ELEC-E8116 Model-based control systems
exercise 7

1. Let the weight of the sensitivity function be given as

$$\frac{1}{W_s} = A\frac{\frac{s}{A\omega_0}+1}{\frac{s}{B\omega_0}+1}, \quad 0 < A << 1, B >> 1$$

Sketch a schema for the magnitude plot of the frequency response and investigate its characteristics. What is the slope in the inceasing part of the curve? What is the magnitude at frequency ω_0?

Generate a second order model, where the slope is twice as large as in the previous case. Investigate again the characteristics. What is the magnitude at frequency ω_0?

2. Consider the angular frequencies $\omega_B, \omega_c, \omega_{BT}$ which are used to define the bandwidth of a controlled system. State the definitions. Prove that when the phase margin is less than 90 degrees ($PM < \pi/2$) it holds $\omega_B < \omega_c < \omega_{BT}$. Interpretations?

3. In solving the discrete-time LQ problem an essential step is to find a "first control step" by minimizing the cont

$$J_{N-1} = \frac{1}{2}x_{N-1}^T Q x_{N-1} + \frac{1}{2}u_{N-1}^T R u_{N-1} + \frac{1}{2}\left(Ax_{N-1} + Bu_{N-1}\right)^T S_N \left(Ax_{N-1} + Bu_{N-1}\right)$$

Do it.

4. The discrete time LQ problem and its solution can be given as

$$x_{k+1} = A_k x_k + B_k u_k, \quad k > i$$
$$J_i = \frac{1}{2}x_N^T S_N x_N + \frac{1}{2}\sum_{k=i}^{N-1}\left(x_k^T Q_k x_k + u_k^T R_k u_k\right), \quad \text{(final state free)}$$
$$S_N \geq 0, \quad Q_k \geq 0, \quad R_k > 0$$

$$S_k = \left(A_k - B_k K_k\right)^T S_{k+1} \left(A_k - B_k K_k\right) + K_k^T R_k K_k + Q_k$$
$$K_k = \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1} A_k, \quad k < N$$
$$u_k^* = -K_k x_k, \quad k < N$$
$$J_i^* = \frac{1}{2} x_i^T S_i x_i$$

Show that the Riccati equation can also be written in the form

$$S_k = A_k^T \left[S_{k+1} - S_{k+1} B_k \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1} \right] A_k + Q_k, \ k < N, \ S_N \text{ given}$$

(The "Joseph-stabilized form" of the Riccati equation)