Aalto-yliopisto
Teknillinen korkeakoulu

EXERCISE 4

ELEC-E8409 HIGH VOLTAGE ENGINEERING

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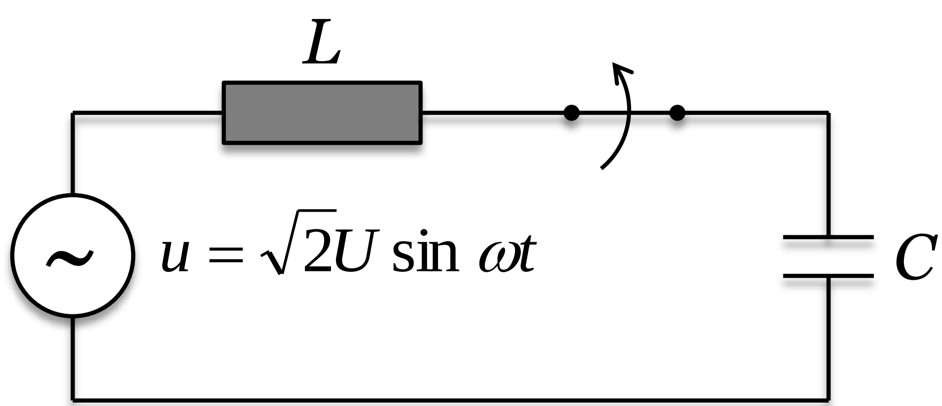
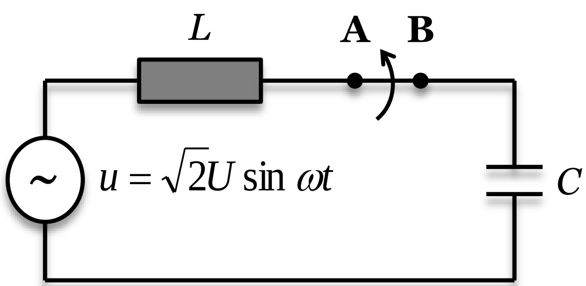
Question 1

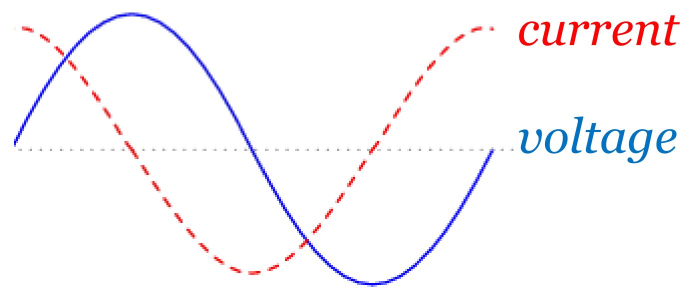
- Disruption of capacitive current occurs when an idle (no-load) cable or a storage capacitor is disconnected. Derive the maximum recovery voltage between the switch terminals.



Some general thoughts from model solutions:

Capacitive load: input voltage is -90° out of phase with load current *(current leading voltage)*.





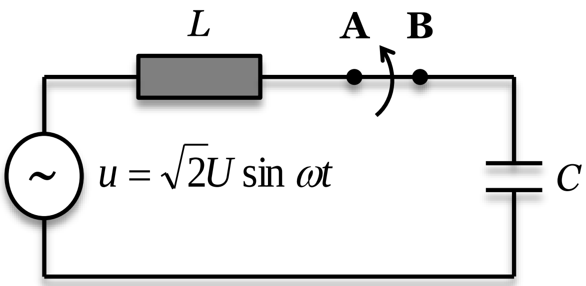
$$\underline{U} = \underline{ZI}$$

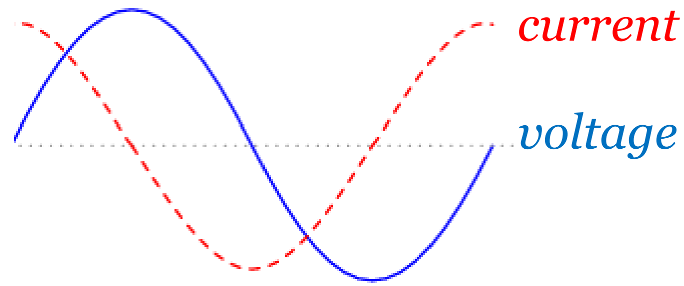
$$\sqrt{2}U \sin\left(\omega t + \frac{\pi}{2}\right) = \left(\frac{1}{\omega C} - \omega L\right) i_c$$

$$\omega t - \varphi = \omega t - \left(-\frac{\pi}{2}\right)$$

$$\Rightarrow \quad i_c = \frac{\sqrt{2}U}{\frac{1}{\omega C} - \omega L} \sin\left(\omega t + \frac{\pi}{2}\right)$$

A more power system engineer way:



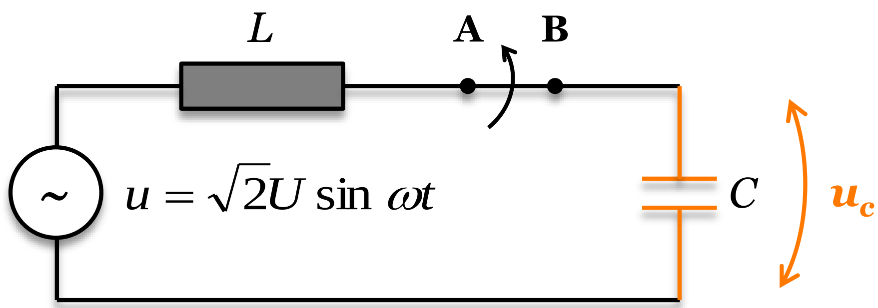


$$\overline{U} = \overline{Z}\,\overline{I}$$

$$\overline{U} = \left(j\omega L + \frac{1}{j\omega C} \right)\overline{I} = \left(j\omega L - j\frac{1}{\omega C} \right)\overline{I} = j\left(\omega L - \frac{1}{\omega C} \right)\overline{I}$$

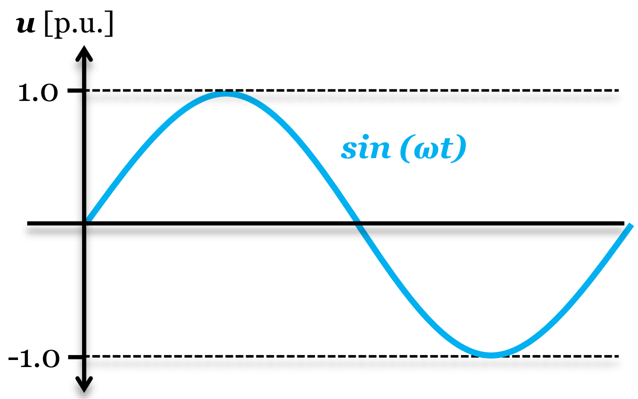
$$j\overline{U} = j \times j \times \left(\omega L - \frac{1}{\omega C} \right)\overline{I} = \left(\frac{1}{\omega C} - \omega L \right)\overline{I}$$

$$\rightarrow \overline{I} = \frac{j\overline{U}}{\left(\frac{1}{\omega C} - \omega L \right)} \xrightarrow{\text{time domain}} i_c = \frac{\sqrt{2}U}{\left(\frac{1}{\omega C} - \omega L \right)} \sin\left(\omega t + \underbrace{\frac{\pi}{2}}_{\times j = +90^\circ} \right)$$



Voltage over capacitance *C*

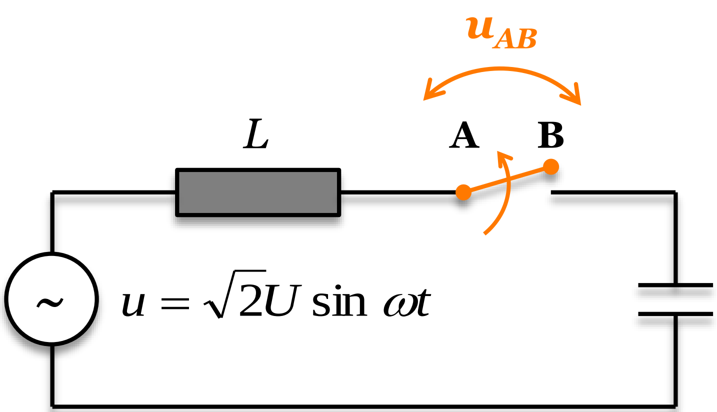
$$u_c = \frac{\frac{1}{\omega C}}{\frac{1}{\omega C} - \omega L}\sqrt{2}U \sin \omega t = \frac{\sqrt{2}U}{1-\omega^2 LC}\sin \omega t$$



Maximum when sin ωt = 1

$$u_{c(\max)} = \frac{\sqrt{2}U}{1-\omega^2 LC}$$

Recovery voltage



Voltage between terminals $u_{AB} = u_{c(\max)} - u$

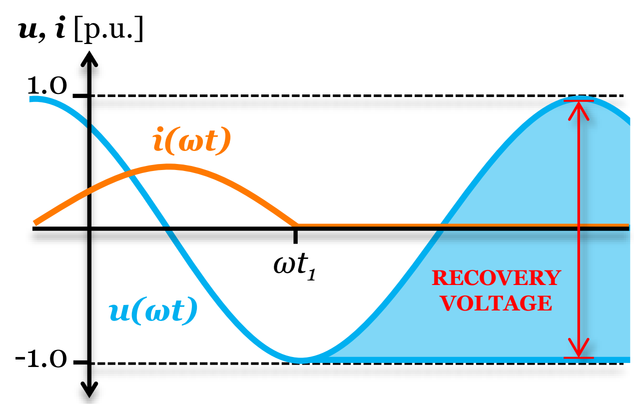
$$u_{AB} = \frac{\sqrt{2}U}{1-\omega^2 LC} - \sqrt{2}U \sin \omega t$$

Maximum when sin ωt = -1

$$u_{AB(\max)} = \frac{\sqrt{2}U}{1-\omega^2 LC} + \sqrt{2}U = \frac{2-\omega^2 LC}{1-\omega^2 LC}\sqrt{2}U$$

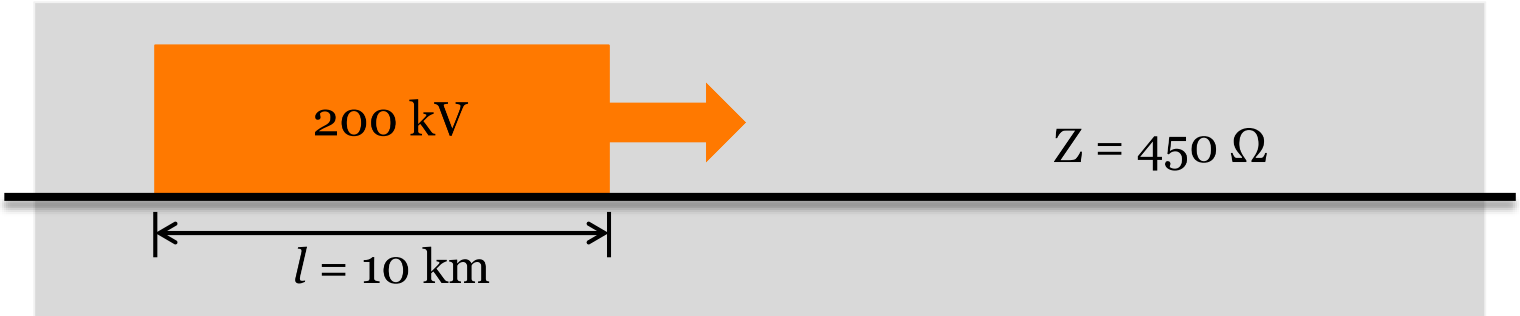
In practice, $\frac{1}{\omega C} >> \omega L$ → $1 >> \omega^2 LC$

$$u_{AB(\max)} = \frac{2-\cancel{\omega^2 LC}}{1-\cancel{\omega^2 LC}}\sqrt{2}U \approx 2\sqrt{2}U$$



Question 2

- An overhead line has characteristic impedance of 450 Ω. A 200 kV rectangular impulse 10 km in length propagates along the line. What is the energy of the impulse? How much of the energy is in the magnetic field and how much is in the electric field?



Impulse duration $t = l/v$ (where $v = c = 3 \cdot 10^8$ m/s)

$$\text{Impulse energy } W = \int_0^t P\,dt$$

$$= P \cdot t = \frac{u^2}{Z} \cdot t = \frac{u^2}{Z} \cdot \frac{l}{v} = \frac{\left(200 \cdot 10^3\right)^2}{450} \cdot \frac{10 \cdot 10^3}{3 \cdot 10^8} = 2962.96 \text{ Ws} \approx 3.0 \text{ kJ}$$

Energy W = 3 kJ

How much of the energy is in the magnetic field and how much is in the electric field?

γ = *capacitance per unit length*
λ = *inductance per unit length*
L = *inductance for length of cable* (λl)

$$Z_w = \sqrt{\frac{l}{c}}$$

Inductance /capacitance

$$v = \frac{1}{\sqrt{LC}}$$

$$W = P \cdot t = i^2 \cdot Z \cdot t = i^2 \cdot \left(\sqrt{\frac{\lambda}{\gamma}} \cdot l \right) \cdot \sqrt{\lambda \gamma} = i^2 \cdot \lambda l = i^2 L$$

Inductive Energy: $W_L = \frac{1}{2} i^2 L = \frac{1}{2} W \approx 1.5\,\mathrm{kJ}$

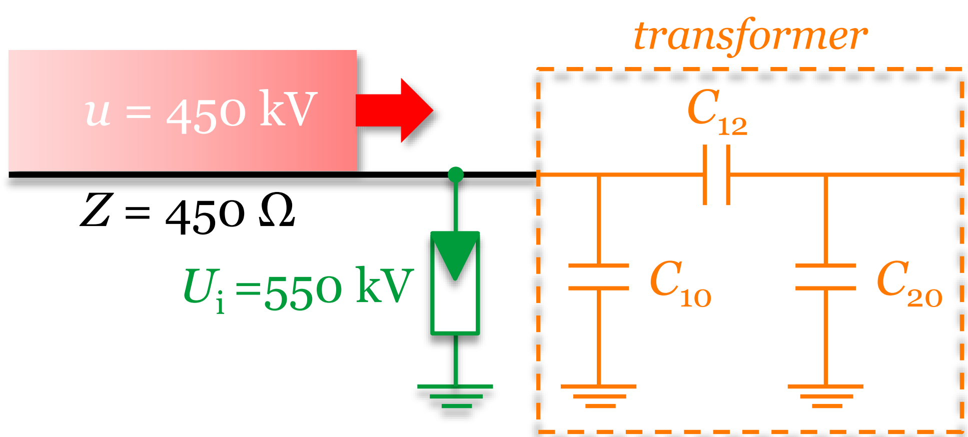
Capacitive Energy: $W_C = W - W_L = \frac{1}{2} W = W_L \approx 1.5\,\mathrm{kJ}$

Energy is distributed half into the electric field and half in the magnetic field

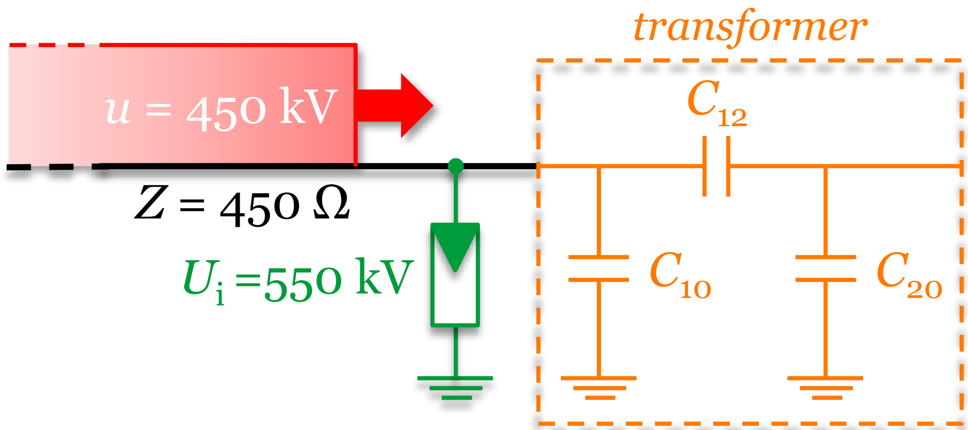
$$dW_{ind} = P\ dt = i \cdot u\, dt = i \cdot \underbrace{L \frac{di}{dt}}_{u} dt = Li\, di \xrightarrow{\text{int}} W = \frac{1}{2} Li^2$$

Question 3

- A step wave with amplitude 450 kV propagates along an overhead line to a 110/20 kV transformer. The characteristic impedance of the line is 450 Ω. The primary winding is protected by a nonlinear resistor type arrester with inception voltage U_i = 550 kV. How long does it take for the arrester to activate? How large capacitors have to be connected to the secondary winding and ground so that the capacitive overvoltage over the transformer does not exceed 75 kV (secondary test voltage)? The transformer can be described as a capacitive equivalent circuit used for impulse voltage testing.



C_{10} =3 nF
C_{12} =5 nF
C_{20} =10 nF



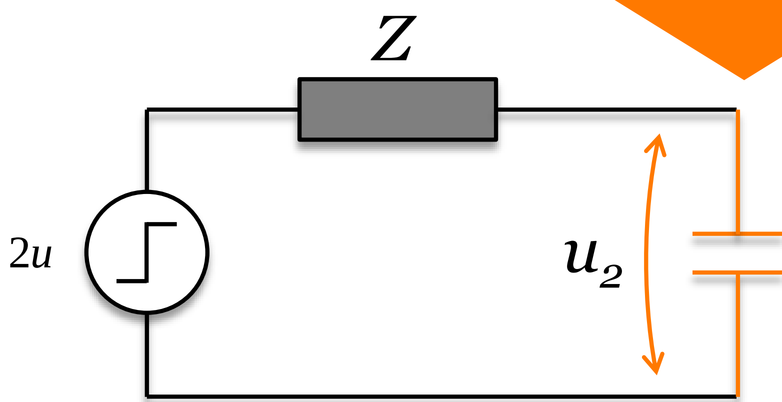
$C_{10} = 3$ nF

$C_{12} = 5$ nF

$C_{20} = 10$ nF

$$series: \frac{1}{C_{12,20}} = \frac{1}{C_{12}} + \frac{1}{C_{20}}$$

$$parallel: C = C_{10} + C_{12,20}$$



$$C = C_{10} + \frac{C_{12} \cdot C_{20}}{C_{12} + C_{20}} = 6.33\,\text{nF}$$

$$u_2 = \frac{\frac{1}{sC}}{\frac{1}{sC} + Z} \cdot \frac{2u}{s} = \frac{2u}{s(ZCs+1)} = \frac{2u}{sZC\left(s + \frac{1}{ZC}\right)} = 2u\,\frac{\frac{1}{ZC}}{s\left(s + \frac{1}{ZC}\right)}$$

$$u_2 = \frac{\frac{1}{sC}}{\frac{1}{sC}+Z}\cdot\frac{2u}{s} = \ldots = 2u\frac{\frac{1}{ZC}}{s\left(s+\frac{1}{ZC}\right)}$$

$$\frac{a}{s(s+a)} \Rightarrow 1-e^{-at}$$

Laplace transform: $L^{-1}\left\{2u\frac{\frac{1}{ZC}}{s\left(s+\frac{1}{ZC}\right)}\right\} = 2u\left(1-e^{-t/ZC}\right)$

$$u_2 = 2u\left(1-e^{-t/\tau}\right) \quad \text{where} \quad \tau = ZC$$

Time to activate: $u_2 = 2u\left(1-e^{-t/\tau}\right) \Rightarrow e^{-t/\tau} = 1-\frac{u_2}{2u} \Rightarrow -\frac{t}{\tau} = \ln\left(1-\frac{u_2}{2u}\right)$

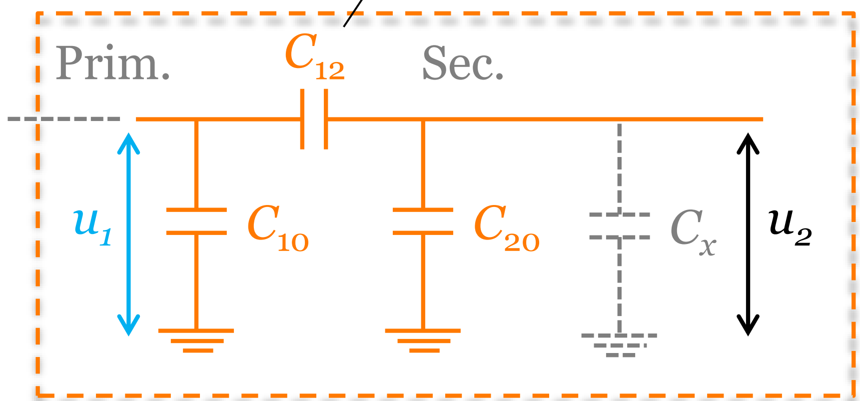
$$\begin{bmatrix} u_i = u_2 = 550\ \text{kV} \\ u = 450\ \text{kV} \end{bmatrix}$$

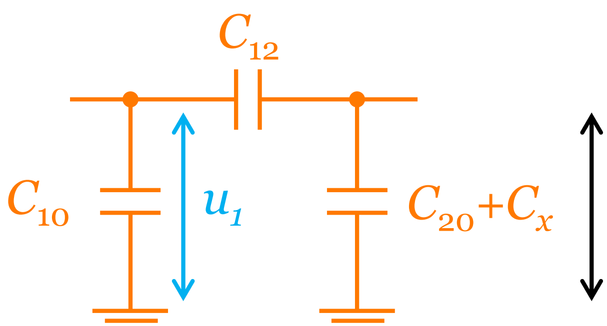
$$t = -\tau\ln\left(1-\frac{u_2}{2u}\right) = 2.69\cdot 10^{-6} \approx 2.7\ \mu\text{s}$$

How large capacitors have to be connected to the secondary winding so that the capacitive overvoltage over the transformer does not exceed 75 kV?

Cap. coupl. between 1.&2.

Here u1 is the same as u2 in previous calculations...





$$u_2 = \frac{\dfrac{1}{s(C_{20}+C_x)}}{\dfrac{1}{sC_{12}} + \dfrac{1}{s(C_{20}+C_x)}} \cdot u_1$$

$$\frac{u_2}{u_1} = \frac{\dfrac{1}{s\left(C_{20}+C_x\right)}}{\dfrac{1}{sC_{12}}+\dfrac{1}{s\left(C_{20}+C_x\right)}} \quad \left(\cdot\frac{s\left(C_{20}+C_x\right)}{s\left(C_{20}+C_x\right)}\right) = \frac{1}{\dfrac{s\left(C_{20}+C_x\right)}{sC_{12}}+1} = \frac{1}{\dfrac{C_{20}+C_x}{C_{12}}+1}$$

$$= \frac{1}{\dfrac{C_{20}+C_x}{C_{12}}+\dfrac{C_{12}}{C_{12}}} = \frac{1}{\dfrac{C_{20}+C_x+C_{12}}{C_{12}}} = \frac{C_{12}}{C_{20}+C_x+C_{12}}$$

Solve for C_x ... $$C_x = \frac{u_1}{u_2}C_{12} - \left(C_{12}+C_{20}\right)$$

$$\begin{bmatrix} u_1 = 550\,\text{kV} \\ u_2 = 75\,\text{kV} \end{bmatrix}$$

$$\boxed{C_x = 21.66\cdot 10^{-9}\ \text{F} \approx 22\ \text{nF}}$$