Aalto University
School of Engineering

GEO – E1050
Finite Element Method in Geoengineering

Lecture 11-12. Other numerical methods

To learn today & next time…

The lectures should give you overview of other numerical methods

1. Discrete element method (DEM, also distinct element method)
   - assumptions
   - solutions
   - problmes & accuracy

2. Smoothed particle hydrodynamics (SPH)
3. Material Point Method (MPM)
4. Particle Finite Element Method in Geoengineering (PFEM)
5. XFEM – eXtended Finite Element Method in Geoengineering (XFEM)
6. ALE , CLE – Coupled Lagrangian – Eurlerian FEM
7. Meshfree methods

Methods on continuous – discontinuous scale

FEM

CLE, ALE
Coupled Lagrangian – Eurlerian FEM

MPM

Smoothed Particle Hydrodynamics (SPH)

Discrete element method (DEM)

Continuous

Classical Meshfree methods

PFEM

XFEM

Discontinuous

Based on continuum mechanics

Not based on continuum mechanics

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XFEM

with thanks to K. Agathos (Aristotle U. of Thessaloniki) and E. Chatzi, (IBK, D-BAUG, ETH Zurich)

XFEM – eXtended Finite Element Method

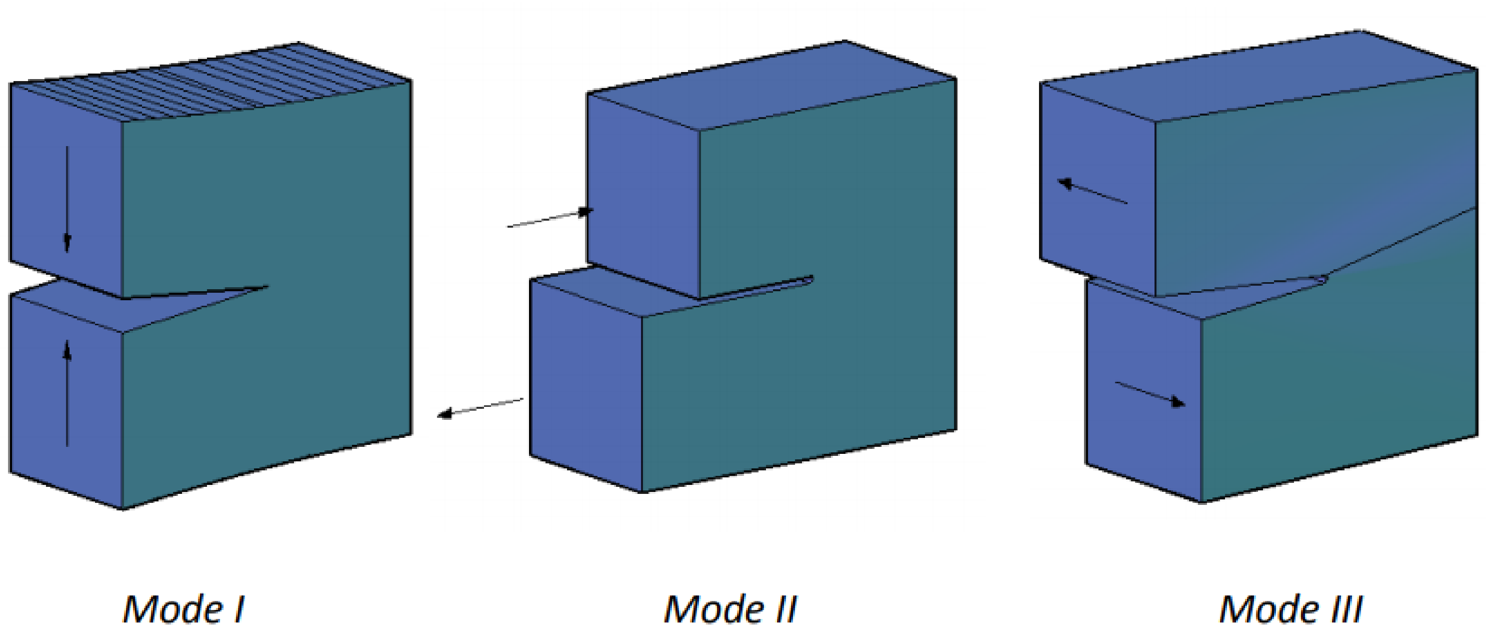
Aim: to introduce discontinuities into continuous FEM

- Strong discontinuity: crack - jump in displacements
- Weak discontinuity – jump in strains

Used to determine displacement, strain and stress fields in structures with cracks and small holes. Allows for discontinuous displacements and strain fields

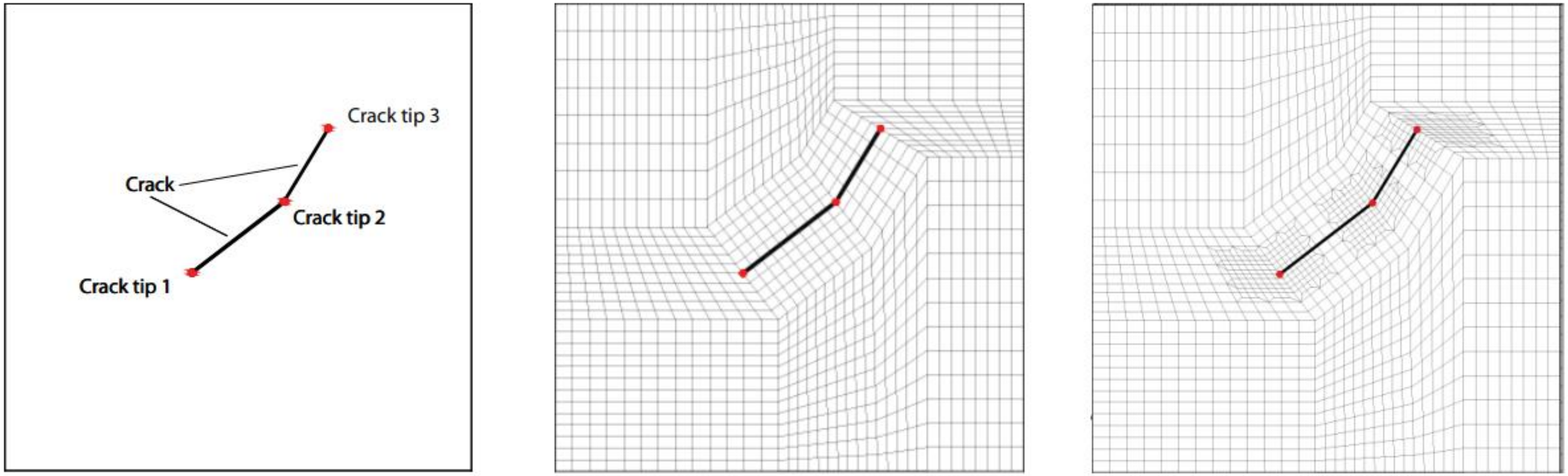
XFEM – eXtended Finite Element Method

Aim: to introduce discontinuities into continuous FEM



XFEM – eXtended Finite Element Method

To model the crack, we need nodes placed across the crack and on the crack tips



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XFEM – Jump enrichment

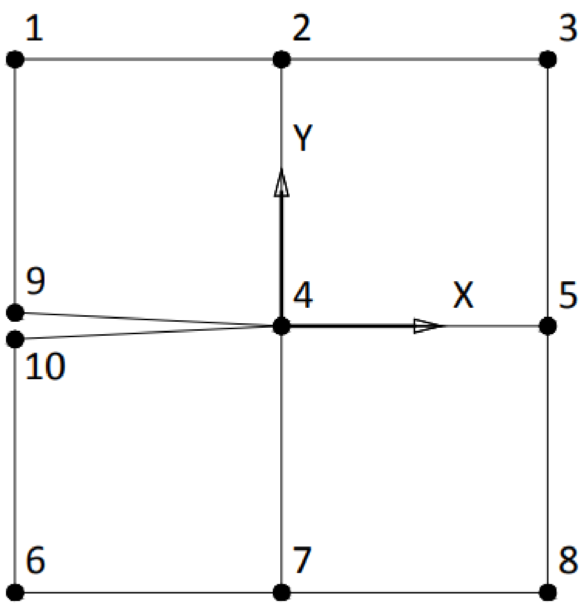
When we have a crack, we have jump in displacements. However, we want to describe it with a continuous mesh, i.e. without physically modelling crack width.

For that, we enrich the element nodes with jump function for displacements. At one side of the node, it has a different value than at the other side of the node.

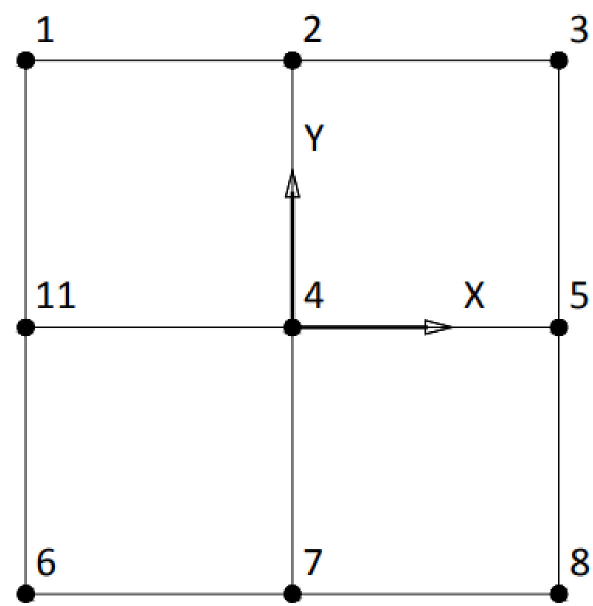
Technically we use Heaviside function $H(x)$ for that…

XFEM – Jump enrichment

In other words, we want to represent the situation in Mesh 1 (physical crack), with Mesh 2



Mesh 1



Mesh 2

XFEM – Jump enrichment

The displacements at any point (and in particular in nodes 9 and 10) are:

$$\underbrace{\Delta\mathbf{u}}_{\text{displacements}} = \sum_{i=1}^{10} \underbrace{\mathbf{N}}_{\text{Shape functions values}} \underbrace{\Delta\mathbf{d}}_{\text{Vector containing increments of displacements of element nodes}}$$

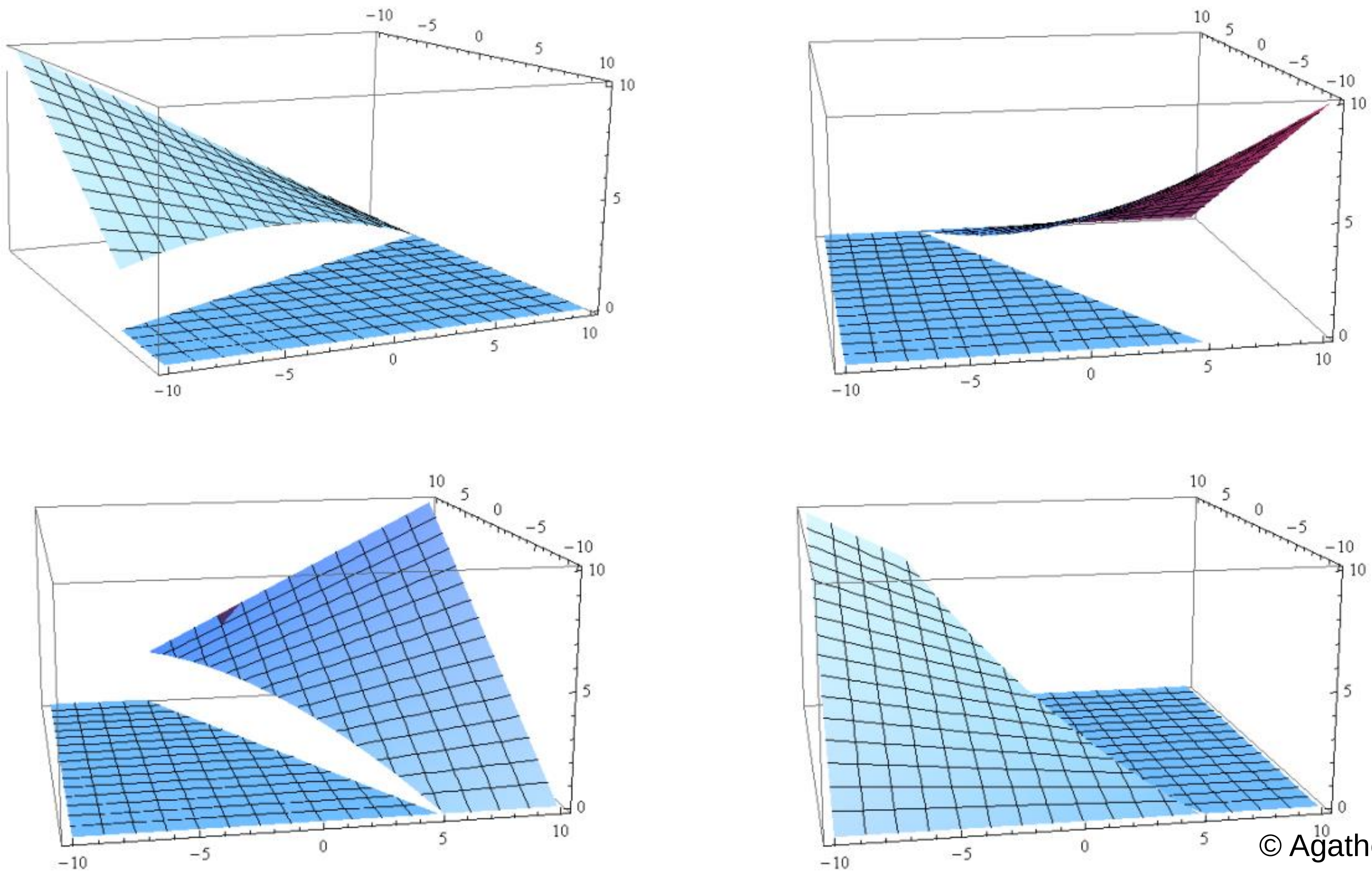
Defining **a**=0,5 (**d**$_9$+**d**$_{10}$) and **b**=0,5 (**d**$_9$-**d**$_{10}$) we get

$$\underbrace{\Delta\mathbf{u}}_{\text{displacements}} = \sum_{i=1}^{8} \underbrace{\mathbf{N}}_{\text{Shape functions values}} \underbrace{\Delta\mathbf{d}}_{\text{Vector containing increments of displacements of element nodes}} + \mathbf{a}(N_9 + N_{10}) + \mathbf{b}(N_9 + N_{10})H(x) =$$

$$= \sum_{i=1}^{8} \underbrace{\mathbf{N}}_{\text{Shape functions values}} \underbrace{\Delta\mathbf{d}}_{\text{Vector containing increments of displacements of element nodes}} + u_{11}(N_{11}) + \mathbf{b}(N_{11})H(x)$$

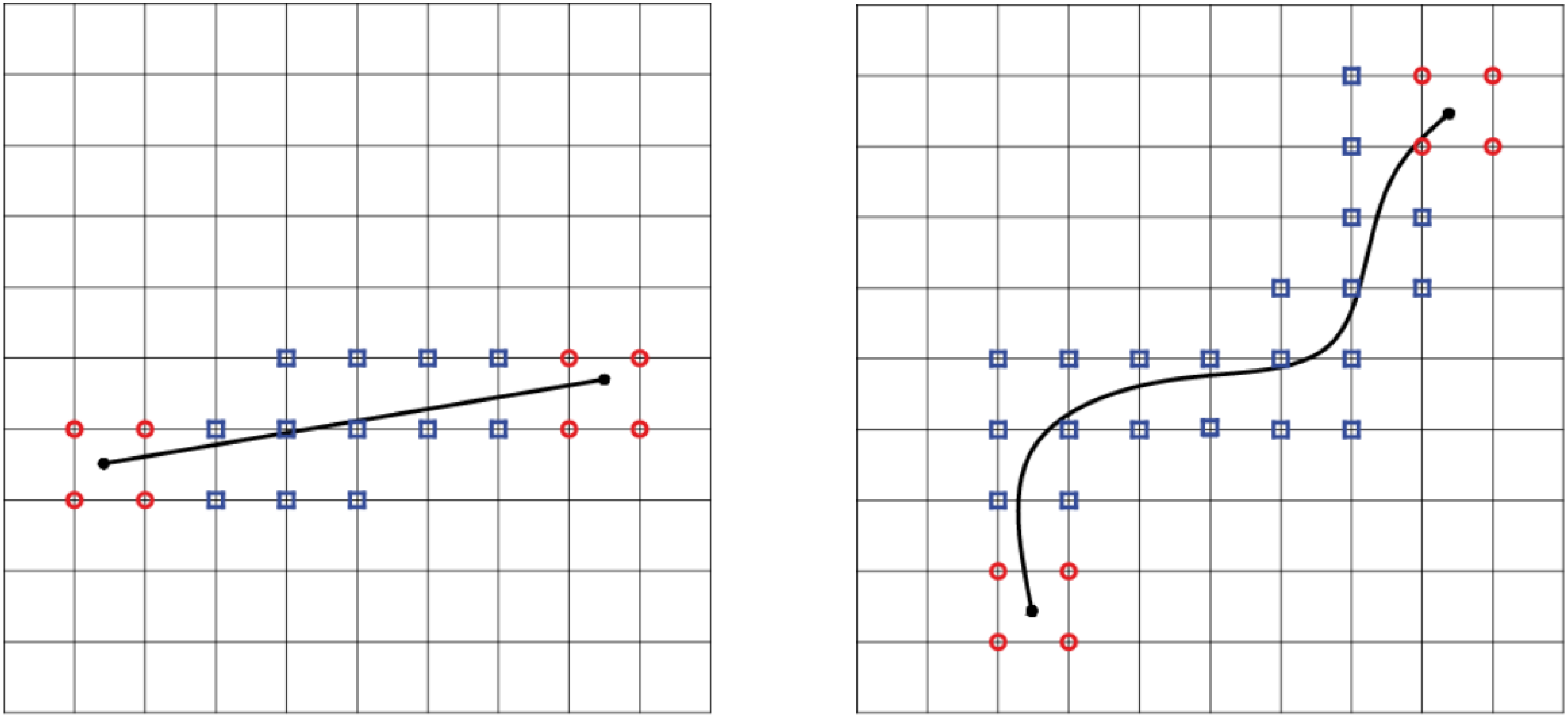
$$H(x)=1 \quad \text{for } y>0 \quad \text{and} \quad -1 \quad \text{for } y<0$$

XFEM – Jump enrichment



Jump enrichment in action ☺

XFEM – Jump enrichment



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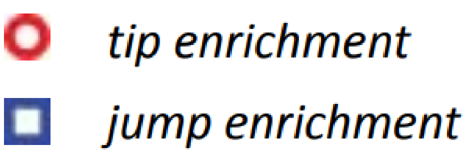
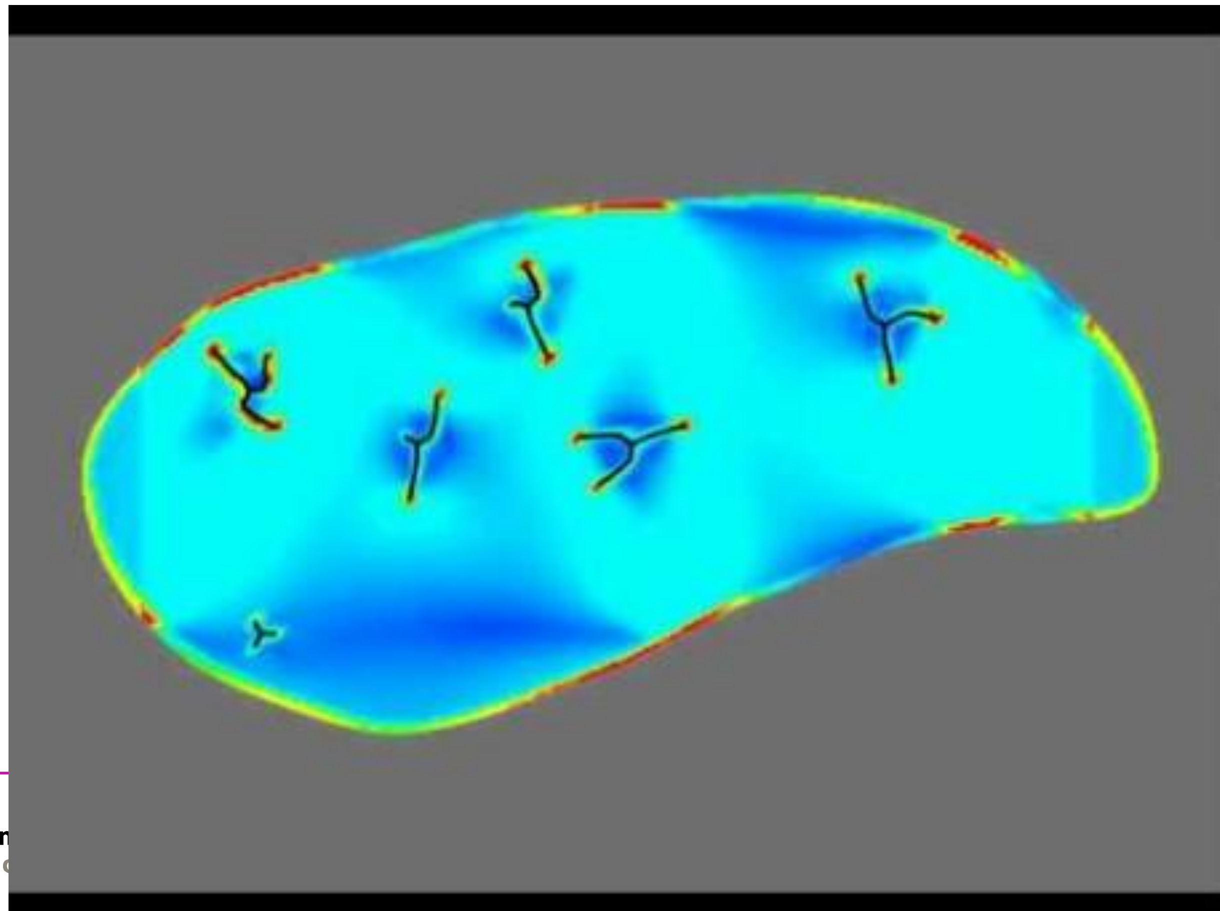


XFEM – abilities

The method – with extensions – can deal with crack propagation, crack branching and intersecting etc.
Also can be used with plasticity and in dynamic problems



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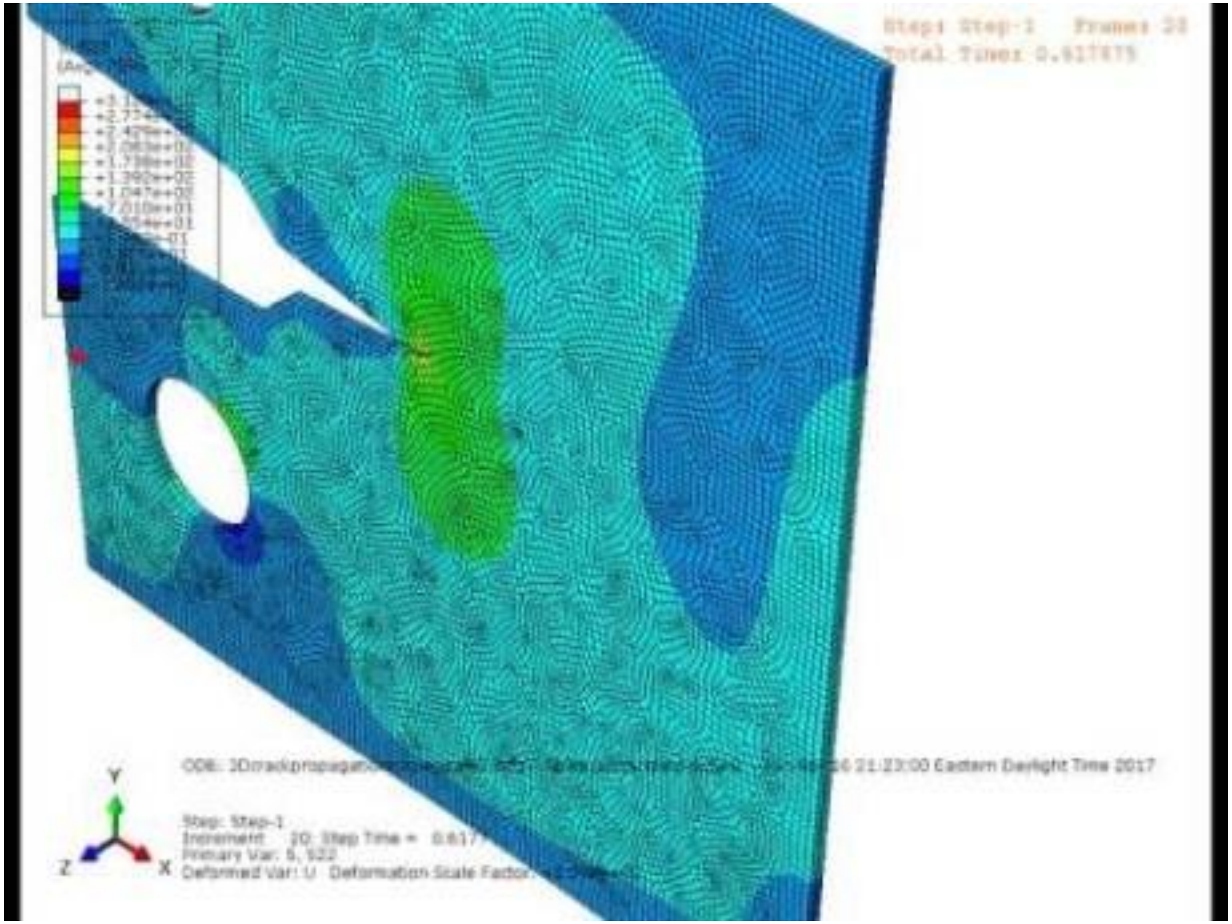
XFEM – abilities



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Thank you