ELEC-E8116 Model-based control systems
/exercises 8

1. Consider a simple integrator:

$$\dot{x}(t) = u(t)$$

Find an optimal control law that minimises a cost-function

$$J = \int_0^1 \left(x^2(t) + u^2(t) \right) dt$$

Further, consider the case, when the optimization horizon is infinite.

2. Consider the 1. order process $G(s) = \dfrac{1}{s-a}$, which has a realization

$$\begin{aligned} \dot{x}(t) &= ax(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

so that the state is the measured variable. It is desired to find the control, which minimizes the criterion

$$J = \frac{1}{2} \int_0^\infty \left(x^2 + Ru^2 \right) dt \quad (R > 0)$$

Calculate the control and investigate the properties of the resulting closed-loop system.

3. Consider a SISO-system. The maximum values of the sensitivity and complementary functions are denoted M_S and M_T , respectively. Let the gain and phase margins of a closed-loop system be *GM* (gain margin) and *PM* (phase margin). Prove that

$$GM \geq \frac{M_S}{M_S - 1} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} \text{ [rad]}$$

$$GM \geq 1 + \frac{1}{M_T} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T} \text{ [rad]}$$