number correlation for a vertical plate

NC creates flow against gravity → near-wall boundary layers → possibility for laminar to turbulence transition → critical Rayleigh number



Critical Rayleigh number

$$\boxed{\mathrm{Ra}_c = 10^9}$$

Since $\boxed{\mathrm{Ra}_c \sim L^3}$ vertical plane wall boundary layers become turbulent at a critical length.

Laminar

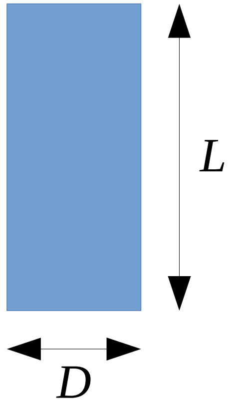
$$\boxed{\bar{Nu}_L = 0.68 + \frac{0.670\,Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}}$$

All conditions

$$\boxed{\bar{Nu}_L = \left[0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2}$$

https://www.youtube.com/watch?v=6ney_Vx00zU

Which way does a can cool faster in the fridge: horizontally or vertically ?



$D = 0.06m,\ L = 0.17m$

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{Buoyancy\ force}{Viscous\ force}$$

These two correlations could be used to approach the problem (ignoring the end effects).

$$\bar{Nu}_D = 0.60 + \frac{0.387\,Ra_D^{1/6}}{[1+(0.559/Pr)^{9/16}]^{8/27}}$$

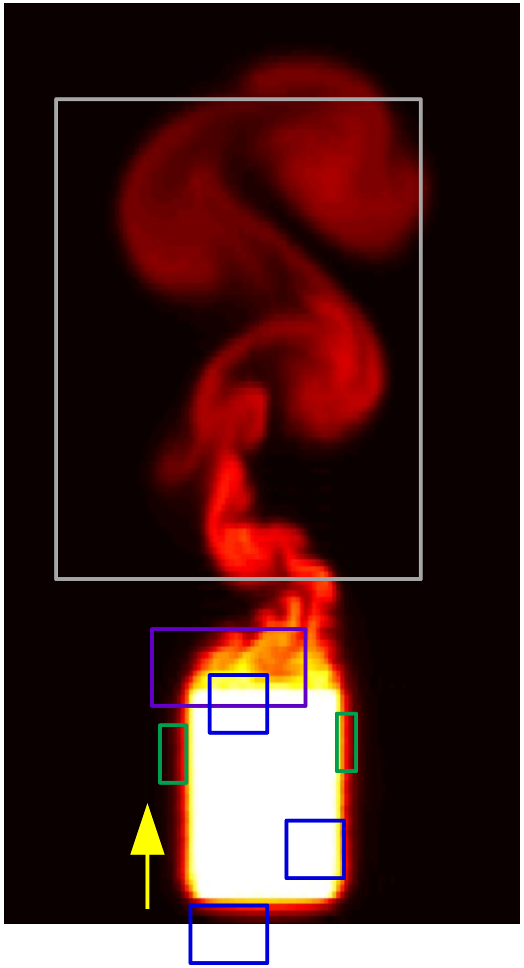
$$\bar{Nu}_L = [0.825 + \frac{0.387\,Ra_L^{1/6}}{[1+(0.492/Pr)^{9/16}]^{8/27}}]^2$$

Some physical steps how heat transfer away from a can in natural convection

Step 1: Conduction from the wall to the fluid and conduction in the thermal boundary layer (TBL).

Step 2: Heated fluid starts rising upwards already when conducting in the TBL

Step 3: Accelerated flow forms **viscous and thermal** boundary layers around the can.



Step 5: fluid rises constantly and the hot air is “**self-transported**” away from the object in a plume which poses fluid dynamical structures (e.g. vortices, turbulence)

Step 4: fluid motion becomes 3d and turbulence starts to transport heat from the top surface

Governing equations

Note: In forced convection the Reynolds number was of very high importance. In natural convection the Rayleigh (and/or Grashof) number is typically the key driving parameter distinguishing also between laminar and turbulent conditions.

Note: here we do not discuss the mixed convection case where Re and Ra may both be relevant.

Important numbers

Grashof number

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{Buoyancy\ force}{Viscous\ force}$$

$L \rightarrow characteristic\ length\ scale\ of\ surface/object$

$\beta \rightarrow thermal\ expansion\ coefficient$

Thermal expansion coefficient

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Rayleigh number (Ra and Gr closely related)

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{Buoyancy\ force}{Viscous\ force}$$

$\alpha = thermal\ diffusivity,\ \nu = kinematic\ viscosity$

Note: Thermal expansion coefficient for ideal gas

$$\boxed{p = \rho RT}$$

$$\boxed{\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial [p/RT]}{\partial T}\right)_p = \frac{p}{\rho R T^2} = 1/T}$$

Note: for other fluids values of the expansion coefficient have been tabulated.

Assumptions in Boussinesq approximation

- density is assumed to have a well defined mean part and a fluctuation part

$$\rho = \rho_o - \left(\frac{\partial \rho}{\partial T}\right)_p \Delta T$$

or

$$\rho = \rho_o \boxed{- \beta \rho_o \Delta T}$$

Fluctuation part of density contributing to a buoyancy force in the Boussinesq approximation

- thermodynamic pressure is assumed to be almost constant
(often a very good assumption because speed of sound is typically high in comparison to other velocities*)

- temperature will then be a function of density

- when temperature of a point in space increases, the density decreases

- it leads to a buoyancy force promoting motion against gravity ("**hot air balloon effect**")

- we can think that pointwise fluctuations of temperature from the mean ($T' = T - T_{ref}$) promote/drive the flow into motion

*__Note__: e.g. in a typical flame pressure is almost constant but density and temperature depend very strongly on position (low density in hot parts).

Governing equations (here 2d) in natural convection in the Boussinesq approximation

Navier-Stokes (momentum)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial x^2}+\nu\frac{\partial^2 u}{\partial y^2}-\boxed{g_x\beta\left(T-T_\infty\right)}$$

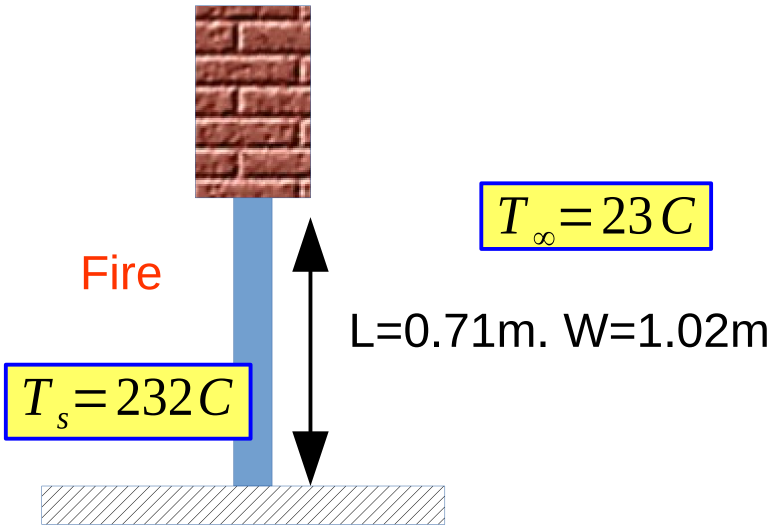
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 v}{\partial x^2}+\nu\frac{\partial^2 v}{\partial y^2}-\boxed{g_y\beta\left(T-T_\infty\right)}$$

Buoyancy force: tries to promote motion into opposite direction from gravity

Convection-diffusion for temperature (energy equation)

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}+\alpha\frac{\partial^2 T}{\partial y^2}$$

Example 9.2: Glass window of a fireplace – relevance HW5



Estimate:
- convection heat rate

$$Ra = \frac{g\beta (T_s - T_\infty) L^3}{\nu \alpha} = 1.813 \cdot 10^9 > Ra_c$$

$$\bar{Nu}_L = [0.825 + \frac{0.387\, Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}]^2 = 147$$

$$\bar{h} = \frac{\bar{Nu}_L k}{L} = 7.0\, W/m^2 K$$

$$q = \bar{h} A_s (T_s - T_\infty) = 1060\, W$$

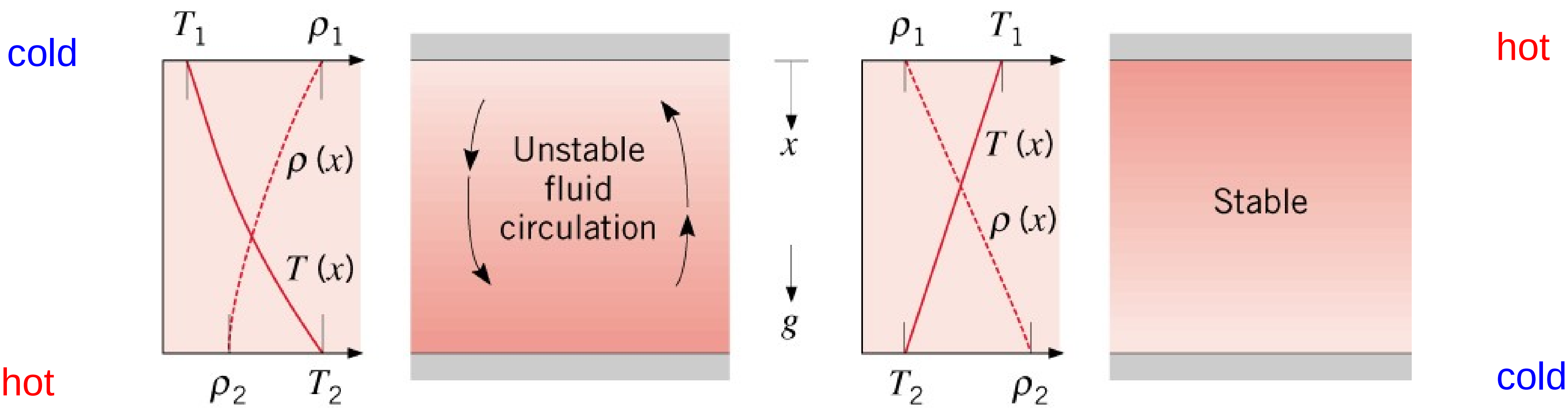
Note 1:

$$q_{rad} = \epsilon A_s \sigma (T_s^4 - T_\infty^4) = 2355\, W$$

Note 2: What would be T on the Opposite side of glass ?

Flow in confinements

Unstable vs stable configurations



$$\frac{dT}{dx} > 0,\ \frac{d\rho}{dx} < 0$$

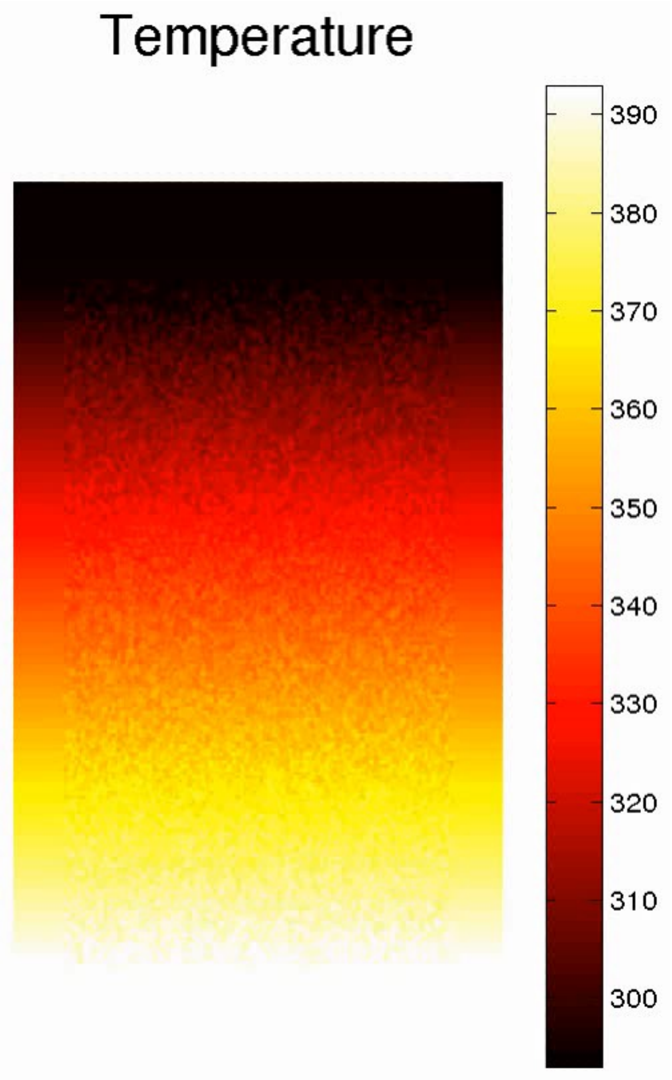
(a)

$$\frac{dT}{dx} < 0,\ \frac{d\rho}{dx} > 0$$

(b)

gravity

Case: Enclosed, tight water-filled kettle on the stove

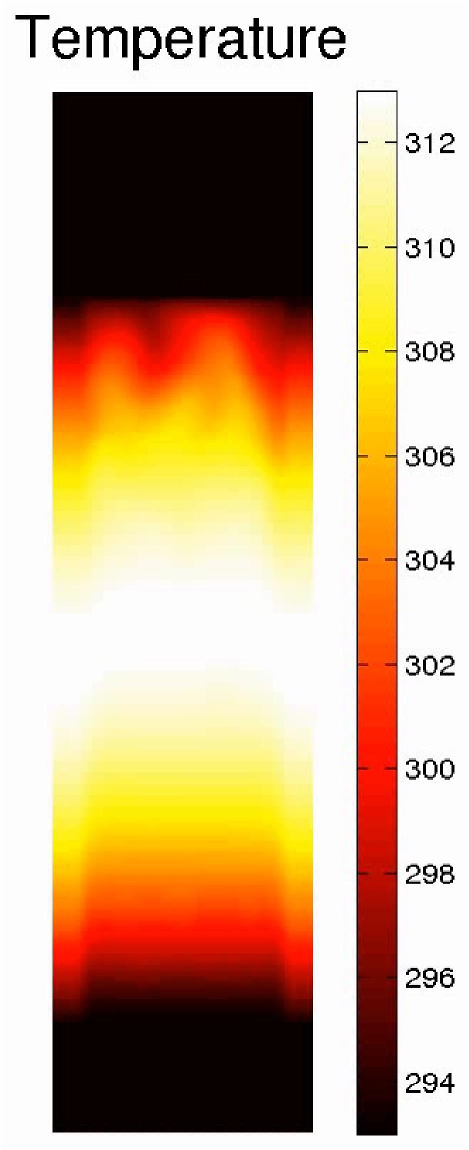


Case: enclosed "kettle" on the stove with space-dependent heating at the walls (linearly decreasing towards the top).

Question 1: Does the schematic on stable vs unstable configuration explain what happens here?

Question 2: Does a steady state solution exist when time$\rightarrow$ infinity?

Case: Enclosed furnace with space-dependent wall heating



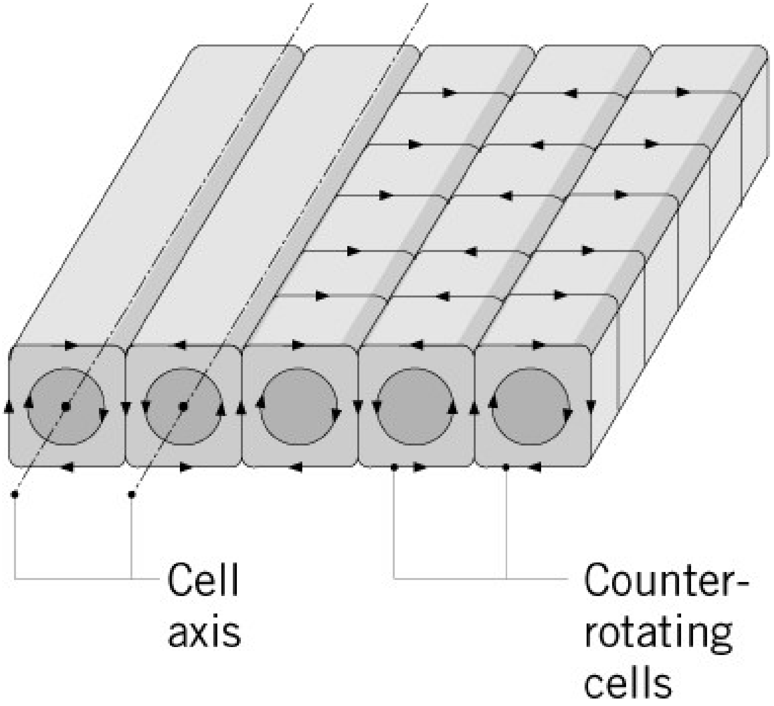
Recall some previous slides:
Stable vs unstable configuration

Case: enclosed “furnace” with space-dependent heating at the walls (cold at top and bottom parts, hot in the center).

Question: Does the schematic picture from the previous slide (stable vs unstable configuration) explain what happens?

Enclosed cavities, heating from below

https://www.youtube.com/watch?v=OM0l2YPVMf8
https://www.youtube.com/watch?v=jFI5KaAqfXI



$$\boxed{Ra < Ra_c = 1708}$$

$$\boxed{\bar{Nu}_L = \frac{\bar{h} L}{k} = 1}$$

Case 2: Thermally unstable but regular cell patterns

$$\boxed{1708 < Ra_L < 5 \cdot 10^4}$$

Case 3: Flow is turbulent

$$\boxed{3 \cdot 10^5 < Ra_L < 7 \cdot 10^9}$$

https://www.youtube.com/watch?v=gSTNxS96fRg&t=56s

Lecture 5.2 Numerical approach from 2018: HW4/heating problem changed to a cooling problem

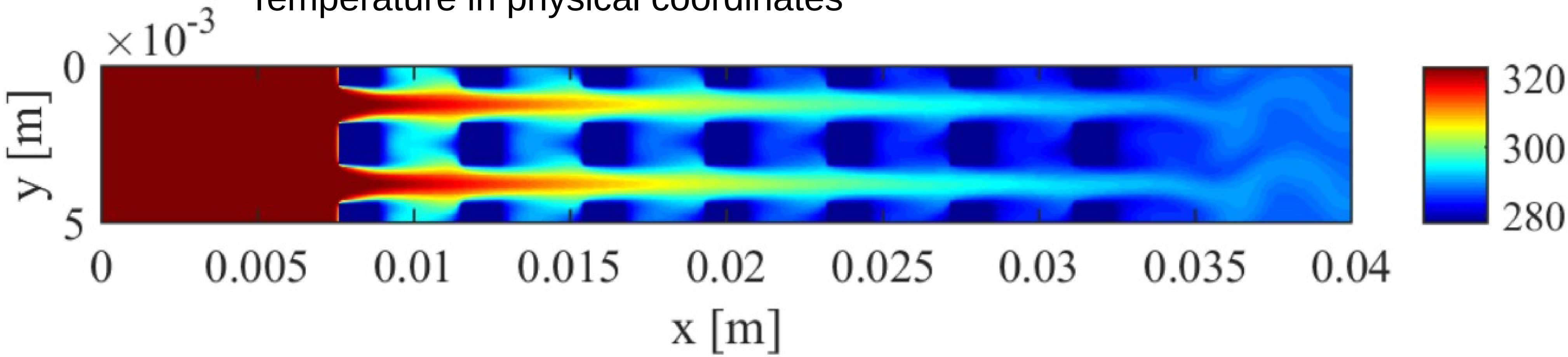
STEP 1: download the `heat2dfins` code from MyCourses and extract – by right clicking mouse - to new folder `Week5`.

STEP 2: make sure you study the fin array now + choose the `VisualizeResultsFull` in `heat2dfins` **`for`** loop.

```
%load VeldataSingle.mat  % single fin
% or instead
load Veldata.mat         % pin fin array
```

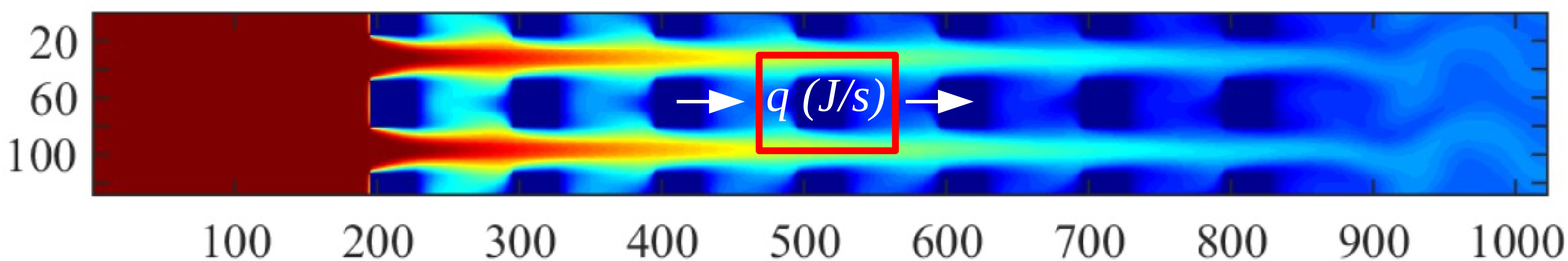
In HW2, we study a cooling problem

Temperature in physical coordinates



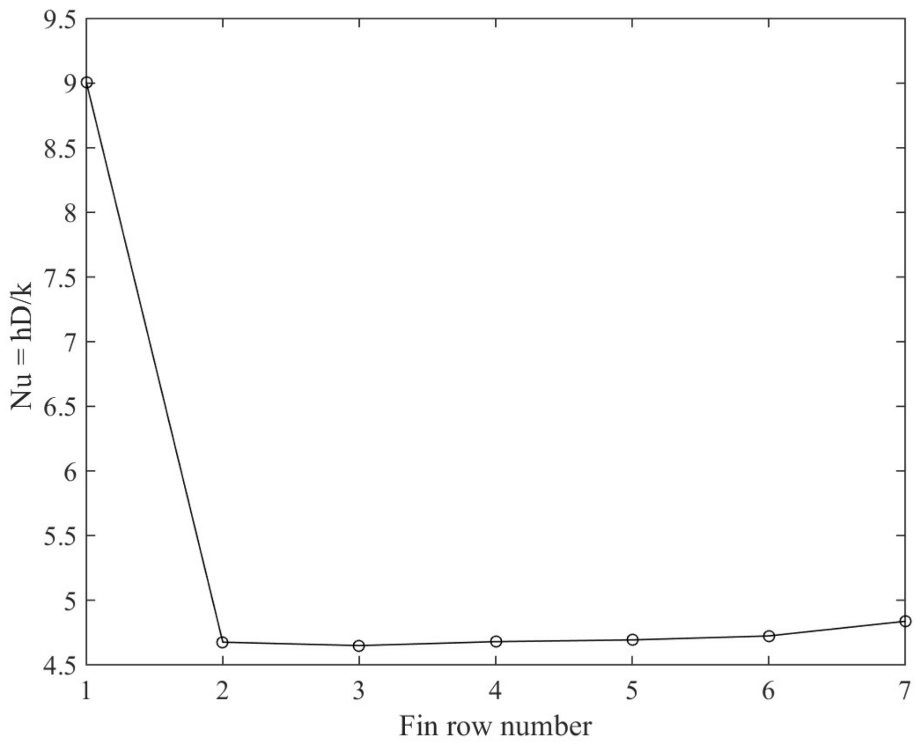
Temperature matrix

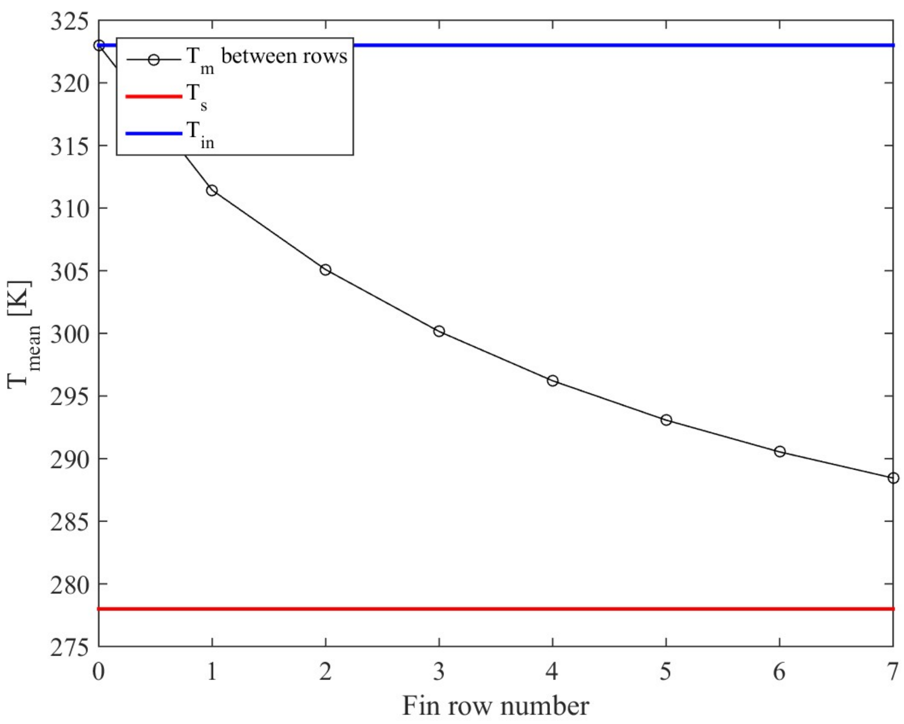
$q = E_{out} - E_{in}$



$$\boxed{q = h_n A_{fin} (T_s - T_n) = c_p \rho U A_{in} (T_{n+1} - T_n)}$$

Note on pin fin row Nusselt numbers and comparison to Hilpert equation for cylindrical fins





$$\mathrm{Nu}_{ave} = \mathrm{Nu} = \frac{h\,D}{k} = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

$C=0.683$ and $m=0.466$

Cylinder correlations (not valid for squares) would indicate that:

$$\mathrm{Nu}_{ave} = 4.65 - 7.2, \text{depending on the reference velocity}$$

→ right order of magnitude, note the role of reference velocity in *Re* definition.

STEP 3: set a warm inflow temperature (e.g. $T_{left} = 350K$) and, for the parameter T_{hot}, assume a rather cold temperature (e.g. 283 K).

STEP 4: play around with the code and try to find how cold must the fins be to reach e.g. 278K target temperature.

STEP 5: reverse-engineer (without CD-equation solver) how cold the fins should be using the equation below making some assumption about h (e.g. constant) and by trying different values of T_s. The task can be solved by Writing 0d code or by using log-mean temperature difference concept.

Hint: you can iterate equation below in a for loop (see HW1) 7 times to get the final temperature after the heat exchanger: $T_{n+1} = T_n + \ldots$
Alternatively, it is possible to solve by just pen and paper by 7 iterations.

$$\boxed{q = h_n A_{fin}(T_s - T_n) = c_p \rho U A_{in}(T_{n+1} - T_n)}$$