

Aalto university

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Exercise sheet 5

Complex Analysis, MS-C1300.

Hand in exercise 1 and 2 for grading. Deadline Monday 9.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as one pdf-file with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Tuesday 10.11 or Wednesday 11.11.

- (1) Let $f = u + iv$ be analytic in a domain D . Assume that

$$v(z) = u(z)^2$$

for all $z \in D$. Prove that f is a constant function. (*Hint:* Differentiate $u(z)^2 - v(z)$ with respect to x and y .) (6p)

- (2) Let $f = u + iv$ where $u(z) = x^3 + axy^2$ and $v(z) = bx^2y + cy^3 + 1$. Determine the values of the real numbers a , b , and c that make f an entire function. Then express f as a polynomial function of z . (*Hint:* Once you have determined a , b , and c start thinking about how $z^n = (x + iy)^n$ looks. Here the binomial theorem is useful.) (6p)

- (3) For each of the following functions determine the largest open in which it is analytic, and compute its derivative in that set:

(a)

$$f(z) = \frac{e^z - 1}{e^z + 1}$$

(b)

$$g(z) = \frac{\sin(\sqrt{iz + 1})}{z^2 + 1}$$

- (4) If a function f is analytic in a domain D and satisfies the differential equation

$$f'(z) - \alpha f(z) = 0$$

in D for a constant $\alpha \in \mathbb{C}$ show that f takes the form $f(z) = Ae^{\alpha z}$ for some constant $A \in \mathbb{C}$. (*Hint:* Study the function $g(z) = e^{-\alpha z} f(z)$.)